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1798

# THE

# ELEMENTS

# OF

# NAVIGATION.

VOLUME THE FIRST.

# THE ELEMENTS OF NAVIGATION;

CONTAINING THE

THEORY and PRACTICE.

With the neceſſary TABLES,

And COMPENDIUMS for finding

The LATITUDE and LONGITUDE at SEA.

To which is added,

## A TREATISE OF MARINE FORTIFICATION.

Compoſed for the Uſe of

The ROYAL MATHEMATICAL School at CHRIST'S HOSPITAL,

The ROYAL ACADEMY at PORTSMOUTH,

And the GENTLEMEN of the NAVY.

IN TWO VOLUMES.

By J. ROBERTSON,

Late Librarian to the Royal Society, and formerly Head-Maſter of the Royal Academy, at Portſmouth.

*The FIFTH EDITION, with ADDITIONS.*

Carefully reviſed and corrected by

WILLIAM WALES,

Maſter of the Royal Mathematical School, Chriſt's Hoſpital, London.

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LONDON,

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TO

JOHN EARL OF

# SANDWICH,

&c. &c. &c.

FIRST COMMISSIONER

OF THE

Boards of Admiralty and Longitude.

MY LORD,

WHILE the public voice is unanimous in applauding your humanity towards the Artificers, in general, of His Majeſty's Dock-yards, and your attention to reſtore the Royal Navy of Britain to the reſpectable ſtate from which it had been ſuffered to decline ſince the laſt War, Philoſophers not only admire theſe noble acts, but likewiſe, your generous encouragement to improve Geographical, Nautical, and Natural Knowledge.

108341
ASTRONOMY

# DEDICATION.

Such exertions of your Lordſhip's extraordinary Mental and Official Abilities, will undoubtedly be tranſmitted with honour to the lateſt poſterity: And your laudable example muſt inſpire a regard for Works intended to promote public utility.

The Author of *The Elements of Navigation*, notwithſtanding the favourable reception which the former impreſſions have met with from Britiſh Mariners, thinks himſelf extremely happy that this improved Edition is permitted to appear under your Lordſhip's Patronage.

That you may long enjoy the Opportunity as well as Inclination of promoting uſeful Arts and Learning, is a hope ſincerely entertained by,

My Lord,

Your Lordship's

moſt obedient

and humble Servant,

Nov. 1. 1772.

John Robertſon.

TO THE

RIGHT WORSHIPFUL

Sir ROBERT LADBROKE, Knt. Alderman,

PRESIDENT;

THE

Worſhipful THOMAS BURFOOT, Eſq.

TREASURER;

And the reſt of the

WORSHIPFUL GOVERNORS

OF

# Chriſt's Hoſpital, London:

This Book, containing the Elements of Navigation, and a Treatiſe on Marine Fortification, firſt publiſhed for the Uſe of the Children in the Royal Mathematical School, when they were under my Care, is, as a grateful acknowledgement for paſt favours, addreſſed by

Your WORSHIPS' moſt humble Servant,

Nov. 1. 1772. John Robertſon.

# ADVERTISEMENT.

IN this Edition, the Editor has carefully corrected the errors which had crept into the former; he has recomputed the Tables in Book V. Art. 308, 309, and 310, of the Sun's Longitude, Right Aſcenſion, and Declination, and has alſo reviſed, as far as his materials extended, the Geographical Table, and added the names of ſuch places as his own obſervations, or thoſe of other perſons, have furniſhed him with; ſo that he flatters himſelf it is the moſt extenſive and correct of any extant. On the whole, he preſumes, this Edition will be found as worthy of the approbation of the public in general, and of ſeamen in particular, as thoſe which were printed under the Author's inſpection.

THE

# P R E F A C E.

*IT having been part of my employment for many years paſt, to inſtruct youth in the theoretical and practical parts of Navigation; I was naturally led to draw up rules and examples fitted to the years and capacity of the ſcholar: ſome of the precepts, from time to time were altered, according as I had obſerved how they were comprehended by the majority of my pupils; until at length I had put together a ſet of materials, which I found ſufficient for teaching this Art.*

*Upon my being intruſted by the governors of Chriſt's Hoſpital (in the beginning of the year* 1748*) with the care of the Royal Mathematical ſchool there, founded by King Charles the ſecond, I had a great opportunity of experiencing the method I had before uſed; and finding it fully anſwered my expectation, I determined to print it for the uſe of that ſchool: but as thoſe children are to be inſtructed in the mathematical ſciences, on which the art of Navigation is founded, I judged it proper, on their account, to introduce the ſubjects of Arithmetic, Geometry, Trigonometry, &c. for which reaſon, this treatiſe is diſtinguiſhed by the title of* Elements of Navigation.

*After my appointment (in the year* 1755*) to be head maſter of the Royal Marine Academy at Portſmouth, founded by King George the ſecond, I alſo found that this book was ſufficiently intelligible to beginners of middling capacities; and therefore, in the ſecond edition, in the year* 1764*, the manner in which it was firſt compoſed was continued, except the removing of the book of Aſtronomy, from being the 8th. into the place of the 5th.; whereby the books of Plane Sailing, Globular Sailing, and Days Works, which together nearly comprehend the art of Navigation, follow in ſucceſſion. There were indeed ſome variations in the modes of expreſſion in a few places; but the additions in every book were made rather to extend the notions of learners, than to ſupply any deficiency wanting in the former edition; except ſome of the additions in the 9th. book, which were not ſo well known at the time of the firſt impreſſion.*

*The work is divided into ten parts or books, each being a diſtinct treatiſe; the preceding ones contain the neceſſary elements which are wanted in thoſe that follow. The demonſtration of the ſeveral propoſitions are given as conciſely as I could contrive, to carry with them a ſufficient degree of evidence: Throughout the whole of the elementary parts, brevity and perſpicuity were conſidered; but the practical parts are more fully treated on, and intended to include every uſeful particular, worthy of the mariner's notice.*

*In the elementary parts, where it is not eaſy to introduce new matter, there will be found the common principles treated in a manner, which, it is apprehended, is better adapted to beginners; and ſuch new lights thrown on ſeveral particulars, as will render them more obvious than in the view wherein they have been commonly ſeen.*

*The treatiſe of Fortification annexed, is the reſult of many years application; and is delivered in a very different mode from what other writers have taken; for among the multitude which I have ſeen, they generally begin with the fortifying of a town, the moſt difficult part of the art, and end with works the moſt eaſy to contrive and execute: Herein the works of the ſimpleſt conſtruction are begun with, and the learner gradually advanced to the fortifying of a town: Indeed the limits choſen for this tract have cauſed ſome articles to be briefly mentioned, and others to be totally omitted; nevertheleſs, it is conceived that, in its preſent ſtate, it may be of conſiderable uſe to Marine Officers, and even furniſh ſome hints not altogether unworthy the notice of the Gentlemen of the Army.*

*The Maritime parts of theſe Elements, contained in the* viith, viiith, *and* ixth *books, are alſo delivered in a manner ſomewhat different from what is ſeen in other writers; who, for the moſt part copying from one another, have not much contributed towards perfecting the art of Navigation; the writers indeed have been many, but the improvers have been very few;* Wright, Norwood, *and* Halley, *having done the moſt of what has been diſcovered ſince a little before the beginning of the* 17*th century: However, in the method here taken, it is apprehended that the proper judges will find ſome few improvements, as well in the art itſelf as in the manner of communicating it to learners.*

*The common treatiſes of Navigation, which, on account of their ſmall bulk and eaſy price, are vended among the Britiſh Mariners, ſeem not to be written with an intention to excite in their readers a* *deſire to purſue the Sciences, farther than they are* handled *in thoſe books; ſo that it is no wonder our ſeamen in general had ſo little mathematical knowledge; for the perſon who could keep a* trite *journal, formed on the moſt eaſy occurrences, has been reckoned a good artiſt; but whenever thoſe occurrences have not happened, the journaliſt has been at a loſs, and unable to find the ſhip's place with any tolerable degree of preciſion; and ſuch accidents have probably contributed to the diſtreſs which many ſhips crews have experienced, and which a little more knowledge among them might have prevented, or at leaſt have leſſened.*

*About the middle of the* 16*th century, Navigation began to be conſidered as an art, in a great meaſure dependent on the Mathematical ſciences; and on ſuch a plan has it been cultivated by the labours of the moſt judicious, who have applied themſelves towards its perfection; and although the art has been enriched by the obſervations of ſome learned men in different nations, yet it has ſo happened, that the chief of the improvements, and particularly the mathematical ones*, were firſt publiſhed in Britain.*

*Into this work are collected moſt, if not all, of the uſeful and curious particulars relating to the art of Navigation; there are alſo interſperſed hiſtorical remarks of inventions, with the names of many eminent men, and their works; theſe were intended as incentives to inſpire learners and our ſeamen with a deſire not only of knowing the things herein treated of from their foundations, but of puſhing their inquiries into ſuch other parts of the ſciences as may procure to themſelves pleaſure, profit, and reſpect, and render them more uſeful to their country by the ſkill reſulting from ſuch acquiſitions.*

*I have always thought that the chief motives which ought to induce a perſon to appear as a writer ſhould be, either that he has ſomething new to publiſh, or that he has arranged the parts of a known ſubject, in a method more regular and uſeful than had been done before; in either of theſe caſes he cannot be a proper judge, unleſs he has ſeen the pieces extant on that ſubject, or at leaſt, thoſe of the moſt eminent authors already publiſhed: On theſe principles I was led to examine what had been done by the different writers on Navigation; and having peruſed moſt of their books, of which I*

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* See the following diſſertation.

*could get information, I had an opportunity of diſcovering the ſteps by which this art has riſen to its preſent perfection, and conſequently of knowing the moſt material parts of the hiſtory of its progreſs: Among other things, I could not avoid remarking a miſtake which has crept into many of the modern books of Navigation; which is, that* Wright's *invention of making a true ſea-chart was ſtolen by* Mercator, *and publiſhed as his own. I ſuſpect this ſtory had its riſe in a book printed in the year* 1675 *by* Edward Sherburne, *intitled "The Sphere of Marcus Manilius made an* Engliſh Poem, with Annotations and an Aſtronomical Appendix."

*My enquiries into theſe matters induced the late learned Dr.* James Wilſon *to review and complete his obſervations on the ſame ſubject, and produced his Diſſertation on the Hiſtory of the Art of Navigation; which he was pleaſed to give me leave to publiſh with the ſecond edition of this work.*

*There are few perſons, however knowing and careful, who may not commit, and overlook, inadvertencies in their own compoſitions, which may be diſcovered by others: therefore at my requeſt the greateſt part of the manuſcript for the firſt edition was read and examined by two of my friends* *, *well acquainted with the theory and practice of Navigation; who, by their judicious obſervations, enabled me to improve ſeveral articles: Some part of the additions to the* 2d *Edition, received much elegance and perſpicuity through the friendly advice and communications of the late learned Dr.* Henry Pemberton, F. R. S.

*The ſecond Edition of theſe Elements having alſo been well received by the Public; Dr.* Wilſon *took the pains to reviſe his Diſſertation, which he improved in many particulars: And I have alſo endeavoured to retain their favourable opinions of my labours, by giving Compendiums for performing the operations of the new methods of finding the latitude and longitude of a ſhip at ſea; and ſome other alterations and additions which I conceived would render this third Edition more generally uſeful.*

Nov. 1, 1772.

---

* William Mountaine, Eſq. F. R. S. and Mr. William Payne.

# THE

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THIS treatiſe in two volumes contains ten books; each is divided into ſeveral ſections, and numbered with the Roman numerals.

The particular articles are numbered by the common figures, each book beginning with the number 1.

The references made from one article of the treatiſe to another is of two kinds.

*Firſt.* When in the ſame book. Then the number of the article referred to, is put in a parentheſis. Thus (27), refers to the article numbered 27 in the ſame book.

*Second.* When in another book. Then the number of the book in Roman figures, and of the article in common figures, is put in a parentheſis. Thus (II. 160) refers to the 160th article of the ſecond book.

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# A

# DISSERTATION

# ON THE

# RISE AND PROGRESS

# OF THE

# Modern ART OF NAVIGATION.

IT has been much diſputed to whom the world was obliged for the mariner's compaſs. A late *Italian* writer indeed contends, after many [*], that the honour of the invention is due to *Flavio Gioja* of *Amalfi* in *Campania*, who lived about the beginning of the 14th century [†], though others ſay it came from the Eaſt, and was earlier known in *Europe* [‡]. However that may be, it is certain, this wonderful diſcovery gave riſe to the preſent art of navigation; which ſeems to have made ſome progreſs during the voyages, that were begun in the year 1420, by *Henry* Duke of *Viſeo* [‖]. This learned Prince, brother to *Edward* King of *Portugal*, was particularly knowing in coſmography, and ſent for one maſter *James* from the iſland of *Majorca*, to teach navigation, and make inſtruments and charts for the ſea [§].

Theſe voyages being greatly extended, the art was improved under the ſucceeding monarchs of that nation. For *Roderic* and *Joſeph*, phyſicians to King *John* the Second, together with one *Martin de Bohemia*, a *Portugueſe* native of the iſland of *Fayal*, ſcholar to *Regiomontanus*, about the year 1485, calculated tables of the Sun's declination, for the uſe of the ſailors, and recommended the aſtrolabe for taking obſervations at ſea [¶].

The famous *Chriſtopher Columbus* is ſaid, before he attempted the diſcovery of *America*, to have conſulted *Martin de Bohemia*, with others, and during the courſe of his voyage to have inſtructed the *Spaniards* in navigation *; for the improvement of which art, the Emperor *Charles* the Fifth afterwards founded a lecture at *Seville* †.

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[*] Suitable to that verſe of *Pannormitana*,

*Prima dedit nautis uſum magneti Amalphis.*

[†] See Signor *Gregorio Grimaldi*'s Diſſertation on this ſubject in the *Memoirs* of the *Etruſcan* Academy of *Cortona*, tom. iii. p. 193, printed at *Rome* in 1742.

[‡] *Hiſtoire des Mathematiques, par M. Montucla, à Paris*, 1758.

[‖] *Marianæ* Hiſt. *Hiſpan.* lib. xx. cap. 11. and lib. xxvi. cap. 17. *Moguntiæ*, 1605.

[§] Decados d'Aſia par J. di Burnos, lib. xvi. 1552.

[¶] *Maffeii* Hiſtor. *Indic.* lib. i. p. 6. printed at *Florence* in 1588.

The variation of the ſea-compaſs could not be long a ſecret. *Columbus*, on the 14th of September 1492, obſerved it, as his ſon *Ferdinand* aſſerts ‡, though others ſeem to attribute that diſcovery to *Sebaſtian Cabot* ‖. And as this variation differs in different places, *Gonzales d' Oviedi* found there was none at the *Azores* §; where ſome geographers have thought fit in their maps to make their firſt meridian to paſs through one of thoſe iſlands; it not being then known, that the variation altered in time.

The uſe of the *Croſs-Staff* now began to be introduced amongſt the Sailors. This very ancient inſtrument being deſcribed by *John Werner* of *Nuremberg*, in his *Annotations* on the firſt book of *Ptolemy's Geography*, printed in 1514; he recommends it for obſerving the diſtance between the Moon and ſome ſtar, in order thence to determine the longitude. *Werner* ſeems to have been the greateſt geometer, as well as aſtronomer, of the time. In 1522, he publiſhed a tract ¶, containing a ſpecimen of the conics, with ſome ſolid problems, and alſo he there determined the preceſſion of the equinox more exactly than it had been done.

But the art of navigation ſtill remained very imperfect, from the conſtant uſe of the plane-chart, the groſs errors of which muſt have often miſled the mariner, eſpecially in voyages far diſtant from the equator. Its precepts were probably at firſt only ſet down on the earlieſt ſea-charts, as that cuſtom is continued to this day; and larger directions have been uſually premiſed by the *Dutch*, to collections of their charts called *Wagoner*'s, from the name of the publiſher: The *Dutch* call theſe collections alſo by many other affected titles, ſuch as *Fiery-Columns*, *Sea-Beacons*, *Mirrors*, *Atlaſſes*, &c.

At length there were publiſhed in *Spaniſh* two treatiſes, containing a ſyſtem of the art, which were in great vogue; the firſt by *Pedro de Medina* at *Valladolid*, 1545, called *Arte de Nauegar*; the other at *Sevile*, in 1556, by *Martin Cortes*, with this title, *Breve Compendio de la Sphera, y de la Arte de Nauegar con nueuos Inſtrumentos y Reglas*. The author of this laſt tract ſays, he compoſed it at *Cadiz* in 1545.

Theſe ſeem to have been the oldeſt writers, who had fully handled this ſubject; for *Medina*, in his dedication to *Philip* Prince of *Spain*, laments, that multitudes of ſhips daily periſhed at ſea, becauſe there were neither teachers of the art, nor books by which it might be learnt; and *Cortes*, in his dedication, boaſts to the Emperor, that he was the firſt who had reduced navigation into a *compendium*, enlarging much on what he had performed *.

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* La Hiſtoria general y natural de las Indias par Gonzalles de Miedo, en Sevilla, 1535. And Deſcriptione de las Indias Occidentale, de Antonio de Herrera, en Madrid, 1601.

† *Hackluyt*, in the dedication of his firſt volume of Voyages, printed in 1599.

‡ In *Columbus*'s life written in *Spaniſh*, which is very ſcarce, but it was printed in *Italian* at *Venice* in 1571.

‖ See *Livio Sanuto*'s *Geographia*, at the ſame place in 1585. Dr. *William Gilbert*, *de Magnete*. *London*, 1600; and *Purchas*'s *Pilgrim*, in 1625, vol. I.

§ *Cabot*, a *Venetian* by birth, firſt ſerved our King *Henry* the ſeventh, then the King of *Spain*, and laſtly, returning to *England*, he was conſtituted grand pilot by King *Edward* the ſixth, with an annual ſalary of above 160 pounds. Of this famous navigator and his expeditions, many writers have made mention, both foreigners and *Engliſh*, as *Peter Martyr*, *Ramuſeo*, *Herrera*, *Holinſhed*. Lord *Bacon*, and particularly *Hackluyt* and *Purchas*, in their Collections of Voyages.

¶ *Opera Mathematica* at *Nuremberg*, in quarto.

*Medina* gave ridiculous directions, how to gueſs at the place of the horizon, when it could not be ſeen; as alſo he defended the errors of the plane-chart, and advanced againſt the variation of the magnetic needle ſuch abſurd arguments, as *Ariſtotle* and his followers had done to prove the impoſſibility of the Earth's motion. But *Cortes* briefly and clearly made out the errors of the plane-chart, and ſeemed to reflect on what had been ſaid againſt the variation of the compaſs, when he adviſed the mariner rather to be guided by experience, than to mind ſubtle reaſonings. Beſides he endeavoured to account for this variation, in imagining the needle to be influenced by a magnetic pole (which he called the point attractive) different from that of the world, and this notion has been farther proſecuted by others.

However, *Medina*'s book, being perhaps the firſt of its kind, was ſoon tranſlated into *Italian*, *French*, and *Flemiſh* †, ſerving for a long time as a guide to navigators of foreign countries.

But *Cortes* was our favourite author, a tranſlation of whoſe work by Mr. *Richard Eden* was, on the recommendation of that great navigator Mr. *Stephen Borrough*, and the encouragement of the Society for making diſcoveries at ſea, publiſhed at *London* in 1561: which underwent various impreſſions ‡, whilſt that of *Medina*, though tranſlated within twenty years after the other, ſeems to have been neglected, notwithſtanding the encomiums beſtowed on it by Mr. *John Frampton*, the tranſlator.

A ſyſtem of navigation at that time conſiſted of ſome ſuch particulars as theſe: An account of the *Ptolemaic* hypotheſis, and the circles of the ſphere; of the roundneſs of the Earth, its longitudes, latitudes, climates, and eclipſes of the luminaries; a kalendar; how to find the prime, epact, &c. and by the laſt the Moon's age, and thence the tides; a deſcription of the ſea-compaſs, not forgetting the loadſtone, with ſomething about the variation, called its north-eaſting and north-weſting, for the diſcovering of which, by night as well as by day, *Cortes* ſaid, an inſtrument might be eaſily contrived; tables of the Sun's declination for four years §, in order to find the latitude, from his meridian altitude; to do the ſame thing by thoſe called the guard-ſtars in the north, and the croſiers in the ſouth; of the courſe of the Sun and Moon; the length of the days; of time and its diviſions; to find the hour of the day, and by the nocturnal that of the night; and laſtly, a deſcription of the ſea-chart, on which to diſcover where the ſhip is, they made uſe of a ſmall table, that ſhewed, upon an alteration of one degree in the latitude, how many leagues were run on each rhumb, together with the departure from the meridian. Beſides ſome inſtruments were deſcribed, eſpecially by *Cortes*; as one to find the place and declination of the ſun, with the days and place of the Moon; certain dials, the aſtrolabe and croſs-ſtaff, with a complex machine to diſcover the hour and latitude at once.

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* The learned Don *Nicolo Antonio*, in his *Bibliotheca Hiſpanica*, printed at *Rome* in 1672, tom. i. p. 323. puts down a book, intitled, *Tradado de la Sphera y del marear con el regimento de las alturas*, written by *Franciſco Falero*, a *Portugueſe*, and printed at *Seville* in 1535; but perhaps there is a miſtake in the date. He alſo mentions an edition of *Cortes* in 1551.

† The *Italian* and *French* tranſlations were printed in 1554, the firſt at *Venice*, the other at *Lyons*; the *Flemiſh* edition, I have ſeen, was at *Antwerp* in 1580; perhaps it had been printed before.

‡ In the latter editions ſome miſtakes in the tranſlation are corrected.

§ *Cortes* ſets down the places of the Sun for a twelvemonth, with an equation-table to correct thoſe places, ſerving for many years to come; and alſo another table to find the Sun's declination from his longitude being given.

And after this manner the art continued to be treated, though from time to time improvements were made by the following authors.

As *Werner* had propoſed to find the longitude by obſervations on the Moon; ſo *Gemma Friſius*, in a tract intitled *De Principiis Aſtronomiæ et Coſmographiæ*, printed at *Antwerp* in 1530, adviſed the keeping of the time by means of ſmall clocks or watches for the ſame purpoſe, then, as he ſays, lately invented. He alſo contrived a new ſort of croſs-ſtaff, which he deſcribes in his treatiſe *De Radio Aſtronomico et Geometrico*, printed at the ſame place 1545, and in his additions to *Peter Apian*'s Coſmography, gives the figure of an inſtrument, he calls a *Nautical Quadrant*, as very uſeful in navigation, promiſing to write largely on the ſubject; accordingly, in an edition he made *anno* 1553, of his above-mentioned book *De Principiis Aſtronomiæ*, &c. he delivers ſeveral *nautical axioms*, as he calls them, which with ſome alterations were repeated by his ſon *Cornelius Gemma*, in a poſthumous piece of his father on the *Univerſal Aſtrolabe*, publiſhed in 1556. *Gemma Friſius* died in 1555, aged 45 years.

With us Dr. *William Cunningham*, in his *Coſmographical Glaſs*, printed in 1559, amongſt other things briefly treats of navigation, eſpecially ſhewing the uſe of the *Nautical Quadrant*, much praiſing that inſtrument.

But a greater genius than theſe undertook this ſubject; for the famous mathematician *Pedro Nunez*, or *Nonius*, having ſo early as 1537 publiſhed a book, written in the *Portugueſe* language, to explain a difficulty in navigation propoſed to him by the commander Don *Martin Alphonſo de Suſa*; which was thirty years after printed at *Baſil*, in *Latin*, with the addition of a ſecond book, the whole intitled *de Arte et Ratione Navigandi*; where he expoſes, both truly and learnedly, the errors of the plane-chart; and beſides gives the ſolution of ſeveral curious Aſtronomical Problems, amongſt which is that of determining the latitude from two obſerſervations of the Sun's altitude and the intermediate azimuth being given. He alſo delivers many uſeful advices about the art of navigation, particularly how to perform its operations on the globe. He obſerved, that though the oblique rhumbs are ſpiral lines, yet the direct courſe of a ſhip will always be the arch of ſome great circle, whereby the angle with the meridians will continually change; all that the ſteerſman can here do for the preſerving of the original rhumb is to correct theſe deviations, as ſoon as they appear ſenſible. But thus the ſhip will in reality deſcribe a courſe without the rhumb-line intended; and therefore his calculations for aſſigning the latitude, where any rhumb-line croſſes the ſeveral meridians,

ridians, will be in ſome meaſure erroneous. He alſo again ſets down his method of diviſion of a quadrant by concentric circles *, which he had deſcribed in his ingenious treatiſe *de Crepuſculis*, printed in 1542, imagining it had been practiſed by *Ptolemy*. There were alſo added other tracts of his, but the completeſt edition of his *Latin* works was made by himſelf at *Coimbra* in 1573. His treatiſe of *Algebra*, written in *Spaniſh*, was printed at *Antwerp* ſix years before.

In 1577 Mr. *William Bourne* publiſhed his treatiſe †, intitled, *A Regiment for the Sea*, which he deſigned as a ſupplement to *Cortes*, whom he frequently quotes. Beſides many things common with others, *Bourne* gives a table of the places and declinations of thirty-two principal ſtars, in order to find the latitude and hour; as alſo a larger tide-table than that publiſhed by Mr. *Leonard Digges*, in 1556 ‡. He ſhews, by conſidering the irregularities in the Moon's motion, the errors of the ſailors in finding her age by the epact; and alſo in their determining the hour from obſerving upon what point of the compaſs the Sun and Moon appeared. He adviſes in ſailing towards high latitudes to keep the reckoning by the globe, as there the plane-chart errs moſt. He deſpairs of our ever being able to find the longitude by any inſtrument, unleſs the variation of the compaſs ſhould be cauſed by ſome ſuch attractive point, as *Cortes* had imagined. Though of this he doubts, and as he had ſhewn how to find the variation of the compaſs at all times, he adviſes to keep an account of the obſervations, as uſeful to diſcover thereby the place of a ſhip; which advice the famous *Simon Stevin* proſecuted at large in a treatiſe publiſhed at *Leyden* in 1599, intitled *Portuum inveſtigandorum Ratio Metaphraſto Hugone Grotio*; the ſubſtance of which was the ſame year printed at *London* in *Engliſh*, by Mr. *Edward Wright*, intitled *The Haven-finding Art*.

But the moſt remarkable thing in this ancient tract is, the deſcribing of the way by which our ſailors eſtimated the rate a ſhip made in her courſe, by an inſtrument called the *log*. This was ſo named from the piece of wood, or log, that floats in the water, while the time is reckoned during which the line that is faſtened to it is veering out. The author of this device is not known, and I find no farther mention of it till 1607, in an *Eaſt-India* voyage, publiſhed by *Purchas*; but from that time its name occurs in other voyages, that are amongſt his collections. And henceforward it became famous, being taken notice of, both by our own authors, and by foreigners; as by *Gunter* in 1623, *Snellius* in 1624, *Metius* in 1631, *Oughtred* in 1633, *Herigone* in 1634, *Saltonſtall* in 1636, *Norwood* in 1637, *Fournier* in 1643; and indeed by almoſt all the ſucceeding writers on navigation, of every country. And it continues to be ſtill in uſe as at firſt, though attempts have been often made to improve it, and other contrivances propoſed to ſupply its place. Many of theſe have ſucceeded in quiet water; but proved uſeleſs in a troubled ſea.

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* The admirable diviſion, now ſo much in uſe, is a very great improvement of this; ſo that when the famous Dr. *Edmund Halley*, the Royal Aſtronomer, revived that by adapting it to his *Mural Arch*, ſome body here named it *A Nonius*, in which he has been followed by many.

† He had ten years before publiſhed what he calls *Rules of Navigation*, as appears from his *Almanac*, printed in 1571.

‡ In his treatiſe, intitled, *A Prognoſtication everlaſting*, fol. 23.

A following edition of this book was reviſed by the author, where, in the preface, he ſets forth the groſs ignorance of the old ſhip-maſters, repeating ſome of the inſipid jeſts they made uſe of to juſtify their want of knowledge in their art. Amongſt the additions, he enlarges on the account of the log-line. And at the end ſubjoins an *Hydrographical Diſcourſe touching the five ſeveral Paſſages into Cathay.*

*Bourne* publiſhed other tracts, as one called *Inventions* or *Deviſes*, where he deſcribes a method by *wheel-work* of meaſuring the velocity of a ſhip at Sea, which artifice he attributes to one Mr. *Humfrey Cole.*

At *Antwerp*, in 1581, *Michael Coignet*, a native of the place, publiſhed a ſmall treatiſe, intitled, *Inſtruction nouvelle des Points plus excellents & neceſſaires touchant l'Art de Naviger* [*]. This ſerved as a ſupplement to *Medina*, whoſe miſtakes *Coignet* well expoſed. He there ſhewed, that as the rhumbs are ſpirals, making endleſs revolutions about the poles, numerous errors muſt ariſe from their being repreſented by ſtraight lines on the ſea-charts; and expreſſed his hopes of diſcovering a rule to remedy thoſe errors; ſaying, that moſt of the ſpeculations, delivered by the great mathematician, *Peter Nonius*, for that purpoſe, were ſcarce practicable; and therefore in a manner uſeleſs to ſailors. In treating of the Sun's declination, he took notice of the gradual decreaſe in the obliquity of the ecliptic, a point long diſputed, but now ſettled from the theory of attraction. He alſo deſcribed the croſs-ſtaff with three tranſverſe pieces, as it is at preſent made, which he acknowledged to be then in common uſe amongſt the mariners; but he preferred that of *Gemma Friſius.* He likewiſe gave ſome inſtruments of his own invention, which are now quite laid aſide, except perhaps his nocturnal. As the old ſea-table, mentioned above, erred more and more in advancing towards the poles; he ſet down another to be uſed by ſuch as ſailed beyond the 60th degree of latitude. At the end of the book is delivered a method of ſailing on a parallel of latitude by means of a ring dial, and a 24 hour glaſs; on which the author very much values himſelf.

The ſame year Mr. *Robert Norman* [†] publiſhed a diſcovery, he had long before made, of the dipping of the magnetic needle, in a ſmall pamphlet called *The Newe Attractive*, where he ſhews how to determine its quantity; and in ſpeaking of the loadſtone, he diſputes againſt *Cortes*'s notion, that the variation of the compaſs was cauſed by a point fixed in the heavens, contending that it ſhould be ſought for in the earth, and propoſes how to diſcover its place. He alſo treats of the various ſorts of compaſſes, ſetting forth at large the dangers that muſt ariſe from the then prevailing practice of not fixing, on account of the variation, the wire directly under the *flower-de-luce*; as compaſſes made in different countries have it placed differently. *Bourne* indeed had warned againſt this abuſe, and there are many things common to both authors.

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[*] It had been publiſhed in *Flemiſh*, but the *French* edition is the fulleſt. *Coignet* died in 1623, leaving many mathematical manuſcripts, ſee *Valerii Andreæ Bibliotheca Belgica*, printed at *Louvain* in 1643.

[†] He is commended for an excellent Artiſt by our authors, as *Bourne*, *Burrough*, Sir *Humfrey Gilbert*, *Hues*, *Potter*, *Blundeville*, *Wright*, and Dr. *Gilbert.*

To *Norman*'s piece is always ſubjoined, Mr. *William Burrough's Diſcourſe of the Variation of the Cumpaſs or Magneticall Needle.* The author had been a famous navigator, having uſed the ſea from fifteen years of age, and for his merit promoted to be Controller of the navy by Queen *Elizabeth* *. He ſhews how to determine the variation ſeveral ways, ſetting down many obſervations of it made by an azimuth-compaſs of *Norman*'s invention, but improved by himſelf. He demonſtrates the falſhood of the rules commonly uſed, to find the latitude by the guard-ſtars. He particularizes many errors in the then ſea-charts, occaſioned by the neglect of the variation; adding, *But of theſe coaſtes* (towards the north), *and of the inwarde partes of the countries of* Ruſſia, Muſcovia, &c. *I have made a perfect plat and deſcription, by myne owne experience in ſundrie voiages and travailes, bothe by ſea and lande to and fro in theſe partes, which I gave to her Majeſtie in* anno 1578. And laſtly, he juſtly finds fault with *Coignet*'s inſtrument, called a *nautical hemiſphere*; but ſpeaks too ſeverely againſt the writers of navigation, concluding thus,

*But as I haue alreadie ſufficientlie declared, the cumpas ſheweth not alwaies the pole of the worlde, but uarieth from the ſame diverſly, and in ſayling deſcribeth circles accordyngly. Whiche thing, if* Petrus Nonius, *and the reſt that haue written of Navigation, had jointlie conſidered in the tractation of their rules and Inſtruments, then might they haue been more auaileable to the uſe of Navigation; but they perceivyng the difficultie of the thyng, and that if they had dealt therewith, it would have utterly overwhelmed their former plauſible conceits, with* Pedro de Medina *(who, as it appeareth, hauyng ſome ſmall ſuſpicion of the matter, reaſoneth very clerkly, that it is not neceſſary that ſuch an abſurdity as the Variation, ſhould be admitted in ſuch an excellent art as Navigation is) they haue all thought beſt to paſſe it over with ſilence.*

The *Spaniards* too continued to publiſh treatiſes of the art; particularly at *Seville* was printed in 1585 an excellent *Compendium* by *Roderico Zamorano*; which is written clearly and with brevity, not being incumbered with ſuch idle ſpeculations as abound in *Medina* and *Cortes*. The author was Royal Lecturer at *Seville*, and contributed much to the reforming the ſea charts; as we are told by his ſucceſſor, *Andres Garcia de Ceſpedes*, who alſo publiſhed a treatiſe of navigation at *Madrid*, in 1606.

As globes may be very ſerviceable for the mariner, Mr. *Edward Mullineux* ſet forth in 1592, at the charges of Mr. *William Sanderſon* merchant †, a pair much larger than thoſe the famous geographer *Gerard Mercator* publiſhed in 1541. On the terreſtrial one were deſcribed many new diſcovered countries, and traced out the reſpective voyages round the world by Sir *Francis Drake* in 1577, and Mr. *Thomas Candiſh* in 1586, with the progreſs Sir *Martin Frobiſher* had made towards the north in 1576, to a place called his *Straits*.

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* *Hackluyt*'s Voyages, vol. i. p. 417, printed in 1599.

† Mr. *Sanderſon* was commended for his knowledge as well as generoſity to ingenious men.

Theſe globes were accompanied with a tract containing their uſes written in *Engliſh*; but in 1594 Mr. *Robert Hues* publiſhed a more elaborate one in *Latin*; wherein, amongſt others, he ſolves by the globe the problem of determining the latitude from two heights of the Sun obſerved with the intermediate time being given *; and in the laſt part of his book, he performs the uſual queſtions in navigation, premiſing a diſcourſe on the rhumb-lines, where he attempts to refute what *Gemma Friſius* had aſſerted, who ſays, that they meet in the poles. At the concluſion he highly praiſes a treatiſe of Mr. *Thomas Hariot*, hoping it would be ſoon publiſhed, in which that author had treated of this ſubject upon geometrical principles, with great ſagacity and judgment. But all the manuſcripts of that great mathematician were loſt, except his *Artis Analyticæ Praxis*, which was publiſhed long after his death in 1631; wherein is firſt advanced that idea of algebraic equations, which has been ever ſince followed.

*Hues* † was a perſon of letters, and beſides had been far at ſea. Amongſt other curious particulars, he gives a good account of the attempts that had been made at various times to meaſure the Earth. In the *Epiſtle* to Sir *Walter Rawley* he takes an occaſion to enumerate the many diſcoveries of our mariners in very different parts of the world. His book was received with great applauſe, and has been indeed a pattern for ſuch as afterwards handled the ſame ſubject. It has been often printed abroad, particularly in 1617 with the notes of *John Iſaac Pontanus*, who omitted the *Epiſtle* and the mentioning of *Hariot*. However from this mutilated edition it was tranſlated into *Engliſh* by one Mr. *John Chilmead*, and publiſhed in 1639.

Amongſt our ſailors none were more famous than Captain *John Davis* ‡, who gave name to the ſtraits which he diſcovered; and great matters were expected from his long experience and ſkill. In 1594 he publiſhed a ſmall treatiſe, intitled, *The Seaman*'s *Secrets*. This is written with brevity, though ſomewhat pedantically, and was eſteemed in its time, an eighth edition being printed in 1657; ſo that it ſeems to have ſupplanted *Cortes*. *Davis* treats of plane ſailing, calling it *horizontal*, and ſets down the form of keeping a reckoning at Sea. He likewiſe ſhews how to ſail by the globe, and boaſts of what he intended to do; much commending great circle ſailing, without deſcribing it, as alſo what he calls *paradoxal*; that is, by a projection on the plane of the equator with ſpiral rhumbs, ſaying, he will publiſh a chart for that purpoſe. But above all, he extols the uſe of calculations in the caſes of navigation, and promiſes to handle that ſubject.

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* This problem has been diſcuſſed by Dr. *Henry Pemberton*, in the Philoſophical Tranſactions, vol. li. part. 2d. 1760, p. 910, where he has alſo given ſome improvements in trigonometry.

† There is an account of *Hues* and *Hariot* in *Anthony Wood*'s *Athen. Oxon.* vol. i. printed in 1721, as being both members of that Univerſity.

‡ Several of his voyages are in *Hackluyt*'s and *Purchas*'s collections. He and Captain *Abraham Kendal* are greatly praiſed by Sir *Robert Dudley*, in his *Arcano del Mare*, as keeping a perfect reckoning by the way of longitude and latitude, where are given two of their *Journals*. This *Dudley* was a natural ſon of the great Earl of *Leiceſter*, and had commanded, in 1594, a fleet againſt the *Spaniards*; but retiring to *Florence*, he aſſumed the titles of Duke of *Northumberland* and Earl of *Warwick*. His *Arcano* was printed at that place in two volumes in 1646 and 1647.

At the end of the book is given the figure of a ſtaff of his contrivance, to make a back obſervation. Of this the author is ſo vain as to ſay, *Than which inſtrument (in my opinion) the ſeaman ſhall not finde any ſo good, and in all clymates of ſo great certaintie, the invention and demonſtration whereof, I may boldly challenge to appertain unto my ſelfe (as a portion of the talent which God hath beſtowed upon me) I hope without abuſe or offence to any.*

This inſtrument ſeems to have for ſome time been in uſe; for *Adrian Metius*, in his treatiſe, intitled *Aſtronomiæ Inſtitutio*, printed in 1605, gives a figure of it from an original, in the poſſeſſion of M. *Frederic Hautman*, governor of *Amboyna*. But it ſoon yielded to one of a more commodious form, which is now commonly called *Davis*'s *Quadrant* *; as if it was alſo of his invention, and that perhaps only becauſe a back obſervation is made by both inſtruments, ſo the quadrant itſelf was at firſt ſtyled a *Staff* and *Back-Staff*.

The famous traveller Signor *Pietro della Valle* paſſing, in 1623, from *Ormus* to *Surat* aboard an *Engliſh* veſſel, where obſerving this quadrant much practiſed by the ſeamen, as it was quite new to him, takes an occaſion to ſhew its uſe very diſtinctly, and ſays, they told him, that it had been lately invented and called *David*'s *Staff* † from its author. Alſo Captain *Charles Saltonſtall*, in his *Navigation*, deſcribes it under the name of a *Back-Staff*; and in Captain *Thomas James*'s famous voyage for diſcovering a north-weſt paſſage, begun in 1631, amongſt the many inſtruments, he carried along with him, are mentioned two of Mr. *Davis*'s *Back-ſtaves*, which were doubtleſs theſe quadrants.

Contemporary with *Davis* was Mr. *Richard Polter*, who, it is ſaid, had been a principal maſter aboard the *Royal Navy*. He wrote a very ſmall book intitled *The Pathway to Perfect Sailing*, where, from an obſervation he made in 1586, he would infer ‡, that different loadſtones communicated different degrees of variation to the magnetic needle, and therefore deſpiſes the publiſhing obſervations of that kind, as needleſs. His book was not printed till 1644, nor did it deſerve to be publiſhed at all, as it abounds with miſtakes, and is written fantaſtically, obſcurely and arrogantly.

But all this while the plane-chart, notwithſtanding its errors were frequently complained of, continued to be followed; as its uſe is eaſy, and ſerves tolerably well in ſhort voyages, eſpecially near the equator.

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* It is called by the *French Quartier Anglois*.

† *David Staff, che in lingua Ingleſe vale à dir legnodi David Viaggi*; *Part* 3. *Letter* 1. *à Roma*. This author not only praiſes the Captain *Nicholas Woodcock*, and other officers, but alſo the common ſailors, for their care and ſkill; and ſays, the *Portugueſe* loſe great number of ſhips for not being ſo exact in their obſervations as the *Engliſh*.

‡ Perhaps he ſhould have thence concluded the variation altered, as was diſcovered afterwards.

However, a way to remedy theſe errors had, for ſome time, been inquired after. And *Gerrard Mercator* ſeems to be the firſt, who conceived the means of effecting this, in a manner convenient for ſeamen, by continuing to repreſent both the meridians and parallels of latitude by parallel ſtraight lines, as in the plane-chart, but gradually augmenting the diſtances between the degrees of latitude in advancing from the equator towards either pole, that the rhumbs alſo might be extended into ſtraight lines, ſo that a ſtraight line drawn between any two places, laid down in this chart by their longitudes and latitudes, ſhould make an angle with the meridians, expreſſing the rhumb leading from one to the other. But though *Mercator*, in 1569, ſet forth an univerſal map thus conſtructed *, it does not appear upon what principles he proceeded; probably, by obſerving in a globe furniſhed with rhumbs, what meridians the rhumbs paſſed at each degree of latitude. That he knew not the genuine principles, I ſhall make evident; our countryman, Mr. *Edward Wright*, was certainly the firſt who diſcovered them.

*Wright* inſinuates, but without ſufficient grounds, that this enlargement of the intervals between the parallels had been ſuggeſted before by *Cortes* †, and even by *Ptolemy* himſelf.

As to *Cortes*, he ſpeaks of the number of the degrees of latitude, and not the extent of them; for his expreſſion amounts to no more than this, that the degrees of latitude are to be numbered from the equator, and conſequently both northwards and ſouthwards from that line the numbers affixed to them muſt continually increaſe; and from any place having latitude (ſuppoſe Cape *St. Vincent* in *Spain*, which is his inſtance) the degrees of latitude will be denoted by numbers increaſing towards the pole, and decreaſing towards the equator. He had before expreſsly directed, that they ſhould be all equal by meaſurement on a ſcale of leagues adapted to the map ‡.

The paſſage in *Ptolemy* ‖, referred to by *Wright* §, does indeed relate to the proportion between the diſtances of the parallels and meridians, but contains no ſhadow of *Mercator*'s ſcheme: for inſtead of propoſing any gradual enlargement of the diſtances of the parallels in a general chart; that paſſage relates only to particular maps, and is more diſtinctly explained in the firſt chapter of his laſt book; where he adviſes explicitly not to confine a ſyſtem of ſuch maps to one and the ſame ſcale, but to plan them out by a different meaſure, as occaſion ſhall require, with this only caution; that the degrees of longitude ſhould in each bear in ſome meaſure that proportion to the degrees of latitude, which the magnitude of the reſpective parallels bear to a great circle of the ſphere; and ſubjoins, that in particular maps, if this proportion be obſerved in regard to the middle parallel, the inconvenience will not be great, though the meridians ſhould be ſtraight parallels to each other; wherein his deſign is plainly no other, than that the maps ſhould in ſome ſort repreſent the figures of the countries they are drawn for. *Mercator*, who drew maps for *Ptolemy*'s tables[*], underſtood him in no other ſenſe, thinking it an improvement not to regulate the meridians by one parallel, but by two; one diſtant from the northern, and the other from the ſouthern extremity of the map by a fourth part of the whole depth; whereby in his maps, though the meridians are ſtraight lines, they are generally drawn inclining to each other towards the pole.

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* See his life, written by his intimate friend, *Gaulterus Ghymmius*, which was prefixed to an enlarged edition of his *Atlas*, publiſhed at *Duiſburg*, in 1593, by *Ramoldus* his ſon, a year after his father's death. *Gerrard Mercator* was born in 1512.

† See the 2d chapter of *Wright*'s book.

‡ *Part* 3d. cap. 2d. fol. 58.

‖ *Geograph.* lib. ii. cap. 1.

§ In an advertiſement ſet down on his univerſal map, at the end of his ſecond edition of his book; and in this miſtake he has been followed by others.

But *Mercator*'s univerſal map, mentioned above, though the author deſigned it for the benefit of ſailors, was ſo far from being readily adopted, that ſome of the moſt ſkilful amongſt them objected to its uſefulneſs. Thus Mr. *Burrough* ſays of it—*By augmenting his degrees of latitude towards the poles, the ſame is more fitte for ſuche to beholde, as ſtudie in coſmographie, by readyng authours upon the lande, then to bee uſed in Navigation at the ſea.*

And Mr. *Thomas Blundeville*, in his *Briefe Deſcription of Univerſal Mappes and Cardes*, firſt printed in 1589, gives an account of this map, obſerving that *Barnardus Puteanus* of *Bruges* had publiſhed, in 1579, one altogether like it. And though *Blundeville* is ſo particular, as to ſet down numbers expreſſing the diſtances between each parallel of latitude in thoſe maps, yet he ſeems to ſlight them, by ſaying, that no better rules than thoſe given by *Ptolemy* can be deviſed. But what is delivered by this *geographer* about the conſtruction of a general map, is a very indifferent performance, altogether unworthy the author of the *Almageſt*, and not in the leaſt correſponding with the ſagacity ſhewn in two treatiſes on the *Planiſphere* and *Analemma*, which the *Arabians* have handed down to us as *Ptolemy*'s[†].

*Maginus* alſo, at the end of his *Geographia Univerſa*, (the former part of which is a tranſlation of *Ptolemy*'s) firſt printed at *Venice*, in 1596, mentions this map of *Mercator*, and gives even a ſketch of it; but ſeems to have no diſtinct conception of the author's deſign.

That *Mercator*'s map was not rightly deſcribed, is manifeſt from the numbers given by *Blundeville*; and that he was ignorant of a genuine method of dividing the meridian, appears from a paſſage in his life, where the writer ſays *Mercator* often aſſured him, that this extending a ſphere into a plane anſwered to the quadrature of the circle, as that nothing ſeemed to be wanting but the demonſtration.

However, our authors now began to entertain favourable thoughts of it, perhaps from the report that Mr. *Wright* was about to treat on that ſubject. Dr. *Thomas Hood*, to the firſt edition which he gave of *Bourne*'s *Regiment* in 1592, added a *Dialogue* of his own, called *The Mariner's Guide*, written only to ſhew the uſe of the plane chart, where he acknowledges and ſets forth its errors, and highly praiſes *Mercator*'s, ſaying, he had compoſed a treatiſe concerning it; but the indiſtinct account he gives of it, ſhews it would not be this author's lot to render it fit for the uſe of navigation. And Mr. *Blundeville*, in the following editions of his above-mentioned tract, omitted the commendation he had given of *Ptolemy*'s method of delineating an *univerſal map*.

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[*] In an edition he made of *Ptolemy*'s *Geography*, in 1584.

[†] Theſe were publiſhed by *Fed. Commandinus*, one at *Venice*, in 1518, the other at *Rome*, in 1562.

*Mercator*'s ſcheme was not indeed contrived for repreſenting the parts of a country in a juſt proportion to each other; but is appropriated to the uſe of mariners, who ſail upon rhumbs by the guidance of the compaſs; which our countryman, Mr. *Edward Wright*, perfected [*], by diſcovering a true way of dividing the meridian. An account of this he ſent from *Caius* college, in *Cambridge*, where he was then a fellow, to his friend the above-mentioned Mr. *Blundeville*, containing a ſhort table for that purpoſe, with a ſpecimen of a chart ſo divided, together with the manner of dividing it. All which *Blundeville* publiſhed, in 1594, amongſt his *Exerciſes*, in that part which treats of navigation [†]; where he has well delivered what had been before written on that art; inſomuch that his book was long in great repute, a ſeventh edition having been printed in 1636. To the ſecond edition, *Anno* 1606, and following ones, was added his former diſcourſe of univerſal maps.

In 1597, the Reverend Mr. *William Barlowe*, in his *Navigator*'s *Supply*, gave a demonſtration of this diviſion as communicated by a friend; ſaying, *This manner of carde has been publiquely extant in print theſe thirtie yeares at leaſt* [‡], *but a cloude (as it were) and thicke miſte of ignorance doth keepe it hitherto concealed: And ſo much the more, becauſe ſome who were reckoned for men of good knowledge, have by glauncing ſpeeches (but never by any one reaſon of moment) gone about what they could to diſgrace it.*

This book of *Barlowe*'s contains deſcriptions of ſeveral inſtruments for the uſe of navigation, the principal of which is an *azimuth compaſs*, with two upright ſights [§]; and as the author was very curious in making experiments on the loadſtone, he diſcourſes well and largely on the ſea-compaſs; and ſtill farther handles that ſubject in a tract he publiſhed ſome years after, intitled, *Magnetical Advertiſements*.

At length, in 1599, Mr. *Wright* himſelf printed his famous treatiſe, intitled, *The Correction of certain Errors in Navigation*, which had been written many years before; where he ſhews the reaſon of this diviſion [‖],

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[*] Some of our modern writers have ſaid, *Mercator* took the hint from *Wright*, but that is a miſtake; for *Mercator*'s map was publiſhed thirty years before *Wright*'s book, who frequently refers to it. See *Edward Sherburn*'s tranſlation of the firſt book of *Manilius*, in 1675, p. 86.

[†] Chap. 29.

[‡] He ſhould have ſaid 28 only.

[§] Many of theſe inſtruments are in the *Arcano del Mare*, together with the demonſtration above mentioned.

[‖] Maps with their meridians thus divided had been publiſhed at *Amſterdam* by *Jodocus Hondius*, who, when in *London*, working as an engraver, learnt the manner of doing it from Mr. *Wright*'s *Manuſcript*; the fourth chapter of which he had tranſcribed into one of his maps. *Hondius* afterwards in his letters, both to Mr. *Briggs*, and alſo to Mr. *Wright*, begged pardon for not having acknowthe manner of conſtructing his table, and its uſes in navigation, with other improvements: A book, as Dr. *Halley* ſays, *well deſerving the peruſal of all ſuch as deſign to uſe the ſea*[*].

In the preface, *Wright* complains of the obſtinacy of our mariners, for not liking an improvement in their art, ſaying, that they were like thoſe whoſe ignorance Maſter *Bourne* had expoſed, repeating *Bourne*'s very words.

"Though this great improvement in navigation by *Wright* has been embraced and followed by all proper judges; yet ſome undiſcerning perſons have of late, even amongſt us, found fault with it, particularly *Henry Wilſon*, author of a Treatiſe on Navigation, by a propoſal for a *curvilinear ſea-chart*, in 1720; and the Rev. Mr. *Weſt*, of *Exeter*, in a poſthumous piece, printed in 1762. But their *cavils* were ſufficiently obviated; thoſe of the firſt by Mr. *Haſelden*, in his *Mercator*'s *Chart*, and in his *Reply*, both printed in 1722; and of the ſecond, by Mr. *William Mountaine*, in the Philoſophical Tranſactions, vol. LIII. p. 69. *Anno* 1763."

In 1610 a ſecond edition of Mr. *Wright*'s book was publiſhed, and dedicated to Prince *Henry*, his royal pupil [†], where the author inſerted farther improvements; particularly, he propoſed an excellent way of determining the magnitude of the Earth; at the ſame time recommending, very judiciouſly, the making our common meaſures in ſome ſettled proportion to that of a degree on its ſurface, that they might not depend on the uncertain length of a barley-corn.

Some of his other improvements were; The Table of Latitudes for dividing the meridian, computed to minutes; whereas before it was but to every tenth minute, and the ſhort table ſent by him to *Blundeville* to degrees only: An inſtrument, he calls the Sea-rings, by which the variation of the compaſs, altitude of the Sun, and time of the day, may be determined readily at once in any place, provided the latitude be known: The correcting of the errors ariſing from the excentricity of the eye in obſerving by the croſs-ſtaff: A total amendment in the Tables of the declinations and places of the Sun and ſtars, from his own obſervations, made with a ſix-foot quadrant, in the years 1594, 95, 96, and 97: A ſea-quadrant, to take altitudes by a forward or backward obſervation, and likewiſe with a contrivance for the ready finding the latitude by the height of the pole-ſtar, when not upon the meridian. And that his book might be the better underſtood by beginners, in this edition is ſubjoined a tranſlation of the above-mentioned *Zamorano*'s *Compendium*; he correct-

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acknowledged the obligation. See *Wright*'s preface, where he complains of *Hondius*'s proceedings; and farther relates, how his book, a copy of which having been preſented to the Earl of *Cumberland*, had liked to have come out under the name of a famous navigator, whom, from ſome circumſtances there mentioned, I imagine to have been *Abraham Kendal*.

[*] *Philoſophical Tranſactions*, for 1696, N° 219.

[†] In 1657 a 3d edition was publiſhed by Mr. *Joſeph Moxon*, where the dedication is unadviſedly left out, and at the end is added by the editor the above-mentioned *Haven-finding Art*, as alſo *Wright*'s univerſal map, improved by the diſcoveries made ſince his time.

ing ſome miſtakes in the original, and adding a large table of the variation of the compaſs obſerved in very different parts of the world, to ſhew it is not occaſioned by any magnetical pole.

This excellent perſon was allowed fifty pounds a year (no inconſiderable ſum at that time) by the *Eaſt India* Company, for reading a lecture of navigation; he alſo projected the conveying water to *London*, but was prevented from executing his ſcheme by deſigning men, which is frequently the caſe. Whilſt he led a ſtudious and retired life, his reputation was ſo far known, that Queen *Elizabeth* granted, in 1589, a *diſpenſation* for his abſence from the univerſity, in order to accompany the Earl of *Cumberland* in the expedition to the *Azores*; as I am informed by Sir *James Burrough*, Maſter of *Caius* College, whoſe fine taſte in architecture, part of the new buildings in *Cambridge* ſhew, they rendering the reſt of thoſe buildings a diſgrace to that famous ſeat of learning, which has produced many great men, as, (to mention here only mathematicians) *Wright*, *Briggs*, *Oughtred*, Dr. *Pell*, *Foſter*, *Horrox*, *Bainbridge*, Biſhop *Ward*, Dr. *Wallis*, Dr. *Barrow*, *Rooke*, Sir *Iſaac Newton*, *Cotes*, and Dr. *Brook Taylor*.

*Wright*'s improvements on *Mercator*'s chart became ſoon known abroad.

In 1608 were publiſhed the *Hypomnemata Mathematica* of the abovementioned *Simon Stevin*, compoſed for the uſe of Prince *Maurice*. In the part concerning navigation, the author, having treated of ſailing on a great circle, and ſhewn how to draw mechanically the rhumbs on a globe, ſets down *Wright*'s two tables of latitude and of rhumbs, in order to deſcribe thoſe lines more accurately; and in an appendix he commends *Hues*, ſhews a miſtake committed by *Nonius* in relation to the rhumbs, and pretends to have diſcovered an error in *Wright*'s latter table; but *Wright* himſelf, in the ſecond edition of his book, has fully anſwered all *Stevin*'s objections, demonſtrating that they aroſe from his groſs way of calculating.

And in 1624 the learned *Willebrordus Snellius*, Profeſſor of the Mathematics at *Leyden*, publiſhed his *Typhis Batavus*[*], a treatiſe of navigation on *Wright*'s plan, written ſomewhat obſcurely. In the introduction are praiſed *Nonius*, *Mercator*, *Stevin*, *Hues*, and *Wright*. But ſince what had been performed by our artiſts on this ſubject, is not there particularly declared, as are the improvements made by the others; it has happened that ſome have attributed *Wright*'s principal diſcovery to this author. Thus *Albert Girard*, who in 1634 publiſhed a *French* tranſlation of *Stevin*'s Works with notes, in one of them obſerves, that *Snellius* had calculated, what he calls, *Tabulæ Canonicæ Parallelorum*, to minutes as far as 70 degrees, whereas *Wright* had ſet forth in 1610 ſuch a table ſo calculated to 89 degrees 59 minutes; notwithſtanding which M. *de Lagni*, in the Memoirs of the Royal Academy of Sciences at *Paris* for 1703, treating of the *Corrected Chart*, ſays, *c'eſt Willebrord Snellius qui*

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[*] In 1617 had been publiſhed his *Eratoſthenes Batavus*, where is given an account of his meaſuring the earth.

*en eſt l'inventeur*. But the *French* writers now acknowledge our countryman to have been its author*.

*Snellius* was followed in *Holland* by *Adrian Metius*, in a treatiſe, intitled, *Primum Mobile*, printed at *Amſterdam*, in 1631; and in *France* by the learned *Peter Herigone*, in his *Curſus Mathematicus*, where, in the dedication of the fourth tome to the Marſhal *Baſſompire*, the author ſays, *Artem navigandi in cenſu Mathematices non repoſuere plerique noſtrum, neque ſanè in hunc ordinem aſcribi meruit, quandiu cœcâ tantùm nautarum praxi celebrata eſt; nunc verò cùm inventis tabulis loxodromicis (quas nos primùm Gallis exhibemus) formam certam firmaſque leges acceperit ſine injuriâ omitti non poteſt*. But to return to our countrymen.

Mr. *Wright*, in the 12th chapter, having ſhewn how to find the place of a ſhip on his chart, obſerved, the ſame might be performed more accurately by calculation; but conſidering, as he ſays, that the latitudes, and eſpecially the courſes at ſea, could not be determined ſo preciſely, he forbore ſetting down particular examples; as the mariner may be allowed to ſave himſelf this trouble, and only mark out upon his chart, when truly conſtructed, the ſhip's way after the manner then uſually practiſed.

However, in 1614†, Mr. *Raphe Handſon*, among his nautical queſtions ſubjoined to a tranſlation of *Pitiſcus's Trigonometry*, ſolved very diſtinctly every caſe of navigation, by applying arithmetical calculations to *Wright*'s table of latitudes, or of meridional parts, as it has ſince been called.

And beſides, though the method *Wright* diſcovered for determining the change of longitude by a ſhip ſailing on a rhumb, is the adequate means of performing it; *Handſon* propoſed two ways of approximation for that purpoſe, without the aſſiſtance of *Wright*'s diviſion of the meridian line. The firſt was computed by the arithmetical mean between the co-ſines of both latitudes; the other by the ſame mean between their ſecants, as an alternative, when *Wright*'s book was not at hand, though this latter is wider from the truth than the firſt; and farther he ſhewed by the aforeſaid calculations, how much each of theſe *compendiums* deviates from the truth, and alſo how erroneouſly the computations on the principles of the plain-chart differ from them all.

There is another method of approximation, by what is called The Middle Latitude‡, which, though it errs more than that by the arithmetical mean between the co-ſines; yet being leſs operoſe, is that generally uſed by our ſailors; notwithſtanding the arithmetical mean between the logarithmic co-ſines, equivalent to the geometrical mean between the co-ſines themſelves, had been ſince propoſed by Mr. *John Baſſat*||, which in high latitudes is ſomewhat preferable.

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* —— *c'eſt qu'on appelle les cartes réduites, invention admirable, de la quelle on eſt redevable à* Edward Wright, *quoiqu'on l'ait ſouvent attribuée à* Mercator. Hiſt. de l'Acad. Royale des Sciences, An. 1753, p. 275.

† It was reprinted in 1630.

‡ *Gunter's* works, firſt printed in 1623.

|| About 1670, in a dialogue which was publiſhed after the author's death, in an appendix to the *Pathway to perfect Sailing*. *Baſſat* had been a teacher of

The computation by the middle latitude, will always fall ſhort of the true change of longitude; that, by the geometrical mean, will always exceed; but that, by the arithmetical mean, fall ſhort in latitudes above 45 degrees, and exceed in leſſer latitudes. However, none of theſe methods, when the change in latitude is ſufficiently ſmall, will deviate greatly from the real change in longitude.

About this time logarithms * began to be introduced into the practice of the mathematics; and as they are of excellent uſe in the art of navigation, we ſhall here ſay ſomething about their original.

Theſe were invented by *John Napier*, Baron of *Marchiſtoun* in *Scotland*, as appears from his treatiſe, intitled, *Mirifici Logarithmorum Canonis Deſcriptio*, firſt printed in 1614 †. Soon after, the author communicated to Mr. *Henry Briggs*, Profeſſor of geometry at *Greſham* college in *London* ‡, another form of logarithms; with which Mr. *Briggs* was ſo well pleaſed, that he immediately ſet about computing a very large table of them, which he publiſhed in 1624, with his *Arithmetica Logarithmica* ‖. But in the mean time, as a ſpecimen, he printed in 1617 a few copies for his own uſe and that of his friends, of a very ſmall one, not exceeding a thouſand natural numbers.

From this table Mr. *Edmund Gunter*, Mr. *Briggs*'s colleague in Aſtronomy, computed one of artificial ſines and tangents to every minute of the quadrant, which he publiſhed in 1620, being the firſt of its kind §. And when he made an edition of his works three years after, both theſe tables were ſubjoined to his book.

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of navigation at *Chatham*, and well made out what he undertook, that a ſhip would return to the place it departed from, by ſailing on the ſame rhumb, contrary to what *Fuller* and others had maintained. At the end of this diſcourſe, he applies his compendium to the three principal problems in ſailing.

* The foundation of logarithms is a property of two ſeries of numbers, one in arithmetical, the other in geometrical proportion; which property is declared by *Archimedes* in his *Arenarius*.

† In 1619 was made, after the author's death, a ſecond edition, with his farther improvements in Spherical Trigonometry.

‡ He was in 1619 appointed by Sir *Henry Saville*, his profeſſor of geometry at *Oxford*.

‖ *Adrian Vlacq* made an edition of this book at *Tergou*, in 1628, where the table of logarithms was continued by him to one hundred thouſand numbers, though the logarithms themſelves are but to ten places, whereas in *Briggs*'s book they were to fourteen. Some copies of *Vlacq*'s tables were purchaſed by our bookſellers, and publiſhed at *London*, with an *Engliſh* explanation premiſed, dated 1631.

§ *Vlacq* alſo publiſhed, at the ſame place, in 1633, his *Trigonometria Artificialis*, with tables of logarithmic ſines and tangents to every tenth ſecond of the quadrant. *Vlacq*'s tables have a great reputation for their exactneſs, as *Sherwin*'s firſt edition in 1706, and *Gardiner*'s in 1742, have amongſt us. M. *de Fontenelle*, in the Hiſtory of the Academy of Sciences for 1717, commends an edition of *Vlacq*'s ſmaller tables, made at *Lyons*, in 1670, as does M. *de la Lande*, in his Aſtronomy, printed at *Paris*, in 1764, tables publiſhed there in 1760.

There he applied to navigation, according to *Wright*'s table of meridional parts; as well as to other branches of the mathematics, his admirable Ruler *, on which were inſcribed the logarithmic lines for numbers and for ſines and tangents of arches. He alſo greatly improved the Sector † for the ſame purpoſes. And he ſhewed how to take a back obſervation by the croſs-ſtaff, whereby the error, ariſing from the excentricity of the eye, is avoided; deſcribing likewiſe an inſtrument of his invention, named by him a Croſs-Bow, for taking altitudes of the Sun or ſtars, with ſome contrivances for the more ready collecting the latitude from the obſervation ‡.

The diſcoveries relating to the logarithms were carried to *France* by Mr. *Edmund Wingate*, who, going to *Paris* in 1624, publiſhed in that city two ſmall tracts in *French* ‖, and dedicated them both to *Gaſton*, the King's only brother. In the firſt he teaches the uſe of *Gunter*'s ruler, and in the other, of the tables of logarithms and artificial ſines and tangents, as modelled according to *Napier*'s laſt form, attributed by *Wingate* to *Briggs*, which is a miſtake; as appears from the dedication of *Napier*'s *Rabdologia*, printed in 1616, and from what Mr. *Briggs* himſelf ſaid in the preface of his *Arithmetica Logarithmica*.

The Reverend Mr. *William Oughtred* projected this ruler into a circular arch, ſhewing fully its uſes in a treatiſe firſt printed in 1633, intitled, *The Circles of Proportion*; where, in an appendix, are well handled ſeveral important points in navigation. It has been made in the form of a Sliding *Ruler*. See *Seth Partridge*'s uſe of the double ſcale in 1662.

As by the logarithmic tables all trigonometrical calculations are greatly facilitated; ſo the firſt author, who, I find, has applied them to the caſes of ſailing, was Mr. *Thomas Addiſon*, in his treatiſe, intitled, *Arithmetical Navigation*, printed in 1625. He alſo gives two traverſe tables with their uſes, the one to quarter points of the compaſs, the other to degrees.

Mr. *Henry Gellibrand*, Mr. *Gunter*'s ſucceſſor at *Greſham* College, publiſhed his diſcovery of the changes in the variation of the compaſs in a ſmall quarto pamphlet, intitled, *A Diſcourſe Mathematical on the Variation of the Magnetical Needle*, printed in 1635. This extraordinary *phenomenon* he found out by comparing the obſervations made at different times near the ſame place by Mr. *Burrough*, Mr. *Gunter*, and himſelf, all perſons of great ſkill and experience in theſe matters. And this diſcovery was ſoon known abroad [*], for father *Athanaſius Kircher*, in his treatiſe intitled *Magnes*, firſt printed at *Rome* in 1641, ſays our countryman Mr. *John Greaves* had informed him of it, and then gives a letter of the famous *Marinus Merſennus*, containing a very diſtinct account thereof. *Gellibrand* had been famous, for the part he bore in the *Trigonometria Britannica* of his deceaſed friend Mr. *Briggs*, which was printed in 1633, at *Tergou*, under the care of *Adrian Vlacq*. *Gellibrand* alſo, in 1635, publiſhed in *Engliſh* an *Inſtitution Trigonometricall*.

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* This Ruler is ſo conſtantly in the practice of our artiſts, that it has got the name of *The Gunter*.

† The uſes of a Sector had been ſhewn by Dr. *Robert Hood*, in a tract he publiſhed in 1598.

‡ This ingenious perſon died 1626, aged 45 years. His works have been ſeveral times reprinted with ſucceſſive additions; the ſecond edition was made in 1636 from his own manuſcript; then from thoſe of Mr. *Samuel Foſter* Profeſſor of Aſtronomy at *Greſham* College, again by Mr. *Henry Bond*, and Mr. *William Leybourn*. The fulleſt and laſt, being the fifth, was in 1673.

‖ Theſe were afterwards printed at *London* in *Engliſh* with improvements.

In 1631 Mr. *Richard Norwood* had publiſhed an excellent treatiſe of *Trigonometry*, adapted to the invention of logarithms, particularly in applying *Napier*'s general canons [†]. The author having, as he ſays, acquired his knowledge in the mathematics at ſea [‡], eſpecially ſhewed the uſe of trigonometry in the three principal kinds of navigation. And towards the farther improvement of that art, he undertook a laborious work for examining the diviſion of the log-line.

As altitudes of the Sun are taken on ſhip-board, by obſerving his elevation above the viſible horizon; to collect from thence the Sun's true altitude with correctneſs, *Wright* obſerves it to be neceſſary, that the dip of the horizon below the obſerver's eye ſhould be brought into the account, which cannot be calculated without knowing the magnitude of the earth. Hence he was led to propoſe different methods for finding this; but complains, that the moſt effectual was out of his power to execute; and therefore contented himſelf with a rude attempt, in ſome meaſure ſufficient for his purpoſe: and the dimenſions of the Earth deduced by him correſponded ſo well with the uſual diviſions of the log-line, that as he wrote not an expreſs treatiſe on navigation, but only for the correcting ſuch errors, as prevailed in general practice, the log-line did not fall under his notice. But Mr. *Norwood*, for regulating this inſtrument upon genuine principles, put in execution the method Mr. *Wright* recommends, as the moſt perfect for meaſuring the dimenſion of the Earth, with the true length of the degrees of a great circle upon it; and, in 1635, actually meaſured the diſtance between *London* and *York*; from whence, and the ſummer-ſolſtitial altitudes of the Sun obſerved on the meridian at both places, he found a degree on a great circle of the Earth to contain 367196 *Engliſh* feet, equal to 57300 *French* fathoms or toiſes, which is very exact; as appears from many meaſures, that have been made ſince that time.

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[*] In the Hiſtory of the Royal Academy of Sciences at *Paris* for 1712, p. 19. it is ſaid by M. *de Fontenelle*, that the learned *Peter Gaſſendi* was the principal diſcoverer of this property; but *Gaſſendi* himſelf acknowledged that he had before received information of *Gellibrand*'s diſcoveries. *Gaſſend*. Oper. vol. ii. p. 152, *Lugd*. 1658.

[†] A very advantageous report of it was made by M. *Mariotte* at a meeting, in 1668, of the Academy. *Du Hamel*, Hiſt. Acad. Scient. p. 51. 1701.

[‡] From a ſailor he became a teacher, ſtyling himſelf before his books, A Reader of the Mathematics in *London*.

Of this affair Mr. *Norwood* gives a full and clear account in his treatiſe, called *The Seaman's Practice*, firſt publiſhed in 1637. There, with unaffected modeſty, he apologizes for the hardineſs of a private perſon's undertaking ſo difficult a taſk; and very cautiouſly points out the true reaſon, how ſo great a mathematician as *Snellius* had failed in his attempt. He alſo ſhews various uſes of his diſcovery, particularly for correcting the groſs errors hitherto committed in the diviſions of the log-line. But ſuch neceſſary amendments have been little attended to by the ſailors, whoſe obſtinacy in adhering to inveterate miſtakes has been always complained of by the beſt writers on navigation. This improvement has at length, however, made its way into practice: few navigators of reputation uſing now the old meaſure of 42 feet to a knot.

Farther, Mr. *Norwood* likewiſe there deſcribes his own excellent method of *ſetting down and perfecting a Sea-Reckoning*, uſing a traverſe-table, which method he had followed and taught for many years; and beſides, ſhews how to rectify the courſe, by the variation of the compaſs being conſidered; as alſo how to diſcover currents, and to make proper allowance on their account.

This treatiſe, and that of *Trigonometry*, were continually reprinted, as the principal books for learning ſcientifically the art of Navigation. What he had delivered, eſpecially in the latter of them, concerning this ſubject, was contracted as a manual for ſailors, in a very ſmall piece, called his *Epitome*, which uſeful performance has gone through numberleſs editions.

No alterations were ever made in *The Seaman's Practice*, till in the 12th edition, printed in 1676, after the author's deceaſe, there began to be inſerted, at page 59, the following paragraph in a ſmaller character [*About the year* 1672, *Monſieur* Picart *has publiſhed an account in* French, *concerning the meaſure of the Earth, a breviate whereof may be ſeen in the* Philoſophical Tranſactions, N° 112, *wherein he concludes one* degree *to contain* 365184 Engliſh *feet*, *nearly agreeing to Mr.* Norwood'*s experiment.*] And this advertiſement is continued in the ſubſequent editions, as I find it in one printed ſo lately as 1732.

*Norwood*'s meaſure therefore, though it was not known to the great Sir *Iſaac Newton* in his youth, was not buried in oblivion, on account of the confuſions occaſioned by our civil wars, as M. *de Voltaire* has been pleaſed to ſay *; on the contrary, it has been conſtantly commended by our writers on navigation: as by Mr. *Henry Bond*, ſoon after its publication, in a note at page 107 of the *Seaman's Kalendar*, which ancient book he reprinted and improved, whoſe uſe, through numberleſs editions, is continued amongſt our ſailors to this day; by Mr. *Henry Phillips* in his *Geometrical Seaman* in 1652, and in his *Advancement of Navigation* in 1657; by Mr. *John Collins* in his *Navigation by the Plane Scale*, in 1659; by the reverend Dr. *John Newton* in his *Mathematical Elements*, in 1660; Mr. *John Seller* in his *Practical Navigation*, in 1669; Mr. *John Brown* in his *Triangular Quadrant* in 1671.

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* *Elemens de la Philoſophie de* Newton, chap. xviii. printed at *Paris* in 1738.

And in the *Philoſophical Tranſactions* for 1676, N° 126, there is given a very particular account of it. Nor had it eſcaped the royal notice; for when King *James*, in 1690, honoured the obſervatory at *Paris* with a viſit, he informed the gentlemen, then preſent, of this meaſure of the Earth; and upon their acquainting his Majeſty how that had been determined by Mr. *Picard*, the King wiſhed the two meaſures might be compared together *.

But that it was not commonly known in *France* is no wonder, ſeeing our books were not then ſo much inquired after as at preſent by that polite and learned people.

In the *Journal des Sçavans* for *December* 1666, it was obſerved of Dr. *Hooke's Micrographia, qu'il eſt écrit en une langue que peu de perſonnes entendent*; but long after, in the ſame *Journal* for *February* 1750, it is ſaid of the *Engliſh* tongue, that it was *une langue que tous les vrais ſavans devroient ſavoir.* And now, as *Norwood* is taken notice of in the latter editions of Sir *Iſaac Newton's Principia*, his name and merit indeed are become univerſally known. Inſomuch that a particular account of his meaſure is given by M. *de Maupertuis*, in the preface to his Treatiſe of the figure of the Earth, printed at *Paris* in 1738; wherein he deſcribes his method of determining the length of a degree on the Earth in *Lapland*; and *Norwood* is mentioned by two learned *Spaniſh* ſea officers, *D. Jorge Juan*, and D. *Antonio d'Ulloa*, in their voyage printed at *Madrid* in 1748, which was undertaken, as they were appointed to accompany the *French* mathematicians, ſent to meaſure a degree near the equator.

About the year 1645 Mr. *Bond* publiſhed in *Norwood's Epitome* a very great improvement in *Wright*'s method, by a property in his meridian line, whereby its diviſions are more ſcientifically aſſigned, than the author himſelf was able to effect; which was from this theorem, That theſe diviſions are analogous to the exceſſes of the logarithmic tangents of half the reſpective latitudes augmented by 45 degrees above the logarithm of the radius.

This he afterwards explained ſomewhat more fully in the third edition of *Gunter*'s works, printed in 1653, where, after obſerving that the logarithmic tangents from 45° upwards increaſe in the ſame manner (as he expreſſes it) that the ſecants added together do, if every half degree be accounted as one whole degree of *Mercator*'s meridional line; his rule for computing the meridional parts appertaining to any two latitudes (ſuppoſed on the ſame ſide of the equator) is laid down to this effect; To take the logarithmic tangent, rejecting the radius, of half each latitude augmented by 45 degrees, and dividing the difference of thoſe numbers by the logarithmic tangent of 45° 30′, the radius being likewiſe rejected, and the quotient will be the meridional parts required, expreſſed in degrees. And this rule is the immediate conſequence from the general theorem, That the degrees of latitude bear to 1 degree (or 60 minutes, which in *Wright*'s table ſtands as the meridional parts for 1 degree) the ſame proportion as the logarithmic tangent of half any latitude augmented by 45 degrees, and the radius neglected, to the like tangent

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* *Du Hamel*, Hiſt. Academ. Regal. Scient. p. 285.

of

of half a degree augmented by 45 degrees, with the radius likewiſe rejected.

But here was farther wanting the demonſtration of this general theorem, which was at length ſupplied by that great mathematician, Mr. *James Gregory* of *Aberdeen*, in his *Exercitationes Geometricæ*, printed at *London* in 1668; and ſince more conciſely demonſtrated, together with a ſcientific determination of the diviſor, by Dr. *Halley*, in the *Philoſophical Tranſactions* for 1695, N° 219, from the conſideration of the ſpirals into which the rhumbs are transformed in the ſtereographic projection of the ſphere upon the plane of the equinoctial; which the excellent Mr. *Roger Cotes* has rendered ſtill more ſimple, in his *Logometria*, firſt publiſhed in the *Philoſophical Tranſactions* for 1714, N° 388.

It is moreover added in *Gunter*'s book, that if $\frac{1}{20}$ of this diviſor (which does not ſenſibly differ from the logarithmic tangent of 45° 1′ 30″ curtailed of the radius) be uſed, the quotient will exhibit the meridional parts expreſſed in leagues: and this is the diviſor ſet down in *Norwood*'s *Epitome*.

After the ſame manner the meridional parts will be found in minutes, if the like logarithmic tangent of 45° 0′ 30″ diminiſhed by the radius be taken, that is, the number uſed by others * being 12633, when the logarithmic tables conſiſt of eight places beſides the index.

This Mr. *Bond*, who introduced ſo uſeful a diſcovery into the art, was a teacher of the mathematics in *London*, and employed to take care of and improve the impreſſions of the current treatiſes of navigation. In an edition of the *Seaman's Kalendar*, p. 103, he declared, he had diſcovered the longitude, by having found out the true theory of the magnetic variation; and to gain credit to his aſſertion, he foretold, that at *London* in 1657 there would be no variation of the compaſs, and from that time it would gradually increaſe the other way, which happened accordingly. Again, in the *Philoſophical Tranſactions* for 1668, N° 40, he publiſhed a table of the variations for 49 years to come.

This joyful news to all ſailors acquired Mr. *Bond* a great reputation; inſomuch that the treatiſe he had compoſed, called *The Longitude found*, was in 1676 publiſhed by the ſpecial command of King *Charles* the Second, and uſhered into the world with the approbation of ſeveral of the moſt eminent mathematicians of that time †.

But it was ſoon oppoſed, there being publiſhed at *London* a book in 1678, called *The Longitude not found*, written by one Mr. *Beckborrow*. And indeed as *Bond*'s hypotheſis did not in any wiſe anſwer its author's ſanguine expectations, the famous Dr. *Halley* again undertook this affair; and from a multitude of obſervations he would conclude, that the magnetic needle was influenced by four poles. His ſpeculations on this ſubject are delivered in the *Philoſophical Tranſactions* for 1683, N° 148, and for 1692, N° 195. But this wonderful *phenomenon* ſeems to have hitherto eluded all our reſearches.

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* See Mr. *Perkins*'s *Treatiſe of Navigation* in Vol. I. of Sir *Jonas Moore*'s *New Syſtem of the Mathematicks*, p. 203, printed at *London* in 1681. *Perkins*'s book was publiſhed by itſelf the year following, under the title of the *Seaman's Tutor*.

† In the *Philoſophical Tranſactions* for the ſame year, N° 130, it is ſaid, the Lord *Brounker*'s name was inſerted by miſtake.

However, that excellent perſon in 1700 publiſhed a general map, on which were delineated curve lines expreſſing the paths, where the magnetic needle had the ſame variation. This was received with univerſal applauſe *, as it may lead to ſome diſcovery in ſo abſtruſe an affair, and at preſent be uſeful on many occaſions in determining the longitude. The poſitions of theſe curves will indeed continually ſuffer alterations; but then they ſhould be corrected from time to time; as they have been for the year 1744, and 1756, by two ingenious perſons, Mr. *William Mountaine* and Mr. *James Dodſon*, Fellows of the Royal Society. The latter died not long after he had been choſen, for his merit, mathematical maſter, at *Chriſt's Hoſpital*, in *London*.

Dr. *Halley* alſo gave, in the *Philoſophical Tranſactions* for 1690, N° 183, a diſſertation on the monſoons, containing many obſervations very uſeful for all ſuch as ſail to places that are ſubject to thoſe winds.

The true principles of navigation having been ſettled by *Wright*, *Norwood*, and *Bond*, many authors amongſt us trod in their ſteps, making ſome little improvements. It would be impoſſible to enumerate each particular. Of the writers already mentioned, *Phillips* and *Collins*, in the title pages of their books, declare what they aimed at; *Phillips* alſo, in his tract called the *Advancement of Navigation*, recommends a pendulum inſtead of a half minute glaſs, to eſtimate the time the log-line is running out. He alſo propoſes to do the ſame thing by wheel-work. Beſides, in the *Philoſophical Tranſactions* for 1668, N° 34, he delivers a better method to determine the tides than what was commonly practiſed; for which purpoſe Mr. *John Flamſteed*, the Royal Aſtronomer, ſtill gave more perfect directions in the ſame *Tranſactions* for 1683, N° 143; as likewiſe he firſt ordered a glaſs lens to be fixed on the ſhade vane, in what is called *Davis*'s quadrant †, which contrivance Dr. *Robert Hook*, Profeſſor of Geometry at *Greſham* College, had before thought of ‡.

*Seller's Practical Navigation*, though without demonſtrations, has the rules of ſailing in the different kinds, as performed by calculation, by the plane ſcale, by the *Gunter*, and by the ſinical quadrant, with various other matters relative to the art; as alſo the uſe of the azimuth-compaſs as now modelled, the ring-dial, the ſea-ring, croſs-ſtaff, *Davis*'s quadrant, plough, nocturnal, inclinatory needle and globe, together with all the neceſſary tables; the whole being delivered in a manner ſo well adapted to the general humour of mariners, that it has undergone numberleſs editions: the laſt, I have ſeen, was in 1739; but ſome late writers ſeem to have abated the run of this book.

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* It is particularly commended in the Hiſtory and Memoirs of the Royal Academy of Sciences at *Paris*, for the year 1701, 1705, 1706, 1708, and 1710. See alſo Mr. *Robins*'s Reflections in his Introduction to Lord *Anſon*'s Voyage round the World, made in 1743, &c. as alſo in the ninth chapter of the firſt book, and eighth of the third.

† See the above-mentioned *Perkins*'s Navigation, page 250.

‡ See Biſhop *Thomas Sprat*'s excellent Hiſtory of the Royal Society in 1667, page 246; and *Hook*'s Poſthumous Works, publiſhed by *Richard Waller*, Eſq. in 1705, p. 557.

As in ſailing eſpecial regard ought to be had to the lee-way a ſhip makes, ſo many authors have touched upon this point; but the allowances uſually made on that account are very particularly ſet down by Mr. *John Buckler*, and publiſhed in a ſmall tract firſt printed in 1702, intitled, *A New Compendium of the whole Art of Navigation*, written by Mr. *William Jones*.

We ought not here to paſs over in ſilence the very uſeful invention of Dr. *Gowin Knight*, which is the making *artificial magnets*, that are of greater efficacy than the natural ones. Though the Doctor has not thought fit to reveal his ſecret; yet others have found it out, who have made it public, particularly the Rev. *John Mitchel*, and Mr. *John Canton*; the firſt in a treatiſe of *Artificial Magnets*, printed in 1750; the other in the *Philoſophical Tranſactions*, vol. XLVII. *Ann.* 1751.

The Earth being now univerſally agreed to be not a perfect globe, but a ſpheroid, whoſe diameter at the poles is ſhorter than any other; the Rev. Dr. *Patrick Murdoch* publiſhed a tract in 1741, where he accommodated *Wright*'s ſailing to ſuch a figure; and Mr. *Colin Maclaurin*, the ſame year, in the *Philoſophical Tranſactions*, N° 461, gave a rule to determine the meridional parts of a ſpheroid, which ſpeculation he farther treats of in his book of *Fluxions*, printed at *Edinburgh*, in 1742.

Though Sir *Iſaac Newton* in his *Principia*, firſt printed in 1686, had demonſtrated from the theory of gravity, that this muſt be the real form of the Earth, as it revolved about an axis; yet in the year 1718 M. *Caſſini* again * undertook from obſervations to ſhew the contrary, and that the earth was a ſpheroid, having its longeſt diameter paſſing through its poles †; and in 1720 M. *de Mairan* advanced arguments, ſuppoſed to be ſtrengthened by geometrical demonſtrations, to confirm farther M. *Caſſini*'s aſſertion. But in the *Philoſophical Tranſactions* for 1725, N° 386, 387, 388, Dr. *Deſaguliers* publiſhed a diſſertation, wherein he made appear the weakneſs of the reaſoning, and the inſufficiency of the obſervations, as they were managed, to ſettle ſo nice an affair. He there alſo propoſed a proper method for adjuſting this point, when he ſays, *If any conſequence of this kind could be drawn from actual meaſuring, a degree of latitude ſhould be meaſured at the equator, and a degree of longitude likewiſe meaſured there; and a degree very northerly, as for example, a whole degree might be actually meaſured upon the* Baltic *ſea, when frozen, in the latitude*

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* In the Memoirs of the Royal Academy of Sciences at *Paris*, his father in 1701, and he in 1713, attempted to prove the Earth was an oblong ſpheroid.

† M. *John Bernouilli* in his *Eſſai d'une Nouvelle Phyſique Celeſte*, printed at *Paris* in 1735, triumphs over Sir *Iſaac Newton*; vainly imagining theſe precarious obſervations could invalidate what Sir *Iſaac* had demonſtrated.

*of ſixty degrees.* There, according to M. *Caſſini's laſt ſuppoſition, a degree would be* 56653 *toiſes*; *whereas at the equator it would be of* 58019 *toiſes, the difference being* 1366 *toiſes, about the two and fortieth part of a degree, which muſt be ſenſible*; *and likewiſe the degree of longitude would according to him be of* 56817 *toiſes, leſs by* 1202, or the *forty-eighth part, than a degree of* latitude at the ſame place.

On this admonition, in 1735, there were ſent from *France* two ſets of mathematicians, members of the Royal Academy of Sciences; one towards the pole, the other to the equator, in order to meaſure, at each place, the length of a degree on the meridian. The report they brought home, quite overſet what had been urged in favour of the oblong figure; a degree towards the north, in the latitude of 66° 20′, being found to contain about 57438 toiſes, and near the equator but 56750.

This unwelcome news cauſed a degree to be again meaſured in *France*, which at length came out to be conſonant with thoſe which had been brought from very diſtant parts of the world. Thus theſe mathematicians confirmed by painful obſervations, what Sir *Iſaac Newton* had, as M. *de Maupertuis* uſed to ſay, determined in his elbow-chair; Sir *Iſaac* making the length of a degree under the pole to be 57382, and at the equator 56637 toiſes. And perhaps no obſervations can be exact enough to determine this matter more preciſely.

But let us mention ſome of the foreign writers on navigation.

At *Rome*, in 1607, came forth a treatiſe, intitled, *Nautica Mediterranea*, written in *Italian* by *Bartolomew Creſcenti*, the Pope's engineer. The author miſſes no opportunity of expoſing the errors of *Medina*; but ſcarce gives any thing of his own, except a machine for meaſuring the way a ſhip made.

As the Jeſuits have treated of moſt branches of learning, ſo this art has not been beneath their conſideration; the three following authors having been of their ſociety.

At *Paris*, in 1633, Father *George Fournier*, publiſhed an *Hydrography*, principally relating to navigation. The author would perſuade us, that one of *Dieppe* had corrected the plane chart; and that the *Hollanders* learnt of the *French* the making charts ſo corrected; whereas this had been engraved long before at *Amſterdam*, by *Iodocus Hondius*, and others.

*John Baptiſt Riccioli*, in his *Geographia & Hydrographia Reformata*, printed at *Bologna* in 1661, inſerts a treatiſe of navigation, collecting his materials from almoſt every writer, as he does in his *Almageſt* and *Chronology*, which is indeed the chief merit of his works.

Father *Millet Dechalles* wrote on this ſubject after a more maſterly manner, both in his *Curſus Mathematicus*, firſt printed at *Lyons* in 1674, and in a *French* treatiſe, publiſhed in 1677, intitled *L'Art de Naviger demontré par Principes.*

Theſe three authors, beſides treating of the different kinds of ſailing, abound in methods for taking of altitudes, finding the variation, and eſtimating the way a ſhip makes, &c. They alſo deſcribe a machine reſembling that of *Creſcenti*. *Riccioli* gives a very faulty meaſure of the Earth, made by himſelf; and *Dechalles* adviſes the uſe of a pendulum in reckoning by the log-line, as alſo of wheel-work for the ſame purpoſe, as *Phillips* and *Cole* had done.

But there were writers in *France* between *Fournier* and *Dechalles*. For in 1666, and the following years, there were printed at *Dieppe* ſeveral tracts handling different parts of navigation, compoſed by M. *G. Denys*, which have been often reprinted.

And in 1671 the Sieur *Blondel S. Aubin* publiſhed a book called, *L'Art de Naviger par le Quartier de Reduction*, deſcribing an inſtrument * much in uſe amongſt the *French* ſailors, by which may be performed, as by the ſinical quadrant, the operations of navigation, though not much more ſpeedily than by the traverſe table, and not at all ſo accurately. He alſo publiſhed in 1673 his *Treſor de la Navigation*, where the art is well treated of, particularly by calculations.

M. *Saverien*, in his *Marine Dictionary*, printed at *Paris* in 1758, ſays, that M. *Daſſier* ſeems to have been the firſt of the *French* writers that ſhewed the uſe of *Gunter*'s ſcales [*échelles Angloiſes*] in his *Pilote expert*, printed in 1683.

At *Paris*, ten years after, was publiſhed the firſt part of a pompous work, intitled, *Le Neptune François*, by order of the *French* king, conſiſting of ſea-charts, according to *Wright*'s ſcheme, made from the lateſt obſervations, and reviewed by Meſſ. *Pene*, *Caſſini*, and others. As this contained the charts of *Europe* only, there were added others of different parts of the world, printed the ſame year at *Amſterdam*. The whole was preceded by a diſcourſe of M. *Sauveur*, who had formed ſome of the charts, where he ſhews how to perform the problems of aſtronomy and navigation by ſcales; which diſcourſe had been publiſhed by itſelf at *Paris*, in 1692.

M. *John Bouguer* compoſed, by authority, his *Traité Complet de la Navigation*, firſt printed in 1698, which was well received, as containing moſt of the practices then known; and Father *Pézenas*, Jeſuit and Royal Profeſſor of Hydrography at *Marſeilles*, publiſhed there, in 1733, a tract, called, *Elemens de Pilotage*; and at *Avignon*, in 1741, a larger work, intitled, *Practique du Pilotage*. This author ſhews how to find the meridional parts by the *Artificial Tangents*, an old diſcovery amongſt us, declared ſo long ago as 1645, in *Norwood's Epitome*; he alſo has been induſtrious in tranſlating ſeveral of our mathematical books into *French*.

But in 1753 M. *Peter Bouguer*, ſon of the former, publiſhed a very elaborate treatiſe on this ſubject, intitled, *Nouveau Traité de Navigation*, which is written ſenſibly, the author being an excellent mathematician, and famous for other productions. He there gives a variation-compaſs † of his own invention, and attempts to reform the log, as he had done in the Memoirs of the Academy of Sciences for 1747. He is alſo very particular in determining the lunations more accurately than by the common methods, and in deſcribing the corrections of the dead reckonings.

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* It is only a kind of ſkeleton of *Wright*'s univerſal map.

† Many of theſe ſorts of compaſſes have been propoſed at different times, as by M. *Buache*, in the Memoirs of the *French* Academy of Sciences for 1752, page 377; Captain *Chriſtopher Middleton*, in the Philoſophical Tranſactions, N° 450, *Ann.* 1738; and Dr. *Knight*, as improved by the ingenious Mr. *John Smeaton*, *ibid.* N° 495, *Ann.* 1750.

The excellent aſtronomer, M. *de la Caille*, in 1760, made an edition of M. *Bouguer*'s book, which he ſomewhat abridged and improved.

In 1766, came out at *Paris*, a treatiſe, with this title, *Abregé du Pilotage diviſé en deux parties, ou on traite principalement des Amplitudes, des Loxodromies, dans l'hypotheſe de la Sphere et de Spheroide, des marées, des variations de l'aiman.*

The former part of this book was firſt publiſhed in 1693. Here the whole is improved by M. *le Monnier.*

Though the *Spaniards* were the earlieſt writers on navigation, yet they were very backward to adopt its improvements. Indeed *Antonio de Naiera* publiſhed at *Liſbon*, in 1628, a treatiſe, intitled, *Navegacion eſpeculativa y practica*; where, though the author rectifies the tables of the Sun and fixed ſtars, from *Tycho Brahe*'s obſervations, he proceeds no farther in the theory of navigation than what had been advanced by *Nonius*, as followed by *Ceſpedes.* But of late, in 1712, was printed at the ſame place, *Arte de Navegar por Manuel Pimental*; where is ſhewn the uſe of *Wright*'s chart, which, in imitation of the *French*, the author calls *Charta Reduzida.* He likewiſe deſcribes *Davis*'s quadrant, and mentions *Norwood* and *Picard*'s meaſures of the Earth. In 1757 a treatiſe was printed at *Cadiz*, intitled, *Compendio de Navigacion para el uſo de los Cavelleros Guardias Marinas*, written by the ingenious gentleman mentioned above, Don *Jorge Juan.* This is a good performance, delivering very diſtinctly the ſeveral parts of the art, as now improved. Some things are here omitted, that uſually occur in books on this ſubject; but for the knowledge of ſuch particulars, references are made to tracts compoſed expreſly for the uſe of the ſociety of gentlemen, deſtined for the ſea-ſervice.

*Bouguer* and *Jorge Juan*, deſcribe and commend the method of dividing inſtruments for taking of angles, publiſhed by *Peter Vernier*, in a treatiſe, intitled, *La Conſtruction, &c. du quadrant nouveau*, printed at *Bruſſels*, in 1631. This diviſion is an improvement of that of *Curtius*, as that of *Fererius* is of the diviſion by diagonals *, and readily follows from the firſt *Lemma* of *Clavius*'s treatiſe on the *Aſtrolabe* †, as has been obſerved by *Pézenas*, in a book he publiſhed at *Avignon* in 1765, intitled, *Aſtronomie des Marins.*

As to their treating of *Wright*'s chart, I mentioned above *Snellius* and *Metius.* To an edition, in 1665, of *Vlacq*'s ſmall tables of logarithms, &c. is added, by *Abraham de Gruel*, one of meridional parts, whoſe uſe he ſhews, with other parts of navigation, in his *Courſe of Mathematics*, written in *Dutch*, and printed at *Amſterdam* in 1676, as had been done by *John Virst*, in his *Flambeau reluiſſant où Threſor de la Navigation*, at the ſame place, in 1677.

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* See Mr. *Robins*'s Mathematical Tracts, where theſe diviſions are largely treated of.

† Firſt printed at *Rome*, in 1693.

The *Dutch* are great navigators, and have been famous for their *Atlaſſes*, before which are premiſed treatiſes of navigation, as has been already obſerved. The oldeſt I have ſeen of theſe, was publiſhed at *Leyden* in 1584, intitled, *Spiegel der Zee-Vaert* (or *Mirror of Navigation*,) by *Lucas Janſz Waghenaer*. In their later *Atlaſſes* there is deſcribed an inſtrument to be uſed after the manner of *Davis*'s quadrant, but where inſtead of circular arches are ſubſtituted ſtraight lines.

Notwithſtanding all the improvements hitherto mentioned, the *ſea-reckonings*, though kept by ſuch as were deemed very ſkilful mariners, are often found widely different from the truth. But this often happens through negligence, as I have heard Dr. *Halley*, who had uſed the ſea, ſay.

Theſe errors would be avoided, if from time to time the latitude and longitude could be determined. The firſt is generally obtained by the meridian altitude and declination of the Sun being given. The declination is got by the help of tables of the Sun, with an eaſy trigonometrical operation.

But even the latitude could not be very exact, before the famous *Kepler* had determined the true form of the Earth's orbit [*]. Hence were fabricated his *Tabulæ Rudolphinæ*. Next, thoſe of Mr. *Thomas Street* were in great requeſt [†]. But they, in their turn, yielded to Dr. *Halley*'s, and his again to thoſe of the accurate and elaborate *Mayer*; which, however, will want to be corrected hereafter: For, as Sir *Iſaac Newton* has ſhewn, that all bodies mutually attract one another, the Earth will be diſturbed in its motion by the actions of ſome of the other planets.

To find the longitude is a much more difficult affair. For this end, at preſent, the ſocieties of learned men in *Europe* offer from time to time rewards to ſuch as ſhall beſt treat of particular ſubjects in mathematicks or phyſicks. Some of theſe have been relating to navigation, when *Polleni*, *Bernouilli*, *Bouguer*, and others have obtained the prizes. And it is hoped, this inſtitution may contribute to the advancement of the art.

Eclipſes of the moon were uſed of old; and *Kepler* recommended thoſe of the Sun as preferable [||].

The ſatellites of *Jupiter* were no ſooner diſcovered by the great *Gallileo* [§], than the frequency of their eclipſes recommended them for this purpoſe; and amongſt thoſe who attempted this ſubject, none were more ſucceſsful than Signor *Dominic Caſſini*.

This great aſtronomer in 1688 publiſhed at *Bologna* tables for calculating the appearances of their eclipſes, with directions for finding thence the longitudes of places; and being invited to *France* by *Lewis* the Fourteenth, he there publiſhed correcter tables in 1693. But the mutual attractions of the ſatellites on one another rendering their motions exceſſively irregular, the tables ſoon run out; inſomuch that they require to be renewed from time to time, which has been performed by ingenious perſons, as Dr. *James Pound*, Dr. *James Bradley*[*], M. *Caſſini* the ſon, and M. *Peter Wargentin*[†]; ſo that now many of the common *Almanacs* ſet down, when theſe eclipſes happen throughout the year.

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[*] In his treatiſe *de Motu Martis*, in 1609.

[†] In his *Aſtronomia Carolina*, in 1661.

[||] *Tabulæ Rudolph.* printed at *Ulm* in 1627, cap. xvi. & xxxi.

[§] In his *Sydereus Nuncius*, firſt printed at *Venice* in 1610.

The Rev. *Nevil Maſkelyne*, D. D. our preſent Royal Aſtronomer, has publiſhed annually, ſince the year 1767, by order of the Commiſſioners of Longitude, a work entitled, *The Nautical Almanac and Aſtronomical Ephemeris*, containing not only the eclipſes of the ſatellites, but alſo many other tables, to enable the mariner to determine the longitude at ſea; particularly tables of the diſtances which the moon's center will have from that of the Sun, and from fixed ſtars, at every three hours, under the meridian of the Royal Obſervatory at *Greenwich*, and which have ſince beeen copied into the *Connoiſance des Temps* for theſe latter years by the editor of that work.

The large reward granted by the *Parliament* for a practical way of diſcovering the longitude at ſea, has put many upon the ſearch: inſomuch that ſeveral idle and abſurd ſchemes have been offered by ignorant and wrong-headed men. But the perfecting the methods propoſed long ago by *John Werner* and *Gemma Friſius*, ſeems at preſent to engage the attention of the public.

The theory of the moon, though much amended by the noble *Tycho Brahe* and Mr. *Jeremy Horrox*[‡], was found to be inſufficient to anſwer this end. But the cauſes of her various irregularities having been diſcovered by Sir *Iſaac Newton*, and her theory thence improved beyond expectation, gave great hopes of ſucceſs; which have ſince been happily fulfilled by means of the improvements which have ſince been made in the methods of computing the ſeveral quantities of theſe inequalities by M. *Euler*, and *Tobias Mayer* of *Gottingen*[§]: The former of theſe gentlemen having been happy in reducing Sir *Iſaac Newton*'s theory into neat analytical expreſſions, of which the latter availing himſelf, was, by a very ſingular addreſs of his own, enabled to bring out the greateſt quantities of the equations with eaſe and exactneſs, and thence to conſtruct tables agreeing to the moon's motion in every part of her orbit, with very ſurpriſing exactneſs. And this ingenious perſon has left behind him tables ſtill more exact *, for which the *Britiſh* Parliament have rewarded his widow with £. 3000, as alſo Mr. *Euler* with £. 300. Theſe tables were publiſhed in 1770, by Dr. *Maſkelyne*.

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[*] He ſucceeded Dr. *Halley* at *Greenwich*, where he made a great number of Aſtronomical Obſervations, which, as they are moſt accurate, it is hoped will not be loſt. He became famous on obſerving and accounting for an apparent motion in the fixed ſtars, and called their aberration, which was immediately exhibited by the great mathematician Dr. *Brook Taylor* according to the exact theory of the Earth's motion. See Mr. *Robins*'s Mathematical Tracts, vol. II. page 276.

[†] *Wargentin*'s tables are much eſteemed; they were firſt publiſhed at *Stockholm* in the *Acta Societatis Regii Scientiarum Upſalenſis* for the year 1741, but ſince more correct from a new copy of the author's at *Paris* in 1759, by M. *de la Lande*. The ingenious author has rendered them yet more correct, and his labours on this head may be ſeen in the *Connoiſance des Temps* for 1766, and the *Nautical Almanacs* for 1771, and 1779.

[‡] This great genius died in 1641, ſcarce 23 years old. See his *Opera Poſthuma*, publiſhed by the famous Dr. *John Wallis* at *London*, in 1673. *Horrox* firſt obſerved the Tranſit of *Venus* over the Sun in 1639. He wrote an account of this *Phenomenon*, which was publiſhed by the great aſtronomer *Hevelius*, at *Dantzic*, in 1661.

[§] *Com. Societ. Reg. Gottingenſ.* tom. II. page 283.

As to the method of *Gemma Friſius*, M. *Huygens* was perſuaded it might be accompliſhed by his inventions of pendulum clocks and watches; a deſcription of the firſt he publiſhed in a ſmall tract, printed at the *Hague*, in 1658; and of the ſecond, as improved, in the *Journal des Sçavans* for the month of *February*, 1675. And great expectations of ſucceſs had been raiſed from ſome trials made in a voyage with theſe watches of the firſt conſtruction, by Major *Holmes*; an account whereof is given in the Philoſophical Tranſactions, *Ann.* 1669. But the various accidents thoſe movements are liable to, ſoon cauſed that way to be laid aſide.

Notwithſtanding which, the ingenious Mr. *John Harriſon* has for many years paſt employed himſelf in contriving a machine, that ſhall be free from all imaginable inconveniencies; and his endeavours were ſo well approved of by gentlemen of the greateſt knowledge in theſe ſubjects, that the commiſſioners for the longitude thought fit to allow him ſome gratifications for his pains. He was afterwards farther conſidered, upon diſcloſing the internal ſtructure of his machine, and the whole reward has ſince been given him by Parliament.

The difficulty of making obſervations at ſea with ſufficient exactneſs for finding the longitude, was feared to be inſurmountable; but attempts have not been wanting to overcome it. In the Hiſtory of the Royal Society, at page 246, we meet with the firſt mention of an invention in theſe words: *A new inſtrument for taking angles by reflection, by which means the eye at the ſame time ſees the two objects both as touching the ſame point, though diſtant almoſt to a ſemicircle; which is of great uſe for making exact obſervations at ſea.* A figure of this inſtrument, drawn by Dr. *Hook*, the inventor, is given in the Doctor's poſthumous works, with a deſcription, at page 503. But here, as one reflection only was made uſe of, it would not anſwer the purpoſe. However, this was at laſt effected by Sir *Iſaac Newton*, who communicated to Dr. *Halley*, about the year 1700, a paper of his own writing, containing a deſcription of an inſtrument with two reflections, which ſoon after the Doctor's death was found among his papers by Mr. *Jones*, who communicated it to the Royal Society, and it was publiſhed in the Philoſophical Tranſactions, N° 465, *Ann.* 1742.

How it happened that Dr. *Halley* never mentioned this in his lifetime, is very extraordinary; ſeeing *John Hadley*, Eſq. † had deſcribed, in N° 430, *Ann.* 1731, an inſtrument grounded on the ſame principles, which is ſo well eſteemed, that our ſhops abound with them, accommodated with *Vernier*'s diviſion, as they are made by our moſt ſkilful workmen; and are now in general uſe amongſt the ſkilful ſeamen of moſt of the maritime nations.

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* See his *Elogium* in the *Nova Acta Eruditorum*, for *March* 1762.

† Mr. *Hadley* being well acquainted with Sir *Iſaac Newton*, might have heard him ſay, *Hook*'s propoſal could be perfected by means of a double reflection. However, Mr. *Hadley*, being a very ingenious perſon, might have hit on the ſame thought; as well as Mr. *Godfrey* of *Penſylvania* to whom the invention of this admirable inſtrument has been aſcribed by ſome gentlemen of that colony: This is not the only caſe, wherein different perſons have produced ſimilar inventions.

Though *Medina*'s method for finding the place of the horizon was abſurd; yet, for this end, ſeveral plauſible ones have been propoſed by ingenious perſons, as Meſſ. *Elton*, *Hadley*, *Godfrey*, and *Leigh*; and that chiefly by applying a level to *Davis*'s quadrant. Their devices are deſcribed in the Philoſophical Tranſactions for 1732, 33, 34, and 37. And, laſtly, an *Horizontal Top*, invented by the late Mr. *Serſon*, who was unfortunately loſt at ſea aboard the *Victory* man of war, has been approved of, and publiſhed by Mr. *Smeaton* in the Philoſophical Tranſactions, vol. XLVII. for 1752, part ii. page 352.

Some methods uſed for obtaining the place of the horizon, and of obſerving with Mr. *Hadley's Reflecting Sector*, are deſcribed by Mr. *Robertſon*, in his *Elements of Navigation*; which treatiſe has deſervedly met with the approbation of the public.

Thus have I endeavoured to trace out the principal ſteps by which the art of navigation has advanced to its preſent height; nor without hopes that the attempt may not prove altogether unacceptable to thoſe whoſe buſineſs or curioſity lead them to be acquainted with this very uſeful branch of the mathematics: on the ſucceſsful practiſing of which depends, in an eſpecial manner, the flouriſhing ſtate of our country.

This Diſſertation, written at firſt by deſire, is now reprinted with alterations. Though I may be thought to have dwelt too long on ſome particulars not directly relating to the ſubject; yet I hope that what is ſo delivered, will not be altogether unentertaining to the candid reader. As to any apology for having handled a matter quite foreign to my way of life, I ſhall only plead, that very young, living in a ſea-port town, I was eager to be acquainted with an art that could enable the Mariner to arrive acroſs the wide and pathleſs ocean at his deſired harbour.

London. JAMES WILSON.

ADVER-

# ADVERTISEMENT.

AS it may be expected that four kinds of readers will look into this book, it was thought convenient to point out to ſome of them, the places where they may meet with what they more particularly want.

First. Thoſe who having made a proficiency in the mathematics, will, it is likely, examine in what manner the ſubjects are here treated, and whether any thing new is contained therein: it is conceived that ſuch readers will find ſome things which may recompence them for their trouble, in almoſt every one of the books.

Secondly. Thoſe learners, who are deſirous of being inſtructed in the art of Navigation in a ſcientific manner, and would chuſe to ſee the reaſon of the ſeveral ſteps they muſt take to acquire it: To ſuch perſons, it is recommended that they read the whole book in the order they find it; or, if the learner is very young, he may omit the IVth and Vth books till after he is maſter of the VIth and VIIth.

Thirdly. That claſs of readers, which, with too much truth may be ſaid, comprehends moſt of our mariners, who want to learn both the elements and the art itſelf by rote, and never trouble themſelves about the reaſon of the rules they work by: As it is probable there ever will be many readers of this kind, they may be well accommodated in this work; thus, if they are not already acquainted with Arithmetic and Geometry, let them read the five firſt rules of Arithmetic, to page 20; thence proceed to the definitions and problems in Geometry, from page 43 to 58. In the book of Trigonometry, read pages 89, 90, 91, 92, 98, 99, and from 104 to 114: the whole of book VI. In book the VIIth they may read to page 35, and as much more as they pleaſe. In book VIII, let them read the ſections III, IV, V, VI, from page 146 to page 182. In book V, they may read ſection III, and as many problems in the Vth and VIth ſections as they can; and let them read the whole of the ninth book.

Fourthly. That ſet of readers who will not be at the pains of learning any thing more than how to perform a day's work; ſuch may herein meet with the practice almoſt independent of other knowledge. Let ſuch perſons make themſelves acquainted with ſection IV. of book VI, and the uſe of the table at page 374; then learn the uſe of the Traverſe Table at the end of book VII, which they will find exemplified between pages 8 and 35, Vol. II; alſo they muſt learn the uſe of the Table of Meridional parts at the end of Book VIII. After which, they may proceed to book IX, where they will find ample inſtructions in all the particulars which enter into a day's work. But with this ſcanty knowledge of things, they will be obliged to omit ſome parts, which it is well worth their pains to be acquainted with.

# THE ELEMENTS OF NAVIGATION.

## BOOK I. OF ARITHMETICK.

### SECTION I.

### *Definitions* and *Principles.*

1. Arithmetick is a ſcience which teaches the properties of numbers; and how to compute or eſtimate the value of things.

2. An Unit or Unity, is any thing conſidered as one.

3. Number, in general, is many units.

4. Digits or Figures are the marks by which numbers are denoted or expreſſed, and are the nine following.

| *Digits,* | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. |
|---|---|---|---|---|---|---|---|---|---|
| *Names,* | One. | Two. | Three. | Four. | Five. | Six. | Seven. | Eight. | Nine. |

And with theſe is uſed the mark 0, called cypher, which of itſelf ſtands for nothing; but being annexed to a digit, alters its value.

*Thus* 40 *ſignifies forty*; *and* 400 *ſtands for four hundred, &c.*

5. Integer, or Whole, Numbers, are ſuch as expreſs a number of things, each of which is conſidered as an unit.

*Thus four pounds, twelve miles, thirty-four gallons, one hundred days, &c. are, in each caſe, called an integer number, or whole number.*

6. Fractional Numbers, are thoſe which expreſs the value of ſome part or parts of an unit.

*Thus one half, one quarter, three quarters, &c. are each the fractional parts of an unit.*

7. **Notation** is the expreſſing by digits or figures any number propoſed in words; and the reading of any number that is expreſſed by figures, is called **Numeration**.

8. **Decimal Notation** is that kind of numbering in which ten units of any inferior name are equal in value to an unit of the next ſuperior.

9. Every number is ſaid to conſiſt of as many places as it contains figures.

10. The value of every digit in any number is changed according to the place it ſtands in; and the reading of any number conſiſts in giving to each figure its right name and value.

11. The right hand place of an integer number is called the place of units; and from this place all numbers begin, whether *whole* or *fractional*; the integers increaſing in order from the unit place towards the left; and the fractions decreaſing in order from the unit place towards the right: and to diſtinguiſh decimal fractions from integers, there is always a point or comma (,) ſet on the left hand ſide of the fractional number; ſo that the integers ſtand on the left hand ſide of the mark, and the fractions on the right hand.

12. For the more convenient reading of numbers, they are divided into periods of ſix places each, beginning at the unit place; and each period into two degrees of three places each, the names and order of which are as follow: where X ſtands for the word tens, C for hundreds, and Th. for thouſands.

13.

| | | Integers | | | | | | | | | | | | Decimal fractions | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Second period | | | | | | First period | | | | | | First period | | | | | | Second period | | | | | | |
| | | Degree | | | Degree | | | Degree | | | Degree | | | Degree | | | Degree | | | Degree | | | Degree | | | |
| &c. | Billions | Th. Millions | Th. Millions | Th. Millions | Millions | Millions | Millions | Thouſands | Thouſands | Thouſands | Hundreds | Tens | Units | Tenths | Hundredths | Thouſandths | Thouſandths | Thouſandths | Millionths. | Millionths | Millionths | Th. Millionths | Th. Millionths | Th. Millionths | Billionths | &c. |
| | | C. | X. | | C. | X. | | C. | X. | | | | | | | | X. | C. | | X. | C. | | X. | C. | | |
| | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9, | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 5 | |

Decimal Fractions are alſo thus named, { Primes, Seconds, Thirds, Fourths, Fifths, Sixths, Sevenths, Eighths, Ninths, Tenths, Elevenths, Twelfths, &c.

The name of the firſt period is Units; of the ſecond, Millions; of the third, Billions; of the fourth, Trillions; &c.

In the above order it may be obſerved, that each degree contains the names of Units, Tens, Hundreds; the firſt degree of a period contains the units of that period, and the ſecond contains the thouſandths thereof: ſo that from hence it will be eaſy to read a number conſiſting of ever ſo many places by the following directions.

14. **Rule.** 1ſt. Suppoſe the number parted into as many ſets or degrees of three places each, beginning at the unit's place, as it will admit of; and if one or two places remain, they will be the units and tens of the next degree.

2d. Beginning at the left hand, read in each degree, as many hundreds, tens, and units, as the figures in thoſe places of the degree expreſs, adding the name thouſands, if in the ſecond degree of a period; and adding the name of the period, after reading the hundreds, tens, and units in its firſt degree.

Thus the integer number in the preceding table will be read.
*Five billions, four hundred thirty two thouſand, one hundred twenty three millions, four hundred fifty ſix thouſand, ſeven hundred eighty nine.*

15. All fractional numbers conſiſt of two parts, which are uſually written one above the other with a line drawn between them: the number below the line, called the *denominator*, ſhews into how many equal parts the unit is divided: the number above the line, called the *numerator*, ſhews by how many of theſe equal parts the value of that fraction is expreſſed.

*Thus* 9 *pence, is* 9 *parts in twelve of a ſhilling; and may be written thus,* $\frac{9}{12}$, *when a ſhilling is the unit.*

16. Thoſe fractions, the denominators of which are 10, or 100, or 1000, or 10000, or 100000, &c. are called *decimal fractions:* but fractions with any other denominators are called *vulgar fractions.*

The vulgar fractions that moſt frequently occur, are theſe:

$\frac{1}{4}$, which is read one fourth, or one quarter.
$\frac{1}{3}$ - - - - - - - one third.
$\frac{1}{2}$ - - - - - - - one half.
$\frac{2}{3}$ - - - - - - - two thirds.
$\frac{3}{4}$ - - - - - - - three fourths, or three quarters.

17. As decimal fractions are parts of an unit divided into either 10, 100, 1000, 10000, &c. parts, according to the places in the fractional number; therefore they are read like whole numbers, only calling them ſo many parts of 10, or of 100, or of 1000, &c.

| | | | |
|---|---|---|---|
| Thus a decimal fraction of | one | places, will be ſo many parts of | 10, Ten. |
| | two | | 100, Hundred. |
| | three | | 1000, Thouſand. |
| | four | | 10000, Ten Thouſand, |
| | &c. | | &c. |

18. Cyphers on the right hand of integers increaſe their value; on the left hand of a decimal fraction diminiſh its value: but on the left hand of integers, or on the right hand of fractions, do not alter their value.

| | | | | |
|---|---|---|---|---|
| *Thus* | 8 *is* 8 *units.* | *And* | ,8 *is* 8 *parts in* 10 | *of an unit ſo divided.* |
| | 80 8 *tens.* | | ,08 8 *parts in* 100 | |
| | 800 8 *hundreds.* | | ,008 8 *parts in* 1000 | |

When a fraction has no integer prefixed, it is convenient to put 0 in the place of units.

19. A MIXED NUMBER, is when a fraction is annexed to a whole number.

*Thus five and a half is called a mixed number, and is written* $5\frac{1}{2}$, *or thus,* 5,5; *which is thus read, five and five tenths.*

20. Like names in different numbers are ſuch figures as ſtand equally diſtant from the place of units; or have the ſame denomination annexed to them.

*Thus all numbers of pounds ſterling are like names, and ſo are all numbers of ſhillings; the like of any numbers of miles, &c.*

21. Beſides the decimal notation explained in article 8, there are other kinds in common uſe; ſuch as the *duodecimal*, in which every ſuperior name contains 12 units of its next inferior name: the *Sexagenary*, or *Sexageſimal*, in which ſixty of an inferior name make one of its next ſuperior. The former is uſed by workmen in the meaſuring of artificers works in building; and the latter is uſed in the diviſion of a *Circle*, and of *Time*.

22. The following characters or marks are frequently uſed in Arithmetical computations, briefly to expreſs the manner of operation.

The mark + (more) belongs to addition; and ſhews that the numbers it ſtands between are to be added together.

*Thus* $12 + 3$ *expreſſes the ſum of* 12 *and* 3; *or that* 3 *is to be added to* 12, *and is thus read,* 12 *more* 3.

The mark — (leſs) is for ſubtraction; and ſhews that the number following it, or on the right hand, is to be taken from the number preceding it, or on the left hand.

*Thus* $12 - 3$, *expreſſes the difference between* 12 *and* 3; *or that* 3 *is to be ſubtracted from* 12, *and is thus read,* 12 *leſs* 3, *or* 12 *leſſened by* 3.

This mark × (into) for multiplication, ſhews that the numbers on each ſide of it are to be multiplied the one by the other.

*Thus* $12 \times 3$, *denotes the product of* 12 *into* 3; *or that* 12 *is to be multiplied by* 3.

Diviſion is expreſſed by ſetting the diviſor under the dividend with a line drawn between them, like a fraction.

*Thus* $\frac{12}{3}$, *expreſſes the quotient of* 12 *by* 3; *or that* 12 *is to be divided by* 3.

This ſign = (equal) ſhews that the reſult of the operation by the numbers or quantities on one ſide of it, is equal either to the numbers or quantities on the other ſide, or to the reſult of the operation by theſe numbers or quantities.

*Thus* $12+3=15$; *and* $12-3=9$; *and* $12\times3=36$; *and* $\frac{12}{3}=4$; *ſeverally ſhews the value of the preceding expreſſions.*

## 23. TABLES of ENGLISH MONEY, WEIGHTS, and MEASURES.

### MONEY.

| Farthings | Pence | Shill. | Pound |
|---|---|---|---|
| 960 | 240 | 20 | 1 £. |
| 48 | 12 | 1s. | |
| 4 | 1d. | | |

*Note*, 1, 2, 3 farthings, are thus written, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.

### TROY WEIGHT.

| Grains | Pennyw^ts | Ounces | Pound |
|---|---|---|---|
| 5760 | 240 | 12 | 1 lb. |
| 480 | 20 | 1 oz. | |
| gr. 24 | 1 dwt. | | |

*Note*, Gold and Silver are weighed by Troy Weight.

### AVOIRDUPOISE WEIGHT.

| Drams | Ounces | Pounds | Hund. | Ton |
|---|---|---|---|---|
| 573440 | 35840 | 2240 | 20 | 1 |
| 28672 | 1792 | 112 | 1 C. | |
| 256 | 16 | | 1lb. | |
| 16 | 1 oz. | | | |

*Note*, Proviſions, Stores, &c. are weighed by the Avoirdupoiſe, or great weight.

### WINE MEASURE.

| Solid inch. | Pints. | Gall. | Hogſh. | Pipe | Tun |
|---|---|---|---|---|---|
| 58212 | 2016 | 252 | 4 | 2 | 1 |
| 29106 | 1008 | 126 | 2 | 1 P. | |
| 14553 | 504 | 63 | 1 Hhd. | | |
| 231 | 8 | 1 | | | |
| | | 42 | 1 Tierce. | | |
| | | 84 | 1 Puncheon. | | |

### DRY MEASURE.

| Pints | Gall. | Pecks | Buſh. | Quarter |
|---|---|---|---|---|
| 512 | 64 | 32 | 8 | 1 |
| 64 | 8 | 4 | 1 | |
| 16 | 2 | 1 | | |
| 8 | 1 | | | |

*Note*, 4 buſh. = 1 Comb: 10 qrs. = 1 Wey: 12 Weys = Laſt of Corn. 36 Buſhels = 1 Chaldron of Coals.

### CLOTH MEASURE.

4 Nails = 1 Quarter of a Yard.
4 Quarters = 1 Yard.
5 Quarters = 1 Engliſh ell.
3 Quarters = 1 Flemiſh ell.
6 Quarters = 1 French ell.
a Span = 9 inches.
a Hand = 4 inches.

### LONG MEASURE.

| Barley corns | Inches | Feet | Yards | Poles | Furl. | Mile. |
|---|---|---|---|---|---|---|
| 190080 | 63360 | 5280 | 1760 | 320 | 8 | 1 |
| 23760 | 7920 | 660 | 220 | 40 | 1 | |
| 594 | 198 | $16\frac{1}{2}$ | $5\frac{1}{2}$ | 1 | | |
| 108 | 36 | 3 | 1 | | | |
| 36 | 12 | 1 | | | | |
| 3 | 1 | | | | | |

Alſo 3 miles make 1 league.
And 20 leagues or 60 Sea miles make a degree.
But a degree contains about $69\frac{1}{2}$ miles of ſtatute meaſure.
A fathom = 6 feet = 2 yards.

### TIME.

| Seconds | Minutes | Hours | Days | Year |
|---|---|---|---|---|
| 31556937 | 525948 | 8766 | $365\frac{1}{4}$ | 1 |
| 86400 | 1440 | 24 | 1 day | |
| 3600 | 60 | 1 hour. | | |
| 60 | 1 | | | |

### Pence Table.

| Pence | Sh. Pence | Pence | Sh. Pence |
|---|---|---|---|
| 20 | 1 . 8 | 70 | 5 . 10 |
| 30 | 2 . 6 | 80 | 6 . 8 |
| 40 | 3 . 4 | 90 | 7 . 6 |
| 50 | 4 . 2 | 100 | 8 . 4 |
| 60 | 5 . 0 | 110 | 9 . 2 |

### Even parts of a Pound Sterling.

| s. d. | is | |
|---|---|---|
| 10 . 0 | $\frac{1}{2}$ | of a Pound Sterling. |
| 6 . 8 | $\frac{1}{3}$ | |
| 5 . 0 | $\frac{1}{4}$ | |
| 4 . 0 | $\frac{1}{5}$ | |
| 3 . 4 | $\frac{1}{6}$ | |
| 2 . 6 | $\frac{1}{8}$ | |
| 2 . 0 | $\frac{1}{10}$ | |

| d. | is | | or | |
|---|---|---|---|---|
| 6 | $\frac{1}{2}$ | of a Shilling. | $\frac{1}{40}$ | of a Pound. |
| 4 | $\frac{1}{3}$ | | $\frac{1}{60}$ | |
| 3 | $\frac{1}{4}$ | | $\frac{1}{80}$ | |
| 2 | $\frac{1}{6}$ | | $\frac{1}{120}$ | |
| $1\frac{1}{2}$ | $\frac{1}{8}$ | | | |
| $0\frac{3}{4}$ | $\frac{1}{16}$ | | | |

## 24. SECTION II. ADDITION.

Addition *is the method of collecting ſeveral numbers into one ſum.*

Rule 1ſt. Write the given numbers under each other, ſo that like names ſtand under like names; that is units under units, tens under tens, &c. and under theſe draw a line.

2d. Add up the firſt or right hand upright row, under which write the overplus of the units of the ſecond row, contained in that ſum.

3. Add theſe units to the ſum of the ſecond row, under which write the overplus of the units of the third row, contained in that ſum.

And thus proceed until all the rows are added together.

### Examples.

Ex. I. *Add* 28—76—47—18 *and* 12 *together.*

Theſe numbers being written under each other will ſtand thus. Say 2 and 8 is 10, and 7 is 17, and 6 is 23, and 8 is 31; then, becauſe 10 units in the right hand row make an unit in the next row; therefore in 31 there are 3 units of the ſecond row, and an overplus of 1; write down the 1, and add the 3 to the ſecond row, ſaying, 3 that is carried and 1 is 4, and 1 is 5, and 4 is 9, and 7 is 16, and 2 is 18, in which is one unit of the third row (had there been a 3d) and an overplus of 8; write down the 8, and add the 1 to the third row: but as there is no third row, the 1 carried muſt be written on the left hand of the 8; and 181 will be the ſum of the five given numbers.

28
76
47
18
12

181

Ex. II. *Add* 476—3784—18329—290—75—7638—*and* 46 *together.*

The given numbers ſet in order will ſtand thus

476
3784
18329
290
75
7638
46

The Sum 30638

Ex. III. *Add the numbers,* 10768—3489—28764—289—6438—19 *and* 438 *together.*

The given numbers placed as the rule directs, ſtand thus

10768
3489
28764
289
6438
19
438

The Sum 50205

Ex. IV. *Add theſe numbers together.*

3720,45
25,0036
4179,802
3,6284

Sum 7928,8840

Ex. V. *Add the following numbers together.*

15836,071
20,09
34,7
583,27008

Sum 16474,13108

In the two laſt examples, where there are both integer and fractional numbers, it may be obſerved, that like integer places, and like fractional places, ſtand under each other; and the manner of adding them together, is the ſame as explained in the firſt example.

25. It frequently happens, that numbers are to be added together, the names of which do not increaſe in a tenfold manner, as in the laſt Examples; ſuch as in adding different ſums of money, weights, or meaſures; in which, regard is to be had to the number of thoſe of a lower name, contained in one of its next greater name, as ſhewn in the preceding tables: Examples of which follow.

Ex. VI. *Add the following ſums of money together.*

| £. | s. | d. |
|---|---|---|
| 353 | 14 | 8½ |
| 276 | 10 | 4 |
| 89 | 17 | 5¼ |
| 34 | 12 | 10¾ |
| 754 | 15 | 4½ |

Ex. VII. *Add the following ſums of money together.*

| £. | s. | d. |
|---|---|---|
| 7683 | 08 | 2¾ |
| 954 | 19 | 9½ |
| 682 | 10 | 7¾ |
| 63 | 15 | 6¼ |
| 9384 | 14 | 2¼ |

In theſe two examples the carriage is by 4 in the farthings; by 12 in the pence; by 20 in the ſhillings; and by 10 in the pounds.

Ex. VIII. *Add the following Troy Weights together.*

| lb. | oz. | dwt. | gr. |
|---|---|---|---|
| 218 | 10 | 13 | 18 |
| 176 | 9 | 19 | 23 |
| 85 | 11 | 17 | 11 |
| 24 | 8 | 15 | 21 |
| 506 | 5 | 07 | 01 |

Carry for 24, 20, 12, 10.

Ex. IX. *Add the following Avoirdupoiſe Weights together.*

| Tons. | Cwt. | qrs. | lb. | oz. |
|---|---|---|---|---|
| 535 | 17 | 3 | 22 | 11 |
| 94 | 19 | 1 | 27 | 13 |
| 158 | 12 | 0 | 18 | 15 |
| 7 | 15 | 2 | 13 | 08 |
| 797 | 05 | 0 | 26 | 15 |

Carry for 16, 28, 4, 20, 10.

Ex. X. *Add the following parts of Time together.*

| Weeks | Da. | Ho. | Min. | Sec. |
|---|---|---|---|---|
| 21 | 4 | 18 | 37 | 59 |
| 11 | 6 | 13 | 25 | 47 |
| 19 | 3 | 23 | 59 | 28 |
| 38 | 4 | 08 | 22 | 39 |
| 91 | 5 | 16 | 25 | 53 |

Carry for 60, 60, 24, 7, 10.

Ex. XI. *Add the following parts of a Circle together.*

| Deg. | ′ | ″ | ‴ | |
|---|---|---|---|---|
| 176 | 32 | 59 | 43 | 25 |
| 85 | 59 | 27 | 31 | 59 |
| 114 | 28 | 45 | 59 | 14 |
| 67 | 12 | 38 | 24 | 47 |
| 444 | 13 | 51 | 39 | 25 |

Carry for 60, 60, 60, 60, 10

## Explanation of Example VI.

Three farthings and 1 farthing is 4 farthings, and 2 farthings is 6 farthings; which is a penny halfpenny; ſet down ½ and carry 1.

Then 1 and 10 is 11, and 5 is 16, and 4 is 20, and 8 is 28 pence; which is 2 ſhillings and 4 pence: ſet down 4, and carry 2.

Again, 2 and 12 is 14, and 17 is 31, and 10 is 41, and 14 is 55 ſhillings; which is 2 pounds 15 ſhillings; ſet down 15 ſhillings, and carry 2 pounds. The reſt is eaſy.

## 26. SECTION III. SUBTRACTION.

SUBTRACTION *is the method of taking one number from another, and ſhewing the remainder, or difference, or exceſs.*

The *ſubducend* is the number to be ſubtracted, or taken away.

The *minuend* is the number from which the ſubducend is to be taken.

RULE 1ſt. Under the minuend write the ſubducend, ſo that like names ſtand under like names; and under them draw a line.

2d. Beginning at the right-hand ſide, take each figure in the lower line from the figure ſtanding over it, and write the remainder, or what is left, beneath the line, under that figure.

3d. But if the figure below is greater than that above it, increaſe the upper figure by as many as are in an unit of the next greater name; from this ſum take the figure in the lower line, and write the remainder under it.

4th. To the next name in the lower line, carry the unit borrowed, and thus proceed to the higheſt denomination or name.

### EXAMPLES.

| | | | |
|---|---|---|---|
| Ex. I. | *From* | 436565874 | *the minuend,* |
| | *Take* | 249853642 | *the ſubducend,* |
| | *Remains* | 186712232 | *the difference.* |

Here the five figures on the right of the ſubducend may be taken from thoſe over them: but the 6th figure, viz. 8, cannot be taken from the 5 above it. Now as an unit in the 7th place makes 10 in the 6th place, therefore borrowing this unit makes the 5, 15; then ſay, 8 from 15 leaves 7, which ſet down; and ſay 1 carried and 9 is 10, 10 from 6 cannot be had, but 10 from 16 leaves 6, ſet it down; then 1 carried and 4 is 5, 5 from 13 leaves 8; ſet it down: then 1 carried and 2 is 3, 3 from 4 leaves 1.

| | | |
|---|---|---|
| Ex. II. | *From* | 7620908 |
| | *Take* | 3875092 |
| | *Remains* | 3745816 |

| | | |
|---|---|---|
| Ex. III. | *From* | 327,9563 |
| | *Take* | 49,8697 |
| | *Remains* | 278,0866 |

| | | |
|---|---|---|
| Ex. IV. | *From* | 30007,295 |
| | *Take* | 2536,876 |
| | *Leaves* | 27470,419 |

| | | |
|---|---|---|
| Ex. V. | *From* | 5000,0000 |
| | *Take* | 479,6378 |
| | *Leaves* | 4520,3622 |

| | | £. | s. | d. |
|---|---|---|---|---|
| Ex. VI. | *Borrowed* | 24 | 14 | 6½ |
| | *Paid* | 18 | 12 | 4¼ |
| | *Remains* | 6 | 02 | 2¼ |

| | | £. | s. | d. |
|---|---|---|---|---|
| Ex. VII. | *Lent* | 294 | 15 | 9¼ |
| | *Received* | 89 | 18 | 10¾ |
| | *Remains* | 204 | 16 | 10½ |

Ex. VIII. *In Sexageſimals.*

| | ° | ′ | ″ | ‴ | iv |
|---|---|---|---|---|---|
| *From* | 76 | 28 | 37 | 49 | 32 |
| *Take* | 65 | 29 | 16 | 53 | 45 |
| *Leaves* | 10 | 59 | 20 | 55 | 47 |

Ex. IX. *In Sexageſimals.*

| | ° | ′ | ″ | ‴ | iv |
|---|---|---|---|---|---|
| *From* | 218 | 46 | 32 | 50 | 18 |
| *Take* | 149 | 52 | 47 | 53 | 29 |
| *Leaves* | 68 | 53 | 44 | 56 | 49 |

27. QUESTIONS to exerciſe Addition and Subtraction.

QUEST. I. *The ſhare of Jack's prize money was* 148£. 17*s.* 6*d*½; *and Tom received as much, beſide* 7£. 18*s. ſmart money: How much money did Tom receive?*

| | £. | s. | d. |
|---|---|---|---|
| Tom's prize money | 148 | 17 | 6½ |
| Smart money | 7 | 18 | 0 |
| Tom received | 156 | 15 | 6½ |

QUEST. III. *What year was King George born in, he being* 67 *years old in the year* 1749?

| | |
|---|---|
| Current year | 1749 |
| Age | 67 ſubtr. |
| Year born in | 1682 |

QUEST. V. *A ſeaman who had received* 46£. 17*s.* 6*d. for wages, prize money, &c. meeting with bad company was tricked out of* 18 *guineas: Now John had reckoned to pay his wife's debts of* 13£. 16*s.* 6*d. and his landlady's bill of* 16£. 12*s. Required whether he can fulfil his intentions, and what the difference will be?*

| | £. | s. | d. |
|---|---|---|---|
| Money loſt | 18 | 18 | 0 |
| Wife's debt | 13 | 16 | 6 |
| Landlady's bill | 16 | 12 | 0 |
| Total | 49 | 6 | 6 |
| Money received | 46 | 17 | 6 |
| He will want | 2 | 9 | 0 |

QUEST. II. *The Spaniſh invaſion was in the year* 1588, *and the French attempted an invaſion in the year* 1744: *How many years were between theſe fruitleſs attempts?*

| | |
|---|---|
| French | 1744 |
| Spaniſh | 1588 |
| Years between | 156 |

QUEST. IV. *Two ſhips depart from the ſame port, one having ſailed* 835 *miles, is got* 48 *miles a-head of the other: Required the aftermoſt ſhip's diſtance?*

| | |
|---|---|
| The firſt ſhip's diſtance | 835 |
| Their difference | 48 |
| Second ſhip's diſtance | 787 |

QUEST. VI. *Will and Frank talking of their ages in the year* 1749, *Will ſaid he was born in the year of the Rebellion, in* 1715; *and Frank ſaid he remembered he was ten years old the year King George the ſecond was crowned in* 1727: *Required the age of each, and the difference of their ages?*

| | |
|---|---|
| Current year | 1749 |
| Will was born | 1715 |
| Will's age | 34 |
| Current year | 1749 |
| King George crowned | 1727 |
| Years ſince | 22 |
| Frank's age then | 10 |
| Frank's age | 32 |

So Will was oldeſt by two years.

## 28. SECTION IV. MULTIPLICATION.

MULTIPLICATION *is the method of finding what a given number will amount to, when repeated as many times as is repreſented by another number.*

The number to be multiplied, is called the *Multiplicand.*

The number multiplied by, is called the *Multiplier.*

And the number which the multiplication amounts to, is called the *Product.*

Both multiplicand and multiplier are called *Factors.*

Before any operation can be performed in Multiplication, it is neceſſary that the learner ſhould commit to memory the following table.

29. The MULTIPLICATION TABLE.

| times | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | | | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | | | | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | | | | | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | | | | | | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | | | | | | | 64 | 72 | 80 | 88 | 96 |
| 9 | | | | | | | | 81 | 90 | 99 | 108 |
| 10 | | | | | | | | | 100 | 110 | 120 |
| 11 | | | | | | | | | | 121 | 132 |
| 12 | | | | | | | | | | | 144 |

Obſerve, that in multiplying any figure in the upper line by any figure in the left-hand column, the product will ſtand right againſt the figure uſed in the left-hand column, and under that uſed in the upper line. Thus were 6 to be multiplied by 9, ſeek the greater figure 9 in the upper line, and right under it, againſt 6 in the left hand, ſtands 54 for the Product. And ſo of others.

The foregoing table being well known, the work of Multiplication will be performed as follows.

To multiply any number, as 37256
By any ſingle figure, as by 7
Set them as in the margin, and proceed 260792

thus, 7 times 6 is 42, ſet down 2 and carry 4; 7 times 5 is 35 and 4 carried is 39, ſet down 9 and carry 3; 7 times 2 is 14 and 3 carried is 17, ſet down 7 and carry 1; 7 times 7 is 49 and 1 carried is 50, ſet down 0 and carry 5; 7 times 3 is 21 and 5 carried is 26, which ſet down, and the work is done. But for compound Multiplication take the following:

30. RULE 1ſt. Write the Factors ſo, that the right hand place of the Multiplier ſtands under the right hand place of the Multiplicand.

2d. Multiply the Multiplicand ſeverally by every figure of the Multiplier, ſetting the firſt figure of each line under the figure then multiplying by.

3d. Add the ſeveral lines together; and their ſum is the Product.

4th. From the right hand of the Product point off, for fractions, as many places as there are fractional places in both Factors; and thoſe to the left of the mark of diſtinction are integers; thoſe to the right are fractions.

5th. If the number of places in the Product are not ſo many as the number of fractional places in both Factors, make up that number by writing cyphers on the left hand, and to theſe prefix the mark of diſtinction.

### EXAMPLE I. *Multiply* 742 by 53.

The leſſer Factor being written under the greater Factor, as here ſhewn, and a line drawn under them; ſay 3 times 2 is 6, write 6 under the 3; then 3 times 4 is 12, write down 2 and carry 1; and 3 times 7 is 21, and 1 carried is 22, write the 22:- Again, 5 times 2 is 10, write 0 under the 5, and carry 1; and 5 times 4 is 20 and 1 carried is 21, write down 1 and carry 2; then 5 times 7 is 35, and 2 carried is 37, write down the 37: Now add the two lines together found by multiplying by 3 and by 5, and their ſum 39326 is the product required.

| | | |
|---|---|---|
| Multiplicand | 742 | Factors. |
| Multiplier | 53 | |
| | 2226 | |
| | 3710 | |
| Product | 39326 | |

### EXAMPLE II.

| | |
|---|---|
| *Multiply* | 28704 |
| *by* | 8631 |
| | 28704 |
| | 86112 |
| | 172224 |
| | 229632 |
| | 247744224 Product. |

### EXAMPLE IV.

| | |
|---|---|
| *Multiply* | 936,287 |
| *by* | 607,02 |
| | 1872574 |
| | 65540090 |
| | 56177220 |
| | 568344,93474 |

The cyphers in the Multiplier of this example are thus managed. Having multiplied by the 2 as before, ſay 0 times 7 is 0, write 0 under the 0, and proceed to the next figure 7, by which multiply as before, then coming to the ſecond 0, ſay, 0 times 7 is 0, write 0 under the place of the ſecond 0, and proceed to the next figure 6, by which multiply as before.

### EXAMPLE III.

| | |
|---|---|
| *Multiply* | 3684,2795 |
| *by* | 7,594 |
| | 147371180 |
| | 331585155 |
| | 184213975 |
| | 257899565 |
| | 27978,4185230 |

Here, becauſe there are 4 fractional places in the Multiplicand, and 3 in the Multiplier, which together make 7, therefore 7 places are pointed off on the right of the product for fractions.

### EXAMPLE V.

| | |
|---|---|
| *Multiply* | 0,34796 |
| *by* | 0,0258 |
| | 278368 |
| | 173980 |
| | 69592 |
| | 0,008977368 |

Here, becauſe there are 5 fractional places in one Factor, and 4 in the other, there ſhould be 9 fractional places in the Product; and there ariſing but 7, therefore two cyphers are ſet on the left hand to make 9 places.

## 31. SECTION V. DIVISION.

Division *is the method of finding how often one number is contained in another; or may be taken from another.*

The number to be divided, is called the *Dividend.*

The number dividing by, is called the *Diviſor.*

The *Quotient* is the number ariſing from the diviſion, and ſhews how many times the Diviſor is contained in the Dividend.

The operations in Diviſion are performed as follow.

32. Rule 1ſt. On the right and left of the Dividend draw a crooked line; write the Diviſor on the left ſide, and the Quotient, as it ariſes, on the right ſide of the Dividend.

2d. Seek how often the Diviſor may be taken in as many figures on the left hand of the Dividend, as are juſt neceſſary; write the number of times it may be taken, in the Quotient; and there will be as many figures more in the Quotient, as there are figures remaining in the Dividend then not uſed.

3d. Multiply the Diviſor by this Quotient-figure, ſet the Product under that part of the Dividend uſed; ſubtract, and to the right hand of the remainder bring down the next figure of the Dividend: Divide as before; and thus proceed until all the figures of the Dividend are uſed.

4th. If there is a remainder, to its right hand ſide annex a cypher or cyphers, as if brought down from the Dividend, and divide as before; and thus it is that fractions ariſe, viz. from the remainders in diviſion.

5th. When any figure of the Dividend is taken down, or annexed, as before ſhewn, and the Diviſor cannot be taken in the number thus increaſed; put 0 in the Quotient, and take down, or annex, another figure; and proceed in this manner, until the Diviſor can be taken from the number.

6th. When fractions are concerned: From the number of fractional places uſed in the Dividend, take thoſe in the Diviſor; count the number of remaining places from the right of the Quotient, put the mark there; and thoſe to the left are integers, thoſe to the right fractions.

7th. If there ariſe not ſo many places in the Quotient as the 6th article requires, ſupply the places wanting with cyphers on the left, and to thoſe prefix the fractional mark.

Ex. I. *Divide* 3656 £. *among* 8 *perſons.*

```
8)3656(457
  32
  --
   45
   40
   --
    56
    56
    --
     0
```

Set the given numbers as in Art. 1ſt. Now the two left hand figures contain 8; then ſay 8 is contained in 36, 4 times; ſet 4 in the Quotient, and ſay 4 times 8 is 32, ſet 32 under 36, ſubtract, there remains 4, to which bring down the next figure of the Dividend 5, makes 45; then ſay 8 is contained in 45, 5 times; ſet 5 in the Quotient, and ſay 5 times 8 is 40; write 40 under 45, ſubtract, and to the remainder 5 take down 6, the next figure of the Dividend, makes 56; then ſay 8 is contained in 56, 7 times; write 7 in the Quotient, multiply 8 by 7 makes 56, which write under the other 56, and ſubtracting there remains 0: So it may be concluded, that 3656 contains 8, 457 times: Or, if 3656 £. be divided among 8 perſons, the ſhare of each will be 457 £.

Ex. II. *Divide* 3125 *by* 25.
25)3125(125
25
62
50
125
125
0

Ex. III. *Divide* 95269 *by* 47.
47)95269(2027
94
See precept 5th. 126
94
329
329
0

Ex. IV. *Divide* 5859 *by* 124.
124)5859(47,25
496
899
868
310 for the Remaind.
248 See precept 4th.
620
620

Ex. V. *Divide* 337,27368 *by* 6,28.
6,28)337,27368(53,706 for
3140 the Quotient.
See precept 6th.
2327
1884
4433
4396
See precept 5th. 3768
3768

Ex. VI. *Divide* 2,3569 *by* 673,4.
673,4)2,3569(35
20202 Quot. 0,0035
33670 See precepts
33670 4th, 6th, 7th.

In Ex. VI. the 4 fractional places given in the Dividend, and the 0 uſed with the Remainder, make 5 fractional places; from which 1 place in the Diviſor being taken, leaves 4 fractional places for the Quotient; but in the Quotient are only the two places 35, therefore 2 cyphers are prefixed, and makes ,0035, before which, for form ſake, an 0 is ſet for the place of units.

33. When the Diviſor does not exceed the number 12, the Diviſion may be performed in one line; by making the Multiplication and Subtraction mentally, or in the mind, and carrying the Remainder, as ſo many tens, to the next figure.

34. In all operations of Diviſion, it muſt be obſerved, that the Product of the Diviſor by the Quotient figure muſt not exceed that part of the Dividend then uſing; and the Remainder, by ſubtracting the Product, muſt ever be leſs than the Diviſor.

As the Quotient multiplied by the Diviſor makes the Dividend;

So the Product of two numbers being divided by one of them, will give the other; that is, Diviſion is proved by Multiplication, and Multiplication is proved by Diviſion.

## 35. SECTION VI. REDUCTION.

Reduction *is the method of reducing numbers from one name, or denomination, to another; retaining the ſame value.*

Case I. *To reduce a number conſiſting of ſeveral names, to their leaſt name.*

Rule 1ſt. Multiply the firſt, or greater name, by the parts which an unit of that name contains of the next leſs name; adding to the Product the parts of the ſecond name in the given number.

2d. Multiply this ſum by the number of times that an unit of the next leſs name is contained in one of the ſecond name; adding to the Product the parts of the third name contained in the given number: And thus proceed, until the leaſt name in the given number is arrived at.

Ex. I. *In* 23£. 14*s.* 6½*d. how many farthings?*

£. s. d.
23 14 6½
20
474 Shillings.
12
5694 Pence.
4
Anſwer 22778 Farthings.

Ex. II. *In* 8*lb.* 10*oz. of gold, how many grains?*

lb. oz.
8 10
12
106 Ounces.
20
2120 Pennyweights.
24
8480
4240
50880 Grains.

Ex. III. *In a cannon weighing* 2 *Tons,* 14*C.* 3*qrs.* 19*lb. how many pounds?*

T. C. Qrs. lb.
2 14 3 19
20
54 C. weight.
4
219 Qrs.
28
1771
438
6151 Pounds.

Ex. IV. *In* 36 *deg.* 48′. 27″. 56‴. *how many thirds?*

° ′ ″ ‴
36 48 27 56
60
2208 Minutes.
60
132507 Seconds.
60
7950476 Thirds.

An explanation of the firſt Ex. will make all the reſt plain. Since pounds is the greateſt name in the given number, and an unit thereof contains 20 of the next leſs name, or ſhillings; therefore multiply the pounds by 20, ſaying 0 times 3 is 0, to which adding the 4 in the 14s. makes 4; then 2 times 3 is 6, and the one, in the place of tens in the ſhillings, makes 7; then 2 times 2 is 4: Now multiply 474s. by 12, ſaying 12 times 4 is 48, and the 6 in the pence makes 54; write 4 and carry 5; then 12 times 7 is 84 and 5 is 89, &c. Laſtly, multiply the 5694 pence by 4, ſaying 4 times 4 is 16, and the two farthings in the given number is 18; write 8 and carry 1, &c.

36. CASE II. *A number of an inferior name being given; to find how many of each ſuperior denomination are contained in it.*

RULE 1ſt. Divide the given number, by the number of times that one of its units is contained in an unit of the next ſuperior name.

2d. Divide this Quotient by the parts making one of the next name.

3d. Divide this Quotient by the parts making one of the next name: And proceed in this manner, until the higheſt name is obtained.

4th. Then the laſt Quotient, and the ſeveral remainders, will be the parts of the different names contained in the given number.

Ex. I. *In* 22778 *farthings, how many pounds, ſhillings, and pence?*

4)22778( 2 Farthings.

12) 5694( 6 Pence.

2,0) 47,4( 14 Shillings.

23 Pounds.

Anſwer 23 £. 14 s. 6½ d.

Ex. II. *In* 7950476 *thirds of a degree, how many* ° ′ ″ ‴?

6,0)795047,6( 56 Thirds.

6,0)13250,7( 27 Seconds.

6,0) 220,8( 48 Minutes.

36 Degrees.

Anſwer 36°. 48′. 27″. 56‴.

Ex. III. *In* 6151 *pounds, how many Tons, Hundreds, Quarters, Pounds?*

28)6151(219

56

55

28

271

252

19 Pounds.

4)219( 3 Qrs.

2,0)5,4( 14 C.

2 Tons.

Anſwer 2 T. 14 C. 3 Qrs. 19 lb.

Ex. IV. *In* 50880 *grains, how many Pounds, Ounces, Pennyweights, Grs.*

24)50880(2120

48

28

24

48

48

0

2,0)212,0(0 dwt.

12) 106( 10 oz.

8 lb.

Anſwer 8 lb. 10 oz.

Explanation of Ex. I. Since 4 of the given number make one of the next name, pence, then 22778 divided by 4, give 5694 pence, and a Remainder of 2 farthings; then 5694 pence divided by 12, the number of pence in one of the next name, ſhillings, the Quotient is 474 ſhillings, and a Remainder of 6 pence; then 474 ſhillings divided by 20, the number of ſhillings in one of the next name, pounds, the Quotient is 23 pounds, and a Remainder of 14 ſhillings. And by the 4th precept, the anſwer is collected.

A like operation will ſolve the other examples, having regard to the increaſe of the different names.

37. In any Diviſion, if the Diviſor has one or more cyphers on the right hand, thoſe cyphers may be pointed off; but then as many places muſt be pointed off from the Dividend, which places are not to be divided, but annexed to the right hand of the Remainder. See the above examples.

### 38. Case III. *To reduce a vulgar fraction to its equivalent decimal fraction.*

Rule. To the Numerator annex one or more cyphers, divide this by the Denominator, and the Quotient will be the fraction ſought.

If the Diviſion does not end when ſix figures are found in the Quotient, the work need not be carried any farther.

Exam. I. *To reduce $\frac{15}{423}$ to its equivalent decimal fraction.*

Here 423 the Denominator is made the Diviſor, and 15 the Numerator is ſet for the Dividend, to which annexing a cypher or two for fractional places, ſeek how often the Diviſor can be had in 15, the integral part of the Dividend; and as it cannot be taken, put 0 in the Quotient for the place of units: Then taking in one fractional place, ſeek how oft the Diviſor can be had in 150, ſay 0 times, and put another 0 in the Quotient for the place of primes: Now taking in two fractional places to the 15, the Diviſor will be contained in it thrice, and thus proceed until the Diviſion ends, or till 6 places ariſe in the Qotient: But in this example, as the 6th place would be 0, it is omitted, becauſe cyphers on the right hand of decimal fractions are of no ſignification, as will evidently appear, Notation of Fractions being well underſtood.

```
423)15,00(0,03546
     1269
     ----
      2310
      2115
      ----
       1950
       1692
       ----
        2580
        2538
        ----
         420
         ----
```

Ex. II. *Reduce $\frac{1}{2}$ to a decimal fraction.*

2)1,0(0,5 Anſwer.

Ex. III. *Reduce $\frac{1}{4}$ to a decimal fraction.*

4)1,00(0,25 Anſwer.

Ex. IV. *Reduce $\frac{3}{4}$ to a decimal fraction.*

4)3,00(0,75 Anſwer.

Ex. V. *Reduce $\frac{5}{8}$ to a decimal fraction.*

8)5,000(0,625 Anſwer.

Ex. VI. *Reduce $\frac{1}{3}$ to a decimal fraction.*

3)1,00(0,33, &c. Anſwer.

Ex. VII. *Reduce $\frac{7}{12}$ to a decimal fraction.*

12)7,0000(0,5833 &c. Anſw.

39. In the two laſt Quotients, it may be obſerved, that 3 would continually ariſe; ſuch decimal fractions are called circulating, or recurring fractions: Theſe have a peculiar kind of operation belonging to them, which the inquiſitive reader will find in a book intitled *A General Treatiſe of Menſuration**, the third edition, publiſhed in the year 1767; and alſo in other books.

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* By the Author of theſe Elements.

40. Case IV. *To reduce a number conſiſting of different names, to a decimal fraction of its greateſt name.*

Rule 1ſt. Write the given names orderly under one another, the leaſt name being uppermoſt; and on their left ſide draw a line: Let theſe be reckoned as Dividends.

2d. Againſt each name, on the left hand, write the number making one of its next ſuperior name: And let theſe be the Diviſors to the former Dividends.

3d. Begin with the upper one, and write the Quotient of each diviſion as fractions, on the right of the Dividend next below it; then let this mixed number be divided by its Diviſor, &c.

And the laſt Quotient will be the decimal fraction ſought.

Ex. I. *Reduce* 15 *s.* $9\frac{3}{4}$ *d. to the fractional part of a pound ſterling.*

$$\begin{array}{r|l} 4 & 3 \\ 12 & 9{,}75 \\ 20 & 15{,}8125 \\ & 0{,}790625 \\ \hline \end{array}$$

Firſt ſet the three farthings, the 9 pence, the 15 ſhillings and 0 pounds under one another; and againſt the farthings ſet 4, againſt the pence ſet 12, and againſt the ſhillings, 20; then the three with cyphers ſuppoſed to be annexed, being divided by 4, the Quotient ,75 is written on the right hand of the 9 pence; and the mixed number 9,75 with cyphers annexed as they are wanted, being divided by 12, the Quotient ,8125 is written on the right hand of the 15 *s.* then this mixed number 15,8125 being divided by 20, the Quotient 0,790625 £. is the anſwer.

Ex. II. *Reduce* 1 *s.* $2\frac{1}{4}$ *d. to the fractional part of a pound ſterling.*

$$\begin{array}{r|l} 4 & 1 \\ 12 & 2{,}25 \\ 20 & 1{,}1875 \\ & 0{,}059375 \\ \hline \end{array}$$

Anſwer 1 *s.* $2\frac{1}{4}$ *d.* = 0,059375 £.

Ex. IV. *Reduce* 8 *oz.* 15 *dwt.* 18 *gr. to the fractional part of a pound troy.*

$$\begin{array}{rr|l} 24\,\{ & 4 & 18 \\ \{ & 6 & (4{,}5 \\ 20 & & 15{,}75 \\ 12 & & 8{,}7875 \\ & & 0{,}732291 \\ \hline \end{array}$$

Anſ. 8oz. 15dwt. 18 gr. = 0,732291lb.

41. Here becauſe 24 is a number too great to divide by in one line, therefore it is broken into the parts 4 and 6, which multiplied together make 24.

Ex. III. *Reduce* 48′. 17″. 53‴. *to the fractional part of a degree.*

$$\begin{array}{r|l} 60 & 53 \\ 60 & 17{,}883333 \\ 60 & 48{,}298055 \\ & 0{,}804967 \\ \hline \end{array}$$

Anſw. 48′. 17″. 53‴. = 0,804967 Deg

Ex. V. *Reduce* 3 *qrs.* 19 *lb.* 14 *oz. to the fractional part of a C. weight.*

$$\begin{array}{rr|l} 16\,\{ & 4 & 14 \\ \{ & 4 & (3{,}5 \\ 28\,\{ & 4 & 19{,}875 \\ \{ & 7 & (4{,}96875 \\ 4 & & 3{,}709821 \\ & & 0{,}927455 \\ \hline \end{array}$$

Anſw. 3qr. 19lb. 14oz = 0,927455 C.

Here the 16 is broken into the numbers 4 and 4; and 28 into 4 and 7; and 14 is divided by 4; and the Quotient 3,5 by 4, &c.

42. CASE V. *To reduce a decimal fraction of a ſuperior name, to its value in inferior denominations.*

RULE 1ſt. Multiply the given fraction by the number that an unit of its name contains units of the next leſſer name; from the right hand of the Product point off as many places as there are in the given fraction.

2d. Multiply the places, ſo pointed off, by as many as an unit of this name contains of the next leſs name; point off as before.

And thus proceed until the multiplication is made by the leaſt name.

3d. Then the integers, or the numbers on the left of the diſtinguiſhing marks in each Product, will be the parts in each name, which together are equal to the given fraction.

EXAMPLE I. *What number of ſhillings, pence, and farthings, are equal in value to 0,790625£. ſterling.*

Here an unit of the given name £. contains 20 of the next leſs name, ſhillings; then multiplying by 20, and pointing off 6 places on the right, becauſe the given number 0,790625 contains 6 fractional places, the Product is 15,812500 ſhillings; then the fractions of this number, *viz.* 812500 multiplied by 12, the number that an unit of this name contains of the next leſs name, and the product pointed as before, there ariſes 9,750000 pence; the fractions of this number multiplied by 4, gives 3,000000 farthings; then the parts pointed off on the left, *viz.* 15s. 9¾d. are the value of the given fraction.

| | |
|---|---|
| £. | 0,790625 |
| | 20 |
| *s.* | 15,812500 |
| | 12 |
| *d.* | 9,750000 |
| | 4 |
| *far.* | 3,000000 |

EXAMPLE II. *What is the value of* 0,056285£. *ſterling?*

| | |
|---|---|
| £. | 0,056285 |
| | 20 |
| *s.* | 1,1257\|00 |
| | 12 |
| *d.* | 1,5084 |
| | 4 |
| *far.* | 2,0336 |

EXAMPLE III. *What is the value of* 0,58695 *degrees?*

| | |
|---|---|
| Deg. | 0,58695 |
| | 60 |
| Min. | 35,217\|00 |
| | 60 |
| Sec. | 13,02\|0 |
| | 60 |
| Thirds | 0,120 |

EXAMPLE IV. *What is the value of* 0,732291*lb. troy?*

This example worked as above, by multiplying by 12, 20, 24, the value will be found to be

8oz. 15dwts. 18gr. nearly.

EXAMPLE V. *What is the value of* 0,927455 *part of a C. weight?*

By operating as above, multiplying by 4, 28, 16, the anſwer will be

3qrs. 19lb. 14oz. nearly.

43. QUEST.

43. QUESTIONS to exerciſe the preceding rules.

QUEST. I. *A ſloop with the captain and 26 hands take a prize which ſold for 1578 £. of which each ſeaman had 45 £. and the captain the reſt: How much was his ſhare?*

26 Men
45 £. to each
130
104
ſubtract 1170 £. the crew's ſhare.
from 1578 £. the whole prize.
remains 408 £. the captain's ſhare.

QUEST. II. *A boat's crew of 15 men got by plunder 321 £. How much was the ſhare of each?*

15)321(21 £.
30
21
15
remains 6 £. which
mult. by 20*s.* in 1 £.
15)120*s.* (8 *s.*
120
Anſwer 21 £. 8 *s.* to each.

QUEST. III. *A ſeaman, whoſe wages are 35 s. 6 d. a month, returns home at the end of 29 months; he having taken up 12 £. 18 s.: How much has he to receive?*

*s.* *d.*
35 6
mult. by 12
426 pence a month,
mult. by 29 months,
3834
852
12)12354 pence
2,0) 102,9 6*d.*
from 51 £. 9*s.* 6*d.* = wages,
take 12 £. 18s. 0*d.* received,
remains 38 £. 11*s.* 6*d.* to receive.

QUEST. IV. *Six meſs-mates, who propoſe to live well during an Eaſt-India voyage of 22 months, agree to expend among them 5 s. a day, beſides the ſhip's allowance: Now one of them having but 25 s. a month, how will matters ſtand with him at the end of the voyage?*

Now 28 days, at 5 *s.* a day, makes 140 *s.* or 7 £. a month; which for 22 months, is 154 £.

Then a ſixth part of 154 £. is 25 £. 13 *s.* 4 *d.* for each man.

Alſo 25 *s.* a month for 22 months makes 27 £. 10 *s.* for wages; which will overpay his expences, by 1 £. 16 *s.* 8 *d.*

QUEST. V. *In 306 crowns, how many half crowns and pence?*

Anſwer { 612 half crowns.
18360 pence.

QUEST. VI. *In 30 chalders of coals, each of 36 buſhels, how many pecks?*

Anſwer 4320 pecks.

QUEST. VII. *A ſeaman's ſhare of a prize was 14 guineas, 32 moidores, 12 thirty-ſix ſhillings pieces, and 52 piſtoles at 17 s. each: How much ſterling did the whole come to?*

Anſwer 123 £. 14 *s.*

QUEST. VIII. *Suppoſe a ſhip ſails 5½ miles an hour for 14 days: How many degrees and minutes has ſhe ſailed in the whole; 60 ſea miles making one degree?*

Anſwer 30 deg. 48 min.

## SECTION VII. Of PROPORTION:

## Or, THE RULE OF THREE.

44. Four numbers are ſaid to be proportional, when by comparing them together by two and two, they either give equal Products or equal Quotients.

Suppoſe theſe four numbers 3 8 12 32

In comparing them together by multiplication,

The Product of 3 and 8 is 24; of 12 and 32 is 384, unequal.
of 3 and 12 is 36; of 8 and 32 is 256, unequal.
of 3 and 32 is 96; of 8 and 12 is 96, equal.

Therefore 3 8 12 32, are called proportional numbers.

Now let them be compared together by diviſion.

The Quotient of 8 by 3 is 2,6*&c.* of 32 by 12 is 2,6 *&c.* equal.
of 12 by 3 is 4 of 32 by 8 is 4, equal.
of 32 by 3 is 10,6*&c.* of 12 by 8 is 1,5, unequal.

Therefore by this compariſon, the numbers are ſaid to be proportional.

In this kind of comparing four numbers together, there is no need to try for more equal Products, or Quotients, than one ſet of either ſort; for either caſe will determine the proportionality independent of the other.

But it muſt be obſerved, that among four proportional numbers, there will be but one ſet of equal Products, and two ſets of equal Quotients, the ſmaller numbers being Diviſors.

45. When four numbers are to be written as proportionals, they muſt be placed in ſuch order, that the Product of the firſt and fourth be equal to the Product of the ſecond and third.

A queſtion is ſaid to belong to the *Rule of Three*, when three numbers or terms are given to find a fourth proportional, which is the anſwer to the queſtion.

And in order to reſolve ſuch queſtions, the three given terms muſt be firſt placed in a proper order, which is called ſtating the terms of the queſtion.

46. Queſtions in the *Rule of Three* are ſtated, and reſolved by the following precepts.

1ſt. Conſider of what kind the fourth term, or number ſought, will be, whether money, weight, meaſure, time, *&c.* and among the three numbers given in the queſtion let that which is of the ſame kind with what is required be placed for the third term.

2d. From the nature of the queſtion, determine whether the number ſought will be greater or leſs than the number which is placed for the third term.

3d. If the fourth term will be greater than the third, ſet the greater of the remaining two terms for the ſecond, and the leſs for the firſt.

But if the fourth term is to be leſs than the third, ſet the greater of the remaining two terms for the firſt, and the leſs for the ſecond.

Then in either caſe, the given three terms are ſtated.

4th. Reduce thoſe terms which conſiſt of more names than one, to one name; and obſerve that the firſt and ſecond terms are always to be of the ſame name.

5th. Multiply the ſecond and third terms together, divide the product by the firſt term, and the Quotient will be the fourth term, of the ſame name the third term was reduced to.

47. QUEST. I. *If* 4 *yards of cloth coſt* 18 *s. what will* 24 *yards coſt?*

Here it is plain, that the term ſought, or the worth of 24 yards, will be money; therefore the given money 18 *s.* is ſet for the third term; and as the worth of 24 yards muſt be greater than the worth of 4 yards, therefore the 24 is ſet for the 2d term, and the 4 for the 1ſt. Then the 2d term 24 being multiplied by the 3d, 18, the Product is 432, which divided by the 1ſt term 4, the Quotient or 4th term is 108, which are ſhillings, the ſame name of the 3d term; then 108 ſhillings divided by 20, gives 5 £. 8 *s.*

*yds.* *yds.* *s.*  
4 — 24 — 18  
18

192  
24

4)432( 108 ſhillings.

2,0) 10,8 ſhillings.

5 pounds.

Anſwer 5 £. 8 *s.*

QUEST. II. *If I lend* 200 £. *for* 12 *months, how long ought I to have the uſe of* 150 £. *to recompence me?*

Here the anſwer or 4th term is to be time; therefore let 12 months, the given time, be ſet for the 3d term: Now it is evident, that the 150 £. being leſs than the 200 £. muſt be kept a longer time, and ſo the 4th term will be greater than the 3d term: Therefore the 200 is put for the 2d term, and the 150 for the 1ſt. Then the 2d term multiplied by the 3d, the Product will be 2400; which being divided by the 1ſt term, the Quotient 16 is the 4th term; and becauſe the 3d term was months, the 4th term will be months.

£. £. *m.*  
150 — 200 — 12  
12

15,0)240,0(16 months.  
15

90  
90

Anſwer 16 months.

Quest. III. *What will* 1836*lb. of raiſins come to, at the rate of* 6*s.* 8*d. for* 24*lb.?*

Here as money is the thing ſought, money muſt be the 3d term: And as 6*s.* 8*d.* conſiſts of two names, they muſt be reduced to one name, *viz.* pence.

```
lb.     lb.         s.  d.
24 —— 1836 —— 6   8
                    6   8
      1836         12
        80         —
      ———        80d.=3d term.
24)146880(6120d.=4th term.
   144        ———
   ———  | 12)6120
    28  |     ———
    24  | 2,0)51,0 ( 10s.
    ——        ———
     48       25£.
     48
     ——
      0
```

Here the 2d term being multiplied by the 3d, and the Product divided by the firſt, the quotient is 6120 pence; which being valued, gives 25£. 10*s.*

Quest. IV. *If* 20 *yards of cloth,* 5 *quarters wide, will ſerve to hang a room: How many yards of* 4 *quarters wide will ſerve to hang the ſame room?*

Here yards of length are required; then 20 yards muſt be the 3d term.

```
qrs.    qrs.    yds.
 4 —— 5 —— 20
              5
             ——
         4)100(
         ———
Anſwer 25 yards.
```

Quest. V. *What will* 420 *yards of cloth come to, at* 14*s.* 10¾*d. for* 1 *ell Engliſh?*

The term ſought being money, the 14*s.* 10¾*d.* muſt be the 3d term, and be reduced to farthings; alſo the 1ſt and 2d terms are to be reduced to quarters of a yard.

```
Ell Eng. yds.        s.    d.
   1 —— 420 —— 14   10¾
   1      420        14   10¾
   5        4        12
   —      ———        ——
   5     1680     | 178
          715     |   4
         ————     |  ——
         8400     | 715 far.=3d term.
        1680
       11760
      ——————
    5)1201200(
      ———————
     4)240240  farthings=4th term.
      ———————
    12)60060  pence.
      ———————
     2,0)500,5   5 ſhillings.
      ———————
        250   pounds.
    Anſwer 250£. 5s.
```

The Diviſor 5 being a ſingle digit, the Quot. is written under the Divid.

Quest. VI. *A owes to B* 463£. *but compounds for* 7*s.* 6*d. in the pound: How much muſt B receive for his debt?*

Here compoſition money is the thing ſought; then the 3d term muſt be the compoſition money, *viz.* 7*s.* 6*d.*

```
£.     £.          s.   d.
1 —— 463 ——  7    6
        90      | 12
      ————      | —
  12)41670(6d.  | 90 pence.
  2,0)347,2 (12s.
      ———
      173
Anſwer 173£. 12s. 6d.
```

48. As it will be more convenient in moſt caſes to reduce ſuch numbers, or terms, which conſiſt of ſeveral names, to the fractional parts of their greateſt name, than to reduce them to their loweſt name; therefore in the ſolution of ſome of the following queſtions, the inferior parts of the given terms are reduced by Caſe IV. of Reduction; and the anſwers are valued by Caſe V.

QUEST. VII. *If 8lb. of pepper coſt* 4*s.* 8*d.* : *What will* 7 *C.* 3*qrs.* 14*lb. come to at that rate?*

lb. C. qrs. lb. s. d.
8 —— 7 3 14 —— 4 8
4 12
—— ——
31 56
28 882lb. ——
—— 56
262 ——
62 5292
—— 4410
882 ——
—— 8)49392
——
12)6174 6*d.*
——
2,0)51,4 14*s.*
——
25 £.

Anſwer 25 £. 14*s.* 6*d.*

QUEST. IX. *What is the intereſt of* 584 £. *for a year, at* 5 *per cent. per annum: Or at the rate of* 5 £. *for the uſe of* 100 £. *for a year?*

Here intereſt is the term required; therefore 5 £. the intereſt of 100 £. is to be the 3d term: And as the 4th term, or the intereſt of 584 £. is greater than the 3d term; then the 2d term is to be greater than the 1ſt.

£. £. £.
100 —— 584 —— 5
5
——
1,00)29,20( See Caſe V. of Reduction.
——
20
——
4,00
——

Anſwer 29 £. 4 *s.*

QUEST. XI. *What is the intereſt of* 542 £. 10*s. for* 219 *days, at* 5 £. *per cent. per annum?*

To ſolve this queſtion, find the intereſt for 1 year; multiply this intereſt by 219, and divide the Product by 365, the Quotient will be the anſwer; and is 16 £. 5*s.* 6*d.*

QUEST. VIII. *One bought* 4 *Hhds. of ſugar, each containing* 6 *C.* 2*qrs.* 14*lb. at* 2 £. 8*s.* 6*d. for each C. weight: What did the whole come to?*

C. C. qrs. lb. £. s. d.
1 —— 6 2 14 —— 2 8 6

Now 1 C. weight is 112lb.
And 4 Hh. at 6C. 2q. 14lb = 2968lb.
Alſo 2 £. 8*s.* 6*d.* is 582*d.*
Then the Product of the 2d and 3d terms is 1727376.
Which divided by the 1ſt term 112, the Quotient is 15423 pence, whoſe value is 64 £. 5*s.* 3*d.*

QUEST. X. *What is the intereſt of* 387 £. 12*s. for three years and* 4 *months, at* $3\frac{1}{2}$ *per cent. per annum?*

Find the intereſt for 1 year; then thrice that, together with $\frac{1}{3}$ of one year, will be the intereſt ſought.

£. £. £.
100 —— 387,6 —— 3,5
3,5
——
19380
11628
——
100) 1356,60(
13,566 for 1 year.
3
——
40,698 for 3 years.
$\frac{1}{3}$ of 1 year = 4,522 for 4 months.
The ſum 45,220 is the intereſt.
——

Anſwer 45 £. 4*s.* 5*d.*

QUEST. XII. *For how long muſt* 487 £. 10*s. be at ſimple intereſt, at* $4\frac{1}{3}$ £. *per cent. per annum. to gain* 95 £. 1*s.* 3*d.*?

Find what will be the intereſt of 487 £. 10*s.* for 1 year; divide 95 £. 1*s.* 3*d.* by this intereſt, and the Quotient will be $4\frac{1}{2}$ years.

QUEST. XIII. *One bought* 14 *pipes of wine, and is allowed* 6 *months credit: But for ready money gets it* 6*d.* *in a gallon cheaper: How much did he ſave by paying ready money?*

Anſwer 44£. 2 [illegible]

QUEST. XV. *One bought* 3 *tons of oil for* 153£. 9*s.* *which having leaked* 74 *gallons, he would make the prime-coſt of the remainder: How muſt it be ſold per gallon?*

Now 1 T. = 252 Gall. And 3 T. = 756
Subtract the gallons leaked = 74

Remains 682

G. G. £.
Then 682 —— 1 —— 153,45
Anſwer 4*s.* 6*d.* a gallon.

QUEST. XVII. *At* 13£. *for* 100*lb.* *of goods: What will* 895*lb.* *come to, allowing* 4*lb.* *upon every* 100*lb.* *for tret, or waſte?*

Since 4lb. is to be allowed on the 100lb. therefore 104lb. is given for 100.

lb. lb. £.
Then 104 —— 895 —— 13

Anſwer 111£. 17*s.* 6*d.*

QUEST. XIX. *If* 100 *pounds of ſugar be worth* 36*s.* 8*d.* *What will be the worth of* 875*lb.* *rebating* 4*lb.* *upon every* 100*lb.* *for tare?*

Here the buyer has 100lb. on paying for 96lb.

lb. lb. lb.
Then 100 —— 96 —— 875
And the 4th term will be 840lb.
Alſo 100 —— 840 —— 1,833333
Then the 4th term will be 15£. 8*s.* and ſo much will the ſugar come to.

QUEST. XIV. *A clothier ſold* 50 *pieces of kerſey, each piece containing* 34 *ells Flemiſh, at the rate of* 8*s.* 4*d.* *per ell Engliſh: What did the whole come to?*

Anſwer 425£.

QUEST. XVI. *A broker ſold* $\frac{2}{5}$ *of* $\frac{3}{4}$ *of a ſhip for* 147£. 11*s.* 3*d.*: *How much was the whole ſhip valued at?*

Now $\frac{2}{5}$ of $\frac{3}{4} = \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$ by art. 38.
For 2 × 3 = 6, a new numerator.
And 5 × 4 = 20, a new denominator.
Alſo 147£. 11*s.* 3*d.* = 147,5625. art. 40.
ſhare ſhare £.
Then 0,3 —— 1 —— 147,5625 art. 46.

Anſwer 491£. 17*s.* 6*d.*

QUEST. XVIII. *One has cloth which coſt* 2*s.* 8*d.* *a yard: For how much muſt it be ſold a yard on* 3 *months credit, to gain* 25£. *per cent. per annum?*

mon. mon. £. £.
Firſt 12 — 3 — 25 — 6,25
£. £. £.
Secondly 100 — 6,25 —— 0,133333
By multiplying and dividing, the 4th term will be found 2*d.*
Then 2*s.* 8*d.* + 2*d.* = 2*s.* 10*d.* a yard, the ſelling price.

QUEST. XX. *A chapman bought* 81 *kerſeys for* 135£.: *How muſt he ſell them per piece to gain* 15£. *per cent?*

Find how much 135£. will be advanced to, at 15£. per cent.

Then this ſum divided by 81 will be the ſelling price of each piece.

£. £. £. £.
Now 100 — 115 — 135 — 155,25
Then 81) 155,25 (1,916666£.
Anſwer 1£. 18*s.* 4*d.* a-piece.

Quest. XXI. *A merchant who is to receive a ſum of money, is offered ducats at* 6s. 4d. *which are worth but* 6s. 2½d. *or chequins at* 8s. 2d. *each, that are worth but* 8s: *By which ſpecie will he ſuſtain the leaſt loſs?*

6s. 2½d.=74,5d. } the real value.
8s. 0 d.=96
6s. 4 d.=76 } the advan. val.
8s. 2 d.=98

r. val. r. val. ad. val. ad. val.
Then 74,5 —— 96 —— 76 —— 97,93

But the chequins are valued at 98 d. Therefore the ducats are moſt advantageous.

Quest. XXII. *One who had ſold a parcel of cloths at* 2s. 10d. *a yard on* 3 *months credit, found he had gained* 25£. *per cent. per annum: What did the cloth coſt per yard?*

mo. £. mo. £.
Now 12——25——3——6,25
And 100+6,25=106,25 £.
£. £. £.
Then 106,25—100—0,14166

The fourth term to which will be a fraction, the value of which will be 2s. 8d. which is the prime coſt per yard of the cloth.

Quest. XXIII. *A perſon wants* 750 *pieces of foreign coin, each worth* 11s. 4d. *How much will they come to, allowing the broker the worth of* 2 *pieces upon every* 100?

Now 100 —— 102 —— 750 —— 765. He muſt pay for 765 pieces, which will come to 433 £. 10s.

Quest. XXIV. *A gentleman would exchange* 729 *pieces of* 4s. 2d. *each into ſterling money: How much will he receive for them, allowing the broker* 1¼£. *per cent.?*

P. P. £. £.
Now 1—729—0,208333—151,875
the worth of the pieces.
Then 101,25—100—151,875—150£.
He will receive 150£. for them.

Quest. XXV. *A grocer bought* 4¼ *C. of pepper for* 15£. 17s. 4d. *which proving to be damaged, he is willing to loſe* 12½£. *per cent. For how much muſt he ſell it a lb.?*

Since he is to loſe 12½ per cent. he muſt take 87£. 10s. for 100£. Now diminiſh the 15£. 17s. 4d. in this proportion, and this ſum divided by the pounds in 4¼ C. will give 7d. for what each pound is to be ſold at.

Quest. XXVI. *Suppoſe* 42 *gallons of honey be valued at* 2£. *and the duty is* 15£. *per cent. on this value, and a drawback of* 5£. *per cent. on the duty for prompt payment: What will the ready money duty of* 672 *gallons come to?*

Now 42G. : 672G. : : 2£. : 32£.
And 100£. : 15£. : : 32£. : 4,8£.
Alſo 100£. : 95£. : : 4,8£. : 4,56£.

Anſwer 4£. 11s. 2½d.

The Rule of Proportion is of almoſt univerſal uſe in all buſineſs where computation is required; as in buying and ſelling, values of ſtocks and their dividends; the intereſt and diſcount of money; the cuſtoms and duties on goods, &c. But the deſigned brevity of this book will not permit farther illuſtrations.

## SECTION VIII. OF THE POWERS OF NUMBERS, AND OF THEIR ROOTS.

49. *The* POWER *of a number, is a product ariſing by multiplying that number by itſelf, the product by the ſame number, this product by the ſame number again, &c. to any number of multiplications.*

50. The given number is called the firſt power or root.
The Product of the 1ſt power by itſelf, is the ſecond power, or ſquare.
The Product of the 2d power by the 1ſt, is the 3d power, or cube.
The Product of the 3d power by the 1ſt, is the 4th power, &c.

51. Here follow the 1ſt, 2d, and 3d powers of the nine digits.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Roots, or 1ſt power | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Squares, or 2d power | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| Cubes, or 3d power | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 |

Ex. I. *What is the 2d power, or ſquare of the number* 24*?*

24 × 24 = 576 is the 2d power.

Ex. II. *What is the 3d power, or cube of* 38*?*

Now 38 × 38 = 1444 the 2d power.
Then 1444 × 38 = 54872 the 3d power.

The figure, or number, ſhewing the name of any power, is called the index of that power.

Thus 1 is the index of the firſt power: 2 is the index of the 2d power; 3 of the third power, *&c.* Alſo $\frac{1}{2}$ is the index of the ſquare root; $\frac{1}{3}$, the index of the cube root, *&c.*

52. Any number may be conſidered as a power of ſome other number.

Thus 64 may be taken as the 2d power of 8, and the third power of 4, *&c.*

53. The root of a given number, conſidered as a power, is a number which being raiſed to the index of that power, will either be equal to the given number, or approach very near to it.

### 54. *To extract the Square Root of a given number.*

RULE 1ſt. Begin at the unit's place, put a point over it, and alſo over every next figure but one, reckoning to the left for integers, and to the right for fractions; and there will be as many integer places in the root, as there are points over the integers in the given number.

The figure under a point, with its left-hand place, is called a period.

2d. Under the left-hand period write the greateſt ſquare contained in it, and ſet the root thereof in the Quotient; ſubtract the ſquare, and to the remainder bring down the next period, as in Diviſion.

3d. On the left of this Remainder write the double of the Root or Quotient for a Diviſor; ſeek how often this may be had in the Remainder, except the right-hand place; write what ariſeth both in the Root, and on the right of the Diviſor.

4th. Multiply this increaſed Diviſor by the laſt Quotient-figure; ſubtract, and to the Remainder bring down the next period; double the Root for a Diviſor, and proceed as before.

55. Fractional places will ariſe in the Root, by annexing to the Remainders, periods of two cyphers each, and renewing the operation.

Ex. I. *What is the Square Root of* 1444?

Put a point over the units place 4, and alſo over the place of 100s. Now the number conſiſts of 2 periods, and will have 2 integer places in the Root: Then the greateſt Square in 14, the left-hand period, is 9, and its Root is 3; write 9 under the period, and three in the Root; now 9 from 14 leaves 5, to which annex the next period 44; the Root 3 doubled makes 6, which in 54 is contained 8 times, annex 8 to the 3 in the Quotient, and to the Diviſor 6, makes the Root 38 and the Diviſor 68; then 8 times 68 is 544; and there remaining 0, on ſubtraction, it may be concluded, that 38 is the true Root.

$1\dot{4}4\dot{4}$ (38 Root.
9
68) 544
544

Ex. II. *What is the Square Root of* 36372961?

$3\dot{6}3\dot{7}2\dot{9}6\dot{1}$ (6031 Root.
36
1203) 3729
3609
12061) 12061
12061

Ex. IV. *What is the Square Root of* 24681024?

Anſwer 4968.

Ex. VI. *What is the Square Root of* 76395820?

$7\dot{6}3\dot{9}5\dot{8}2\dot{0}$ (8740,4702
64
167 ) 1239
1169
1744 ) 7058
6976
174804 ) 822000
699216
1748087 ) 12278400
12236609
174809402 ) 417910000
349618804
68291196

Ex. III. *What is the Square Root of* 1,0609?

$\dot{1},0\dot{6}0\dot{9}$ ( 1,03 Root.
1
203 ) 0609
609

Ex. V. *What is the Square Root of* 911236798,794365?

Anſwer 30186,699, &c.

$\dot{9}1\dot{1}2\dot{3}6\dot{7}9\dot{8},7\dot{9}4\dot{3}6\dot{6}$ ( 30186,6
601 ) 1123
6028 ) 52267
60366 ) 404398
603726 ) 4220279
597923
&c.

Here the products are omitted, the multiplication and ſubtraction being made in the mind.

In the VIth Example, after all the periods given were brought down, there remained 8220, to which a period of two cyphers was annexed, and the operation renewed, and continued until 4 decimal places were obtained in the Root; every period brought down giving one place.

## 56. *To extract the Cube Root of a given Number.*

Rule 1ſt. Over the unit place of the given number put a point, and alſo over every third figure from the unit place, to the left for integers, and to the right for fractions; and the root will have as many integer places, as there are points, or periods, in the integral part of the given number.

2d. Under the left hand period, write the greateſt Cube it contains, the root of which ſet in the Quotient: Subtract the Cube from the period, and to the Remainder annex the remaining periods; call this the Reſolvend.

3d. To the Quotient annex as many cyphers as there were periods remaining; call this the Root.

4th. Divide the Reſolvend by the Root, add the Quotient to thrice the Square of the Root, let the Sum be a Diviſor to the Reſolvend, and the Quotient-figures annexed to the right of the firſt Root, without the cyphers, will be the Cube Root ſought.

5th. If the ſecond figure of the Root be 1, or 0; then generally 3 or 4 figures of the Root will be obtained at the firſt operation: But if the ſecond figure exceeds 2, it will be beſt to find only two places at firſt.

6th. To renew the operation; ſubtract the Cube of the figures found in the Root from the given number; then form a Diviſor, and divide as directed in the fourth precept; and this will give the Root true to 5 or ſix places: For each operation commonly triples the figures found in the laſt Root.

### Ex. I. *What is the Cube Root of* 9800344?

```
        ·  ·  ·
        9800344(2
        8
        -------
2,00) 18003,44 Reſolvend.
        -------
        9001 = Quotient.
        120000=thrice the Sq. of the R.
        ------
        129001)1800344(14
                129001
                -------
                 510344
                 516004
                 -------
               The Root is 214.
```

Put a point over the unit place 4, another over the place of thouſands, and another over that of millions; and becauſe there are 3 points, there will be 3 places in the Root. Under the left hand period 9, write 8, the greateſt Cube in it, and its Root 2 write in the Quotient, then ſubtracting, the Reſolvend is 1800344: Now becauſe there are two periods remaining, therefore two cyphers annexed to the Root 2, make it 200, by which dividing the Reſolvend, the Quotient is 9001; alſo the ſquare of 200 is 40000, the triple thereof 120000 being added to 9001, makes 129001 for a Diviſor, by which dividing 1800344, the Quotient is 14 nearly, and is taken as 14, becauſe it is much nearer to it than to 13; now 14 being annexed to the former Root 2, makes 214, the Root ſought.
For 214×214×214=9800344.

**Ex. II.** *What is the Cube Root of* 518749442875?

·  ·  ·  ·
518749442875(8
512

8,000) 6749442,875

843680
192000000

192843680( 6749442875 (035
578531040

964132475
964218400

In this example, becauſe 3 periods were remaining, and conſequently 3 places more to be found; therefore in the laſt diviſion a point is put over the 3d place from the right hand, and the Diviſor is firſt to be tried in the Dividend as far as this point, in which as it cannot be taken, 0 is put in the Quotient, *&c.* here the laſt figure 5 is too much, but it is much nearer to 5 than to 4; then 035 annexed to the firſt Root 8, makes 8035 for the Root.

Ex. III. *What is the Cube Root of* 114604290,028?

·  ·  ·  ·
114604290,028
64

4,00) 50604290

126510
480000

606510) 50604290 (8

48
48

384
192

2304
48

18432
9216

110592

Here 480 is taken for the Root at the firſt operation.

Then 114604290,028(48
110592

480) 4012290 The work of the Diviſion is ſuppoſed to be done on a waſte paper.

8358,9 the Quotient.
691200 = triple the Square of 480, *viz.* 230400 × 3.

Diviſor 699558,9) 4012290,028 (5736
34977945

5144955
4896912

248043
209807

38176

To 480 the firſt Root
Add 5,736

Sum 485,736

57. Here, inſtead of bringing down the figures of the Dividend to the Remainders, the Diviſor is leſſened each time, by pointing off a place on the right; but regard is to be had to the carriage which will ariſe from the places thus omitted.

## SECTION IX. OF NUMERAL SERIES.

58. *A rank of three or more numbers that increaſe or decreaſe by an uniform progreſſion, is called a Numeral Series.*

59. If the Progreſſion is made by equal differences, that is by the conſtant addition or ſubtraction of the ſame number; the ſeries is called an *Arithmetic Progreſſion.*

Thus
1 2 3 4 5 6 7 8 9 &c. increaſing by adding 1,
3 6 9 12 15 18 21 24 27 &c. increaſing by adding 3,
49 43 37 31 25 19 13 7 1 &c. decreaſing by ſubducting 6,

are ranks of numbers in Arithmetic Progreſſion: And of ſuch ranks there may be an infinite variety.

60. If the Progreſſion is made by a conſtant multiplication or diviſion with the ſame number, the ſeries is called a *Geometric Progreſſion.*

Thus
1 2 4 8 16 32 64 &c. increaſing by 2,
1 5 25 125 625 3125 15625 &c. increaſing by 5,
6561 2187 729 243 81 27 9 3 &c. decreaſing by 3,
16384 4096 1024 256 64 16 4 1 &c. decreaſing by 4,

are ranks of numbers in Geometric Progreſſion: And of ſuch ranks there may be an infinite variety.

61. The common Multiplier or Diviſor is called the *ratio.*

*Thus* 2 *is the ratio in the* 1*ſt rank,* 5 *in the* 2*d rank,* 3 *is the ratio in the* 3*d rank, and* 4 *in the* 4*th rank.*

62. In any ſeries of terms in Arithmetic Progreſſion, the ſum of any two terms, conſidered as extremes, is equal to the ſum of any two terms taken as means equally diſtant from the extremes.

*Thus in* 3 *terms (where the* 1*ſt and* 3*d are extremes, and the other the mean) viz.* 6 . 9 . 12, then 6+12=9+9=18.
*And in* 4 *terms, viz.* 13 . 19 . 25 . 31.
*Then* 13+31=19+25=44.
*Alſo in the terms* 49 . 43 . 37 . 31 . 25 . 19 . 13 . 7 . 1.
*Then* 49+1=43+7=37+13=31+19=25+25=50.

63. In a ſeries of terms in Geometric Progreſſion, the Product of any two terms conſidered as extremes, is equal to the Product of any two intermediate equidiſtant terms conſidered as means.

*Thus in* 3 *terms, viz.* 5 . 25 . 125. *Or* 3 . 9 . 27.
*Then* 5×125=25×25=625. *Alſo* 3×27=9×9=81.
*And in* 4 *terms* 4 . 8 . 16 . 32.
*Then* 32×4=16×8=128.
*Alſo in the terms* 1 . 4 . 16 . 64 . 256 . 1024 . 4096 . 16384.
*Then* 16384×1=4096×4=1024×16=256×64=16384.

64. In any Arithmetic Progreſſion, the ſum of any two terms leſſened by the firſt term; or their difference increaſed by the firſt term, will be a term alſo in that progreſſion.

*Thus in the Progreſſion* 1 . 3 . 5 . 7 . 9 . 11 . 13 . 15 . 17 . 19 . 21 *&c.*
*Then* $7+11=18$, and $18-1=17$ is a term of the Progreſſion.
*Alſo* $11-7=4$, and $4+1=5$ is a term of the Progreſſion.

65. In any Geometric Progreſſion, the product of any two terms divided by the firſt term; or the Quotient of any two terms multiplied by the firſt term, will give a term alſo in that ſeries.

*Thus in the Progreſſion* 3 . 6 . 12 . 24 . 48 . 96 . 192 . 384 . 768 *&c.*
*Then* $\frac{12\times96}{3}=384$; and $\frac{192}{12}\times3=48$, are terms in the Progreſſion.

66. If over a ſeries of terms in Geometric Progreſſion, be written a ſeries of terms in Arithmetic Progreſſion, the firſt term of which is 0, and common difference is 1, term for term; then any term in the Arithmetic Series, will ſhew how far its correſponding term in the Geometric Series is diſtant from the firſt term.

$$\textit{Thus}\ \begin{cases} 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \ \text{\&c. Arithmetic Series.} \\ 1 \quad 3 \quad 9 \quad 27 \quad 81 \quad 243 \quad 729 \ \text{\&c. Geometric Series.} \end{cases}$$

Here 729 is diſtant from the 1ſt term, 6 terms; 243 is diſtant 5 terms, 81 is diſtant 4 terms.

67. The terms of the Arithmetical Series are called indices to the terms of the Geometric Series.

*Thus* 5 *is the index to* 243; 3 *is the index to* 27; 1 *is the index to* 3; *&c.*

68. PROBLEM I. *In an Arithmetic Progreſſion: Given the firſt term, the common difference, and the number of terms.*
*Required the laſt term.*

RULE. Subtract 1 from the number of terms, multiply the remainder by the common difference; to the product add the firſt term, and the ſum will be the laſt term.

EX. I. *Suppoſe* 1 *and* 9 *to be the firſt and ſecond terms, of an Arithmetic Progreſſion of* 1074 *terms: What is the laſt term?*

Here $9-1=8$ is the com. diff. Now $1074-1=1073$.
And $1073\times8=8584$. Then $8584+1=8585=$laſt term.

EX. II. *A perſon agrees to diſcharge a certain debt in a year, by weekly payments,* viz. *the firſt week* 5*s.* *the 2d week* 8*s.* &c. *conſtantly increaſing each week by* 3*s.*: *How much was the laſt payment?*

| | |
|---|---|
| $5=$1ſt. term. | Now $52-1=51$. |
| $3=$com. diff. | And $51\times3=153$. |
| $52=$N° of terms. | Then $153+5=158$ *s.* $=7$ £. 18 *s.* $=$laſt Payment. |

69. PROBLEM II. *In an Arithmetic Progreſſion: Given the firſt term, laſt term, and the number of terms.*
*Required the ſum of all the terms.*

RULE. Add the firſt and laſt terms together, the ſum multiplied by half the number of terms, gives the ſum of all the terms.

Ex. I. *Required the ſum of the firſt* 1000 *numbers in their natural order.*

Here 1 = 1ſt term, 1 = com. diff.
1000 = N° of terms, its $\frac{1}{2}$ is 500.
Now 1000 + 1 = 1001.
Then 1001 × 500 = 500500 is the ſum required.

Ex. II. *A debt is to be diſcharged in a year by weekly payments equally increaſing; the 1ſt to be* 5*s. and the laſt* 7*£.* 18*s. How much was the debt?*

Here 7*£.* 18*s* = 158*s.* = laſt term.
52 = N° of terms, its $\frac{1}{2}$ is 26.
Now 158 + 5 = 163.
Then 163 × 26 = 4238*s.* = 211*£.* 18*s.* is the ſum of the terms, or debt.

Ex. III. *Suppoſe a baſket and* 500 *ſtones were placed in a ſtraight line, a yard diſtant from one another: Required in what time a man could bring them one by one to the baſket, allowing him to walk at the rate of* 3 *miles an hour?*

Between the baſket and ſtones are 500 ſpaces, which is the number of terms.
Now 500 + 1 = 501. Then 501 × 250 = 125250 = ſum of the terms.
But as he goes backwards and forwards, he walks 250500 yards.
Which divided by 1760 (the yards in 1 mile) gives 142,329 miles.
Which at 3 miles an hour, will take 47 h. 26 min. 35 ſeconds nearly.

70. PROBLEM III. *In a Geometric Progreſſion: Given the firſt term, the ratio and the laſt term.*
*Required the ſum of all the terms.*

RULE. Multiply the laſt term by the common ratio, from the Product ſubtract the firſt term for a Dividend.

Subtract 1 from the ratio for a Diviſor; then divide, and the Quotient will be the ſum of all the terms.

Ex. I. *Suppoſe the firſt term of a ſeries to be* 3, *the ratio* 3, *and the laſt term* 6561: *Required the ſum of all the terms.*

Now 6561 = laſt term. And 3 = ratio.
Mult. by 3 = ratio. Sub. 1
19683 = Product. Rem. 2 = Diviſor.
Subtr. 3 = firſt term.
19680 = Dividend.

Then 2)19680(9840 is the ſum of all the terms.

Ex. II. *Let the firſt term be* 2, *the ſecond term* 10, *and the laſt term* 156250: *Required the ſum of all the terms.*

Here 2) 10 (5 is the common ratio.
Now 156250 × 5 = 781250. And 781250 − 2 = 781248 = Dividend.
Alſo 5 − 1 = 4 the Diviſor.
Then 4) 781248 (195312 is the ſum of all the terms.

71. **Problem IV.** *In a Geometric Progreſſion: Given the firſt term, the ratio, and the number of terms.*

*Required the laſt term.*

**Rule** 1ſt. Write down 6 or 7 of the leading terms in the Geometric Series, and over them their Indices.

2d. Add together the moſt convenient indices to make an index leſs by unity than the number expreſſing the place of the term ſought.

3d. Multiply together the terms of the Geometric Series, belonging to thoſe indices which were added; make the product a dividend.

4th. Raiſe the firſt term to a power whoſe index is one leſs than the number of terms multiplied; make the reſult a Diviſor to the former Dividend, and the Quotient will be the term ſought.

Ex. I. *What is the 12th term of a Geometric Series, the firſt term of which is 3, and ſecond term is 6?*

Now $\frac{6}{3} = 2$ is the common ratio.

$$\text{And}\left\{\begin{array}{l} 0 \quad 1 \quad\; 2 \quad\; 3 \quad\; 4 \quad\; 5 \quad\;\; 6\ \text{\&c. Indices.} \\ 3 \,.\, 6 \,.\, 12 \,.\, 24 \,.\, 48 \,.\, 96 \,.\, 192\ \text{\&c. Geometric terms.} \end{array}\right.$$

Then $6+5=11$, is the index to the 12th term.

And $192 \times 96 = 18432$, is the Dividend.

The number of terms multiplied together is 2; and $2-1=1$, the power to which the firſt term 3 is to be raiſed; but the firſt power of 3 is 3.

Then $\frac{18432}{3} = 6144$ is the 12th term of the given ſeries.

Ex. II. *A Perſon being aſked to diſpoſe of a fine horſe, ſaid he would ſell him on condition of having one farthing for the 1ſt nail in his ſhoes, two farthings for the 2d nail; one penny for the 3d nail; two pence for the 4th; four pence for the 5th; 8 pence for the 6th, &c.; doubling the price of every nail to 32, the number of nails in the four ſhoes: How much would that horſe be ſold for at that rate?*

Here the firſt term is 1, the ratio 2, and the number of terms 32.

Firſt. To find the laſt term.

$$\text{Now}\left\{\begin{array}{l} 0 \quad 1 \quad 2 \quad 3 \quad\; 4 \quad\; 5 \quad\;\; 6 \quad\;\;\; 7 \quad\;\;\; 8\ \text{\&c. Indices.} \\ 1 \,.\, 2 \,.\, 4 \,.\, 8 \,.\, 16 \,.\, 32 \,.\, 64 \,.\, 128 \,.\, 256\ \text{\&c. Geometric terms.} \end{array}\right.$$

And 31 is the index to the 32d term.

Then $8+8=16$; $16+8=24$; $24+7=31$.

The 1ſt term being 1, any power thereof is 1; ſo the 4th article of the rule is uſeleſs in this queſtion.

Now $256 \times 256 = 65536$ is the 17th term.

$65536 \times 256 = 16777216$ is the 25th term.

$16777216 \times 128 = 2147483648$ is the 32d term.

Then 2147483648
2
4294967296
—1 the 1ſt term.
$2-1=1$) 4294967295 the ſum of the terms: or the price, in farthings, of the horſe.

4 | 4294967295
12 | 1073741823 ¾ *f.*
20 | 89478485 - 3 *d.*
4473924 - 5 *s.*

Anſwer 4473924 £. 5*s.* 3¾*d.*

## SECTION X. OF LOGARITHMS.

72. Logarithms *are a ſeries of numbers ſo contrived, that by them the work of multiplication may be performed by addition; and the operation of diviſion may be done by ſubtraction.*

73. Or, Logarithms are the Indices to a ſeries of numbers in Geometrical Progreſſion.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Thus | 0 | 1 | 2 | 3 | 4 | 5 | 6 &c. | Indices or Logarithms. |
| | 1 | 2 | 4 | 8 | 16 | 32 | 64 &c. | Geometric Progreſſion. |
| or | 0 | 1 | 2 | 3 | 4 | | 5 &c. | Indices or Logarithms. |
| | 1 | 3 | 9 | 27 | 81 | | 243 &c. | Geometric Series. |
| or | 0 | 1 | 2 | 3 | 4 | | 5 &c. | Indices or Logarithms. |
| | 1 | 10 | 100 | 1000 | 10000 | | 100000 &c. | Geometric Series. |

Where the ſame Indices ſerve equally for any Geometric Series.

74. Hence it is evident, there may be as many kinds of Indices or Logarithms, as there can be taken kinds of Geometric Series.

But the Logarithms moſt convenient for common uſes, are thoſe adapted to a Geometric Series increaſing in a tenfold Progreſſion, as in the laſt of the examples above.

75. In the Geometric Series 1 . 10 . 100 . 1000 . &c. between the terms 1 and 10, if the numbers 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 were interpoſed, to them might Indices be alſo adapted in an Arithmetic Progreſſion, ſuited to the terms interpoſed between 1 and 10, conſidered as a Geometric Progreſſion: Alſo proper Indices may be found to all the numbers that can be interpoſed between any two terms of the Geometric Series.

But it is evident that all the Indices to the numbers under 10 muſt be leſs than 1; that is, are fractions: Thoſe to the numbers between 10 and 100 muſt fall between 1 and 2; that is, are mixed numbers conſiſting of 1 and ſome fraction: And ſo the Indices to the numbers between 100 and 1000 will fall between 2 and 3; that is, are mixed numbers conſiſting of two and ſome fraction: And ſo of the other Indices.

76. Hereafter, the integral part only of theſe Indices will be called the Index; and the fractional part will be called the Logarithm: And the computing of thoſe fractional parts is called the making of Logarithms; the moſt troubleſome part of this work is to make the Logarithms of the *prime numbers*; that is, of ſuch numbers which cannot be divided by any other number than by itſelf and unity.

77. *To find the Logarithms of prime numbers.*

Rule 1ſt. Let the ſum of the propoſed number and its next leſs number be called A.

2d. Divide 0,868588963 * by A, reſerve the Quotient.

---

* The number 0,868588963 is the Quotient of 2 divided by 2,302585093, which is the Logarithm of 10, according to the firſt form of the Lord *Nepier*, who was the inventor of Logarithms. The manner by which *Nepier*'s Log. of 10 is found, may be ſeen in many books of Algebra; but is here omitted, becauſe this treatiſe does not contain the elements of that ſcience: However, thoſe who have not opportunity to enter thoroughly into this ſubject, had better grant the truth of one number, and thereby be enabled to try the accuracy of any Logarithm in the tables, than to receive thoſe tables as truly computed, without any means of examining the certainty thereof.

3d. Divide the reſerved Quotient by the Square of A, reſerve this Quotient.

4th. Divide the laſt reſerved Quotient by the Square of A, reſerving the Quotient; and thus proceed as long as diviſion can be made.

5th. Write the reſerved Quotients orderly under one another, the firſt being uppermoſt.

6th. Divide theſe Quotients reſpectively by the odd numbers 1 . 3 . 5 . 7 . 9 . 11, *&c.* that is, divide the firſt reſerved Quotient by 1, the 2d by 3, the 3d by 5, the 4th by 7, *&c.* let theſe Quotients be written orderly under one another, add them together, and their ſum will be a Logarithm.

7th. To this Logarithm, add the Logarithm of the next leſs number, and the ſum will be the Logarithm of the number propoſed.

Ex. I. *Required the Logarithm of the number* 2.

Here the next leſs number is 1, and $2+1=3=A$.
And the Square of A is 9. Then;

$$\frac{0,868588963}{3}=,289529654. \qquad \text{And} \qquad \frac{,289529654}{1}=,289529654$$

$$\frac{0,289529654}{9}=,032169962. \qquad \& \qquad \frac{,032169962}{3}=,010723321$$

$$\frac{0,032169962}{9}=,003574440. \qquad \& \qquad \frac{,003574440}{5}=,000714888$$

$$\frac{0,003574440}{9}=,000397160. \qquad \& \qquad \frac{,000397160}{7}=,000056737$$

$$\frac{0,000397160}{9}=,000044129. \qquad \& \qquad \frac{,000044129}{9}=,000004903$$

$$\frac{0,000044129}{9}=,000004903. \qquad \& \qquad \frac{,000004903}{11}=,000000445$$

$$\frac{0,000004903}{9}=,000000545. \qquad \& \qquad \frac{,000000545}{13}=,000000042$$

$$\frac{0,000000545}{9}=,000000060. \qquad \& \qquad \frac{,000000060}{15}=,000000004$$

To this Log. 0,301029994
Add the Log. of 1=0,000000000

Their ſum is the Log. of 2=0,301029994

This proceſs needs no other explanation than comparing it with the rule.

That the manner of computing theſe Logarithms may be familiar to the Reader, the operations of making ſeveral of them are here ſubjoined.

Ex. II. *Required the Logarithm of the number* 3.

Here the next leſs number is 2; and $3+2=5=A$, whoſe Square is 25.

$$\frac{0,868588963}{5}=,173717792. \text{ And } \qquad \frac{,173717792}{1}=,173717792$$

$$\frac{0,173717792}{25}=6948712. \ \& \qquad \frac{6948712}{3}=,002316327$$

$$\frac{0,006948712}{25}=277948. \ \& \qquad \frac{277948}{5}=,000055590$$

$$\frac{0,000277948}{25}=11118. \ \& \qquad \frac{11118}{7}=,000001588$$

$$\frac{0,000011118}{25}=445. \ \& \qquad \frac{445}{9}=,000000049$$

$$\frac{0,000000445}{25}=18. \ \& \qquad \frac{18}{11}=,000000002$$

| | |
|---|---|
| To this Logarithm | 0,176091258 |
| Add the Log. of 2 | 0,301029994 |
| The ſum is the Logarithm of 3 | 0,477121252 |

78. Since the Logarithms are the Indices of numbers conſidered in a Geometric Progreſſion; therefore the ſums, or differences of theſe Indices, will be Indices or Logarithms belonging to the Products, or Quotients, of ſuch terms in the Geometric Progreſſion as correſpond to thoſe Logarithms which were added or ſubtracted (71).

Ex. III. *Required the Log. of* 4.

Now $4=2\times2$.

| | |
|---|---|
| Then to the Log. of 2 | 0,301029994 |
| Add the Log. of 2 | 0,301029994 |
| Sum is the Log. of 4 | 0,602059988 |

Ex. V. *Required the Log. of* 10.

In the original Series, 1 is aſſumed for the Logarithm of 10.

Ex. VII. *Required the Log. of* 8.

Now $8=2\times2\times2$.

Therefore the Log. of 2 taken thrice gives the Log. of the number 8

| | |
|---|---|
| The Log. of 2 is | 0,301029994 |
| Which multiplied by | 3 |
| Gives the Log. of 8 | 0,903089982 |

Ex. IV. *Required the Log. of* 6.

Now $3\times2=6$.

| | |
|---|---|
| Then to the Log. of 3 | 0,477121252 |
| Add the Log. of 2 | 0,301029994 |
| Sum is the Log. of 6 | 0,778151246 |

Ex. VI. *Required the Log. of* 5.

Now 10 divided by 2 gives 5.

| | |
|---|---|
| Then from Log. of 10 | 1,000000000 |
| Take Log. of 2 | 0,301029994 |
| Leaves the Log. of 5 | 0,698970006 |

Ex. VIII. *Required the Log. of* 9.

Now $9=3\times3$.

Therefore the Log. of 3 doubled gives the Log. of 9.

| | |
|---|---|
| The Log. of 3 is | 0,477121252 |
| Which multiplied by | 2 |
| Gives the Log. of 9 | 0,954242504 |

Ex. IX. *Required the Logarithm of* 7.

Here 6 is the next leſs. Then $7+6=13=A$; and $169=$Square of A.

$$\frac{0{,}868588963}{13}={,}066814536. \quad \text{And} \quad \frac{{,}066814536}{1}={,}066814536$$

$$\begin{array}{llll}
\dfrac{0{,}066814536}{169}= & 395352. \ \& & \dfrac{395352}{3}= & 131784 \\[2ex]
\dfrac{0{,}000395352}{169}= & 2339. \ \& & \dfrac{2339}{5}= & 468 \\[2ex]
\dfrac{0{,}000002339}{169}= & 14. \ \& & \dfrac{14}{7}= & 2
\end{array}$$

| | |
|---|---|
| To this Logarithm | 0,066946790 |
| Add the Logarithm of 6 | 0,778151246 |
| The ſum is the Logarithm of 7 | 0,845098036 |

The Log. of 12 is equal to the ſum of the Logs. of 3 & 4, or of 2 & 6.
The Log. of 14 is equal to the ſum of the Logs. of 7 & 2.
of 15 of 3 & 5.
of 16 of 4 & 4, or of 8 & 2.
of 18 of 3 & 6, or of 9 & 2.
of 20 of 4 & 5, or of 10 & 2.

79. The Logs. of the prime numbers 11, 13, 17, 19, are to be found as in the examples I. II. IX. and in like manner is the Log. of any other prime number to be found; but it may be obſerved, that the operation is ſhorter in the larger prime numbers; for any number exceeding 400, the firſt Quotient added to the Logarithm of its next leſſer number, will give the Logarithm ſought, true to 8 or 9 places; and therefore it will be very eaſy to examine any ſuſpected Logarithm in the tables.

80. The manner of diſpoſing the Logarithms, when made, into tables, is various: But in this treatiſe they are ordered as follows.

*Any number under* 100, *or not exceeding two places, and its Logarithm, are found in the firſt page of the table*, where they are placed in adjoining columns; and diſtinguiſhed by the title *Num.* for the common numbers; and by *Log.* for the Logarithms.

Theſe tables are at the end of Book IX.

*A number of three or four places being given, its Logarithm is thus found.*

Seek for a page in which the given number ſhall be contained between the two numbers marked at the top, annexed to the letter N: Then right againſt the three firſt figures of the given number, found in the column ſigned *Num*, and in the column ſigned by the fourth, ſtands the Logarithm belonging to that number of four places.

If the number conſiſted of 3 places only; then theſe places found as before directed, the Logarithm ſtnads againſt them in the column ſigned 0.

Thus, *to find the Logarithm of* 5738. Seek for a page in which ſtands at top N° 5200 to 5800; then in the column ſigned N° find 573, right againſt which in the column ſigned 8 at top or bottom ſtands, 75876, which is the Logarithm to 5738, excluſive of its Index.

81. *A Logarithm being given, its number is thus found.*

Seek for a page in which the three firſt figures of the given Logarithm are found at top annexed to the letter *L*; then in one of the columns ſigned with the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, find a number the neareſt to the given Logarithm; againſt this number in the column ſigned N°, ſtand three figures; to the right of theſe annex the figure with which the column was ſigned at top or bottom, and this will be the number correſponding to the given Logarithm, not regarding the Index.

82. All numbers conſiſting of the ſame figures, whether they be integral, fractional, or mixed, have the fractional parts of their Logarithms the ſame.

If the following examples be well attended to, there will be no difficulty in finding the Logarithm to a propoſed number, or the number to a propoſed Logarithm, within the limits of the table of Logarithms here uſed.

| Num. | Logarithms. | Logarithms. | Numbers. |
|---|---|---|---|
| 5874 | 3,76893 | 0,37295 | 2,360 |
| 587,4 | 2,76893 | 1,28631 | 19,33 |
| 58,74 | 1,76893 | 2,51947 | 330,7 |
| 5·874 | 0,76893 | 3,75062 | 5632, |
| 0,5874 | $\bar{1}$,76893 | $\bar{3}$,18397 | 0,001527 |
| 0,05874 | $\bar{2}$,76893 | $\bar{2}$,43020 | 0,02693 |
| 0,005874 | $\bar{3}$,76893 | $\bar{1}$,85962 | 0.7238 |

83. A general rule to find the Index to the Log. of a given number. *To the left of the Logarithm, write that figure (or figures) which expreſſes the diſtance from unity, of the higheſt-place digit in the given number; reckoning the unit's place* 0, *the next place* 1, *the next place* 2, *the next place* 3, &c.

When there are integers in the given number, the Index is always affirmative; but when there are no integers, the Index is negative, and is to be marked by a little line drawn above it: thus $\bar{2}$.

Thus a number having 1 . 2 . 3 . 4 . 5 *&c.* integer places.

The index of its Logarithm is 0 . 1 . 2 . 3 . 4 . *&c.*

And a fraction having a digit in the place of Primes, Seconds, Thirds, Fourths, *&c.*

Then the Index of its Logarithms will be $\bar{1}$ . $\bar{2}$ . $\bar{3}$ . $\bar{4}$ *&c.*

By the above rule, the place of the fractional comma, or mark of diſtinction, in the number anſwering to a given Logarithm, will be always known.

84. The more places the Logarithms conſiſt of, the more accurate, in general, will be the reſult of any operation performed with them: But for the purpoſes of Navigation, as five places, excluſive of the Index, are ſufficient, therefore the logarithmic tables in this treatiſe are not extended any farther.

## 85. MULTIPLICATION BY LOGARITHMS;

*Or, Two or more numbers being given, to find their Product by Logarithms.*

Rule. Add together the Logarithms of the Factors, and the ſum is a Logarithm, the correſponding number of which is the Product required.

Obſerving to add what is carried from the Logarithm to the ſum of the affirmative Indices.

And that the difference between the affirmative and negative Indices are to be taken for the Index to the Logarithm of the Product.

Ex. I. *Multiply* 86,25 *by* 6,48.

| | |
|---|---|
| 86,25 its Log. is | 1,93576 |
| 6,48 its Log. is | 0,81157 |
| Product 558,9 | 2,74733 |

Ex. III. *Multiply* 3,768 *by* 2,053 *and by* 0,007693.

| | |
|---|---|
| 3,768 its Log. is | 0,57611 |
| 2,053 | 0,31239 |
| 0,007693 | $\bar{3}$,88610 |
| Product 0,05951 | $\bar{2}$,77460 |

The 1 carried from the left hand column of the Logs. being affirmative, reduces $\bar{3}$ to $\bar{2}$.

Ex. II. *Multiply* 46,75 *by* 0,3275.

| | |
|---|---|
| 46,75 its Log. is | 1,66978 |
| 0,3275 its Log. is | $\bar{1}$,51521 |
| Product 15,31 | 1,18499 |

Ex. IV. *Multiply* 27,63 *by* 1,859 *and by* 0,7258 *and by* 0,03591.

| | |
|---|---|
| 27,63 its Log. is | 1,44138 |
| 1,859 | 0,26928 |
| 0,7258 | $\bar{1}$,86082 |
| 0,03591 | $\bar{2}$,55521 |
| Product 1,339 | 0,12669 |

Here 2 being carried to the Index 1, makes 3; which takes off the $\bar{1}$ and $\bar{2}$.

## 86. DIVISION BY LOGARITHMS.

*Or, Two numbers being given, to find how often the one will contain the other, by Logarithms.*

Rule. From the Log. of the Dividend, ſubtract the Log. of the Diviſor; then the number agreeing to the Remainder, will be the Quotient required.

But obſerve to change the Index of the Diviſor from negative to affirmative, or from affirmative to negative: And then let the difference of the affirm. and neg. Indices be taken for the Index to the Log. of the Quotient.

When an unit is borrowed in the left-hand place of the Logarithm, add it to the Index of the Diviſor, if affirmative; but ſubtract it if negative; and let the Index ariſing be changed and worked with as before.

Ex. I. *Divide* 558,9 *by* 6,48.

| | |
|---|---|
| Log. of Divid. 558,9 is | 2,74733 |
| Log. of Diviſor 6,48 is | 0,81157 |
| The Quotient is 86,25 | 1,93576 |

Ex. III. *Divide* 0,05951 *by* 0,007693.

| | |
|---|---|
| Log. of Divid. 0,05951 is | $\bar{2}$,77459 |
| Log. of Diviſor 0,007693 is | $\bar{3}$,88610 |
| The Quotient is 7,735 | 0,88849 |

Ex. II. *Divide* 15,31 *by* 46,75.

| | |
|---|---|
| Log. of Divid. 15,31 is | 1,18497 |
| Log. of Diviſor 46,75 is | 1,66973 |
| The Quotient is 0,3275 | $\bar{1}$,51519 |

Ex. IV. *Divide* 0,6651 *by* 22,5.

| | |
|---|---|
| Log. of Divid. 0,6651 is | $\bar{1}$,82289 |
| Log. of Diviſor 22,5 is | 1,35218 |
| The Quotient is 0,02956 | $\bar{2}$,47071 |

## 87. OF PROPORTION.

1ſt. State the terms of the queſtion (by 46) and let them be written orderly under one another, prefixing to the firſt term the word *As*, to the ſecond *To*, to the third *So*, and under them ſet the word *To*.

2d. Againſt the firſt term, write the arithmetical complement of its Logarithm. See Art. 88.

3d. Againſt the ſecond and third terms, write their Logarithms.

4th. The ſum of thoſe three Logarithms, abating 10 in the Index, will be the Logarithm of the 4th term; which ſought in the tables, the number anſwering to it is the anſwer or term ſought.

88. The arithmetical complement of a Logarithm is thus found. Beginning at the Index, write down what each figure wants of 9, except the laſt, or right-hand figure, which take from 10.

But if the Index is negative, add it to 9; and proceed with the reſt as before.

Ex. I. *Find a fourth proportional number to* 98,45 *and* 1,969 *and* 347,2

| | |
|---|---|
| As 98,45 its * Ar. Co. Log. | 8,00678 |
| To 1,969 | 0,29425 |
| So 347,2 | 2,54058 |
| To 6,944 | 0,84161 |

Ex. III. *What will a gunner's pay amount to in a year at* 2£. 12*s.* 6*d. a month of* 28 *days?*

| | |
|---|---|
| As 28 days its Ar. Co. Log. | 8,55284 |
| To 365 days | 2,56229 |
| So 2£. 12*s.* 6*d.*=2,625£. | 0,41913 |
| To 34£. 4*s.* 5*d.*=34,22£. | 1,53426 |

Ex. V. *What number will have the ſame proportion to* 0,8538 *as* 0,3275 *has to* 0,0131?

| | |
|---|---|
| As 0,0131 its Ar. Co. Log. | 11,88273 |
| To 0,3275 | $\bar{1}$,51521 |
| So 0,8538 | $\bar{1}$,93136 |
| To 21,35 | 1,32930 |

Ex. II. *Find a third proportional number to* 9,642 *and* 4,821.

| | |
|---|---|
| As 9,642 its Ar. Co. Log. | 9,01583 |
| To 4,821 | 0,68314 |
| So 4,821 | 0,68314 |
| To 2,411 | 0,38211 |

Ex. IV. *If* $\frac{3}{4}$ *of a yard of cloth coſt* $\frac{2}{3}$ *of a guinea: How many ells Engliſh for* 3£. 10*s.*?

| | |
|---|---|
| As $\frac{2}{3}$ Guin.=14*s.* Ar. Co. | 8,85387 |
| To 3£. 10*s.*=70*s.* | 1,84510 |
| So $\frac{3}{5}$ ell =0,6 | $\bar{1}$,77815 |
| To 3 ells | 0,47712 |

Ex. VI. *How many yards of ſhalloon of* $\frac{3}{4}$ *ell wide will be enough to line a coat containing* $3\frac{1}{2}$ *ells of* $1\frac{3}{4}$ *yards wide?*

| | |
|---|---|
| As $\frac{3}{4}\times\frac{5}{4}$ yd. w.=0,9375 | 10,02803 |
| To $1\frac{3}{4}$ yd. w.=1,75 | 0,24304 |
| So $3\frac{1}{2}\times\frac{5}{4}$ yd. l.=4,375 | 0,64098 |
| To $8\frac{1}{6}$ yd. long=8,167 | 0,91205 |

where w. ſtands for wide, l. for long.

---

* Ar. Co. Log. ſtands for *the Arithmetical Complement of the Logarithm.*

## OF POWERS AND THEIR ROOTS.

89. *A number being given, to find any propoſed power of that Number.*

Rule 1ſt. Seek the Logarithm of the given number.

2d. Multiply this Logarithm by the Index of the propoſed power.

3d. Find the number correſponding to the Product, and it will be the power required.

90. In multiplying a Logarithm having a negative Index, the Product of that Index is negative.

But the carriage from the Logarithm is affirmative.

Therefore the difference will be the Index of the Product.

And is to be of the ſame kind with the greater, or that which was made the minuend.

Ex. I. *What is the ſecond power of the number* 3,874?

| | |
|---|---|
| To 3,874 its Log. is | 0,58816 |
| The Index is | 2 |
| The power ſought is 15,01 | 1,17632 |

Ex. II. *What is the 3d power of the number* 2,768?

| | |
|---|---|
| The N° 2,768 its Log. is | 0,44217 |
| The Index is | 3 |
| The power ſought is 21,21 | 1,32651 |

Ex. III. *What is the 12th power of the number* 1,539?

| | |
|---|---|
| 1,539 its Log. is | 0,18724 |
| The Index is | 12 |
| The power ſought is 176,6 | 2,24688 |

Ex. IV. *What is the 365th power of the number* 2?

| | |
|---|---|
| 2 Its Log. is | 0,30103 |
| The Index is | 365 |
| | 150515 |
| | 180618 |
| | 90309 |
| | 109,87595 |

In the IVth Ex. the Index of the Product being 109, ſhews that the required power will conſiſt of 110 integer places; of which no more than 4 places are found in theſe tables; therefore the number ſought may be thus expreſſed, 7515 [$\overline{106}$]: That is, 7515 with 106 cyphers annexed.

Ex. V. *What is the 2d power of the number* 0,2857?

| | |
|---|---|
| To 0,2857, its Log. is | $\bar{1}$,45591 |
| The Index of 2d power | 2 |
| The power 0,08162 | $\bar{2}$,91182 |

Here, there being no carriage from the Product of the Log. the whole Product of the negative Index is negative, *viz* $\bar{2}$.

Ex. VI. *What is the 3d power of the number* 0,7916?

| | |
|---|---|
| To 0,7916, its Log. is | $\bar{1}$,89851 |
| The Index of 3d power | 3 |
| The power is 0,4961 | $\bar{1}$,69553 |

Here the carriage from the Product of the Log. is 2; then the Product of the negative Index $\bar{1}$, *viz.* $\bar{3}$, being leſſened by 2, leaves $\bar{1}$, the Index of the Product.

## 91. *A number being given, to find any propoſed Root of it.*

Rule 1ſt. Seek the Logarithm of the given number.

2d. Divide this Logarithm by the denominator of the Index of the propoſed root.

3d. The number correſponding to the Quotient will be the root.

When the Index to the Logarithm to be divided is negative, and leſs than the Diviſor, or Denominator of the root. Then

Increaſe the negative Index by as many units, borrowed, as ſhall be equal to the Diviſor, and the Quotient will give $\bar{1}$ for the Index.

Carry the units borrowed as tens to the left-hand place of the Logarithm, and then divide that Logarithm as in whole numbers.

Ex. I. *What is the Square Root of the number* 1501?

2)3,17638

Root ſought is 38,74 1,58819

Ex. II. *What is the Cube Root of the number* 2121?

3)3,32654

Root ſought 12,85 1,10885

Ex. III. *What is the Root, of which* 176,6 *is the* 12*th power?*

12)2,24699

Root ſought is 1,539 0,18725

Ex. IV. *What is the Root, of which* 2 *is the* 365*th power?*

365)0,30103

Root ſought is 1,002 0,00082

Ex. V. *What is the Square Root of the number* 0,08162?

Log. of 0,08162, div. by 2)$\bar{2}$,91180

Gives Root 0,2857 to $\bar{1}$,45590

Here the Diviſor 2 can be taken in the Index $\bar{2}$, and gives for the Quotient $\bar{1}$.

Ex. VI. *What is the Cube Root of the number* 0,496?

Log. of 0,496 div. by 3)$\bar{1}$,69548

Gives Root 0,7916 to $\bar{1}$,89849

Here the Diviſor 3 cannot be taken in the Index $\bar{1}$; then $\bar{2}$ borrowed makes with $\bar{1}$, $\bar{3}$; in which the Diviſor 3 will go $\bar{1}$: the 2 borrowed carried to the 6, *&c.* makes 269548, which divided by 3, gives 89849.

END OF BOOK I.

# THE ELEMENTS OF NAVIGATION.

## BOOK II.

## OF GEOMETRY.

### SECTION I.

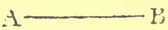
### *Definitions* and *Principles*.

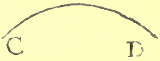
1. GEOMETRY is a ſcience which treats of the deſcriptions, properties and relations of magnitudes in general: Or of ſuch things where length, or where length and breadth, or length, breadth, and thickneſs, are conſidered.

2. A POINT is that which is without parts or dimenſions.

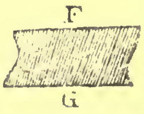
3. A LINE is length without breadth: It is called a RIGHT LINE when it is the ſhorteſt diſtance between two points, as AB: Or a CURVED LINE when it is not the ſhorteſt diſtance, as CD.

*A line is uſually denoted by two letters, viz. one at each end*, as AB or CD.





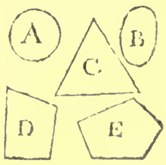
4. A SUPERFICIES or SURFACE is that magnitude which has only length and breadth, and is bounded by lines: as FG.

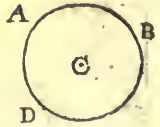


5. A SOLID is that magnitude which has length, breadth and thickneſs.

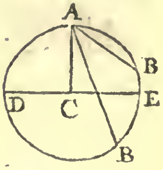
6. A FIGURE is a bounded ſpace, the limits or bounds of which may be either lines or ſurfaces.

7. A PLANE, or a PLANE FIGURE, is a ſuperficies which lies evenly, or perfectly flat, between its limits, and may be bounded by one curve line; but not with leſs than three right lines, as A, B, C, D, or E, *&c.*





8. A **Circle** is a plain figure, bounded by an uniformly curved line, called the Circumference, as ABD, which is every where equally diſtant from one point, as C within the figure called the **Center**.



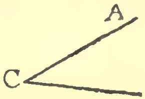
9. A **Radius** is a right line drawn from the center to the circumference as CA, CD or CE.
*All the radii of the ſame circle are equal.*

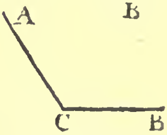
10. An Arc is any part of the circumference.
*As the arc* AB, *or the arc* AD.

11. A Chord is a right line joining the ends of an arc, as AB, and is ſaid to ſubtend that arc; it divides the circle into two parts, called Segments.

12. A Diameter is a chord paſſing through the center, as DE, and divides the circle into two equal parts, called Semicircles.

13. The Circumference of every circle is ſuppoſed to be divided into 360 equal parts, called Degrees; each degree into 60 equal parts, called Minutes; each minute into 60 equal parts, called Seconds, *&c.*



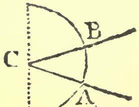


14. A Plane Angle, is the inclination of two lines on the ſame plane meeting in a point, as ACB.

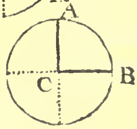
A right lined angle is formed by two right lines. The point where the lines meet is called the *angular point*, as C.

The lines which form the angle are called *legs.*
*Thus* CA *and* CB *are the legs of the angle* ACB.

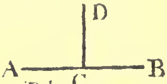
*An Angle is uſually marked by three letters, viz. one at the angular point, and one at the other end of each leg; but that at the angular point is always to be read the middle letter*, as ACB, or BCA.



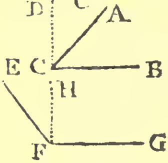
15. The meaſure of a right lined angle is an arc, as BA contained between the legs CB, CA, including the angle, the angular point C being the center of that arc.



16. A Right Angle is that, the meaſure of which is a fourth part of the circumference of a circle, or ninety degrees. *Thus the angle* ACB *is a right angle.*



17. A Perpendicular is that right line which cuts another at right angles; or which makes equal angles on both ſides. *Thus* DC *is perpendicular to* AB, *when the angles* DCA *and* DCB *are equal, or are right angles.*

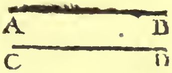


18. An Acute Angle, as ACB, is that which is leſs than a right angle DCB.

19. An Obtuſe Angle, as EFG, is that which is greater than a right angle HFG.

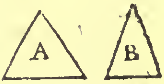
Acute and obtuſe angles are called Oblique Angles.

20. Parallel Lines, are right lines in the ſame plane, which do not incline to one another, as AB, CD.



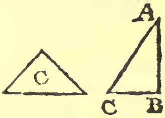
21. A Triangle is a plane figure bounded by three lines.

22. An Equilateral Triangle is that in which the three lines, or ſides, are equal, as A.



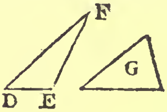
23. An Isosceles Triangle is that which has only two equal ſides, as B or C.

24. A Right angled Triangle, as ABC, is that which has one right angle B.



25. An Obtuse angled Triangle, as DEF, has one obtuſe angle E.

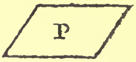
26. An Acute angled Triangle, as G, has all its angles acute.



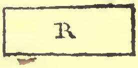
27. A Quadrangle, or Quadrilateral, is a plane figure bounded by four right lines, or ſides.

*A Quadrangle is uſually expreſſed by letters at the oppoſite angles.*

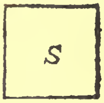
28. A Parallelogram is a quadrangle the oppoſite ſides of which are parallel and equal, as P.



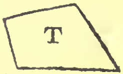
29. A Rectangle is a parallelogram with right angles: and in which the length is greater than its breadth, as R.



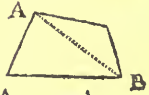
30. A Square is a parallelogram having four equal ſides and right angles, as S.



31. A Trapezium is a quadrangle the oppoſite ſides of which are not parallel, as T.

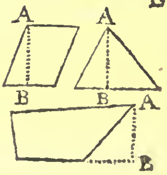


32. The Diagonal of a quadrangle, is a line, as AB, drawn from one angle, to its oppoſite angle.



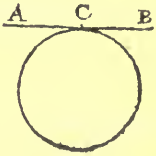
33. The Base of a figure, is the line it is ſuppoſed to ſtand on.

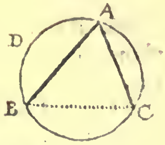
34. The Altitude or Height of a figure, is the perpendicular diſtance AB, between the baſe and the vertex, or part moſt remote from the baſe.



35. Congruous Figures, are thoſe which agree, or correſpond, with one another, in every reſpect.

36. A Tangent to a circle is a right line, as AB, touching its circumference, but not cutting; and the point C, where it touches, is called the *point of contact.*





37. An ANGLE, BAC, *in a Segment*, CADB, is, when the angular point is in the circumference of the ſegment, and the legs including the angle paſs through the ends B and C, of the chord of the ſegment.

*Such an angle is ſaid to be in a circumference; and to ſtand on the arc,* BC, *included between the legs,* AB *and* AC, *of the angle.*

38. Right lined figures, having more than four ſides, are called *Polygons*; and have their names from the number of their angles, or ſides; as thoſe of five ſides are called Pentagons; of ſix ſides, Hexagons; of ſeven ſides, Heptagons; of eight ſides, Octagons, *&c.*

39. A regular Polygon is a figure with equal ſides, and equal angles.

40. A figure is ſaid to be inſcribed in a circle, when all the angles of that figure are in the circumference of the circle.

41. A figure is ſaid to circumſcribe a circle, when every ſide of the figure is touched by the circumference of the circle.

42. A Propoſition is ſomething propoſed to be conſidered; and requires either a ſolution or anſwer, or that ſomething be made out, or proved.

A Problem is a practical propoſition, in which ſomething is propoſed to be done, or effected.

A Theorem is a ſpeculative propoſition, or rule, in which ſomething is affirmed to be true.

A Corollary is ſome concluſion gained from a preceding propoſition.

A Scholium is a remark on ſome propoſition; or an exemplification of the matter which it contains.

An Axiom is a ſelf-evident truth, or principle, that every one aſſents to upon hearing it propoſed.

A Poſtulate is a principle, or condition, requeſted; the ſimplicity or reaſonableneſs of which cannot be denied.

In Mathematics, the following Poſtulates and Axioms, are ſome of the principal ones that are generally taken for granted.

When a propoſition, from ſuppoſed premiſes, aſſerts ſuch and ſuch conſequences; and ſubjoins, *And the contrary:* it is to be underſtood, that if the conſequences be aſſumed as premiſes; then what were firſt taken as premiſes, would become conſequences.

Thus, in Article 95, it is premiſed, *that if two parallel right lines are cut by another right line*, there reſults this conſequence; *The alternate angles are equal.* And the contrary means; that *where equal alternate angles are made by a right line cutting two other right lines; the right lines ſo cut, are parallel lines.*

Poſtulates.

# Poſtulates.

43. I. That a right line may be drawn from any given point to another given point.

44. II. That a given right line may be continued, or lengthened at pleaſure.

45. III. That from a given point, and with any radius, a circle may be deſcribed.

# Axioms.

46. I. Things equal to the ſame thing, are equal to one another.

47. II. If equal things are added to equal things, the ſums or wholes will be equal. But if unequals be added, the ſums are unequal.

48. III. If equal things are taken from equal things, the remainders, or differences, are equal: but are unequal, when unequals are taken.

49. IV. Things are equal which are double, triple, quadruple, *&c.* or half, third part, &c. of one and the ſame thing, or of equal things.

50. V. Things which have equal meaſures, are equal. And the contrary.

51. VI. Equal circles have equal radii.

52. VII. Equal arcs in equal circles have equal chords, and are the meaſures of equal angles. And the contrary.

53. VIII. Parallel right lines have each the ſame inclination to a right line cutting them.

In what follows, it is to be underſtood, that right lines (*viz.* ſtraight lines) are drawn by the edge of a ſtraight ruler: circles or arcs, are deſcribed with one foot of a pair of compaſſes, the other foot reſting on the point which is taken for the center; and the diſtance of the feet, or points, of the compaſſes is taken as the radius: alſo, that the point marked out by a letter is to be underſtood, when the reference is made to that letter.

54. It is alſo taken for granted, that a line or diſtance can be taken between the compaſſes, and may be transferred or applied from one place to another. Alſo, that one figure can be applied to, or laid upon another, or conceived to be ſo applied.

In any problem, when a line, angle, or figure is ſaid to be given; that line, angle, or figure muſt be made, before any part of the operation is performed.

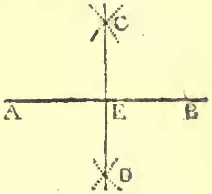
SEC-

## SECTION II.

## *Geometrical Problems.*

### 55. PROBLEM I.

*To biſect, or divide into two equal parts, a given line* AB.



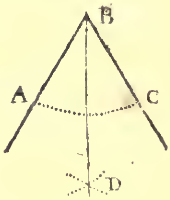
OPERATION. 1ſt. From the ends A and B*, with one and the ſame radius, greater than half AB, deſcribe arcs cutting in C and D. (45)

2d. A ruler laid by C and D, gives E, the middle of AB, as required.

*The proof of this operation depends on articles* 101, 99.

### 56. PROBLEM II.

*To biſect a given right lined angle* ABC.



OPERATION. 1ſt. From B, deſcribe an arc AC.

2d. From A and C*, with one and the ſame radius, deſcribe arcs cutting in D. (45)

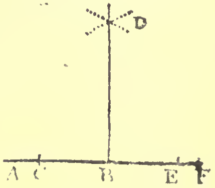
3d. A right line drawn through B and D will divide the angle into two equal parts, as required.

*The proof depends on article* 101.

### 57. PROBLEM III.

*From a given point* B, *in a given right line* AF, *to draw a right line perpendicular to the given line.*

CASE I. *When* B *is near the middle of the line.*



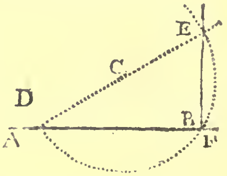
OPERATION. 1ſt. On each ſide of B, take the equal diſtances BC, and BE. (54)

2d. On C and E * deſcribe, with one radius, arcs cutting in D. (45)

3d. A right line drawn through B and D will be the perpendicular required. (43)

*The proof depends on article* 103.

58. CASE II. *When* B *is at, or near the end of the given line.*



OPERATION. 1ſt. On any convenient point C, taken at pleaſure, with the diſtance, or radius CB, deſcribe an arc DBE, cutting AF in D, B. (45)

2d. A ruler laid by D and C will cut this arc in E.

3d. A right line drawn through B and E will be the perpendicular required.

*This depends on article* 130.

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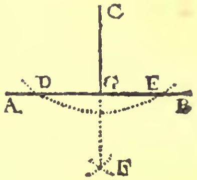
* That is, firſt deſcribe an arc from one point; then deſcribe an arc from the other point with the ſame opening of the compaſſes.

## 59. PROBLEM IV.

*To draw a line perpendicular to a given right line* AB, *from a point* C *without that line.*



CASE I. *When the point* C *is nearly oppoſite to the middle of the given line.*

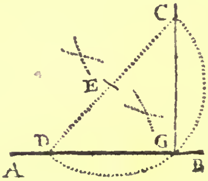
OPERATION. 1ſt. On C, with one radius, cut AB in D and E. (45)

2d. On D and E, with one radius, deſcribe arcs cutting in F. (45)

A ruler laid by C and F gives G; then draw CG, and that will be the perpendicular required.

*This depends on articles* 101, 99.

60. CASE II. *When* C *is nearly oppoſite to one end of the given line* AB.



OPERATION. 1ſt. To any point D in AB, draw the line CD. (43)

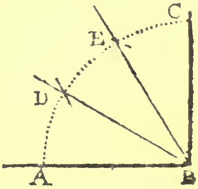
2d. Biſect the line CD in E. (55)

3d. On E, with the radius EC, cut AB in G. Then CG being drawn, will be the perpendicular required.

*This depends on article* 130.

## 61. PROBLEM V.

*To triſect, or divide into three equal parts, a right angle* ABC.



OPERATION. 1ſt. From B, with any radius BA, deſcribe the arc AC, cutting the legs BA, BC, in A, C.

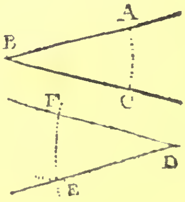
2d. From A, with the radius AB, cut the arc AC in E, and from C, with the ſame radius, cut AC in D.

3d. Draw BE, BD, and the angle ABC will be divided into three equal parts.

*This depends on article* 193.

## 62. PROBLEM VI.

*At a given point* D, *to make a right lined angle equal to a given right lined angle* ABC.



OPERATION. 1ſt. From D and B, with one radius deſcribe the arcs EF, and AC, cutting the legs of the given angle in the points A, C.

2d. Transfer the diſtance AC to the arc EF, from F to E. (54)

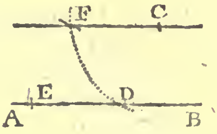
3d. Lines drawn from D, through E and F, will form the angle EDF equal to the angle ABC.

*This depends on article* 101.

## 63. PROBLEM VII.

*To draw a line parallel to a given right line* AB.

CASE I. *When the parallel line is to paſs through a given point,* C.



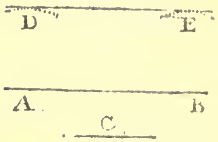
OPERATION. 1ſt. From C, with any convenient radius, deſcribe an arc DF, cutting AB in D.

2d. Apply the radius CD from D to E; and from E, with the ſame radius, cut the arc DF in F.

3d. A line drawn through F and C will be parallel to AB.

*This depends on article* 101, 95.

64. CASE II. *When the parallel line is to be at the given diſtance* C *from* AB.



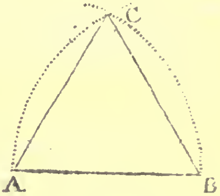
OPERATION. 1ſt. From the points A and B, with the radius C, deſcribe arcs D and E.

2d. Lay a ruler to touch the arcs D and E, and a line drawn in that poſition is the parallel required.

*This operation is mechanical.*

## 65. PROBLEM VIII.

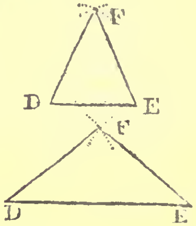
*Upon a given line* AB, *to make an equilateral triangle.*



OPERATION. 1ſt. From the points A and B, with the radius AB, deſcribe arcs cutting in C. (45)

2d. Draw CA, CB, and the figure ABC is the triangle required. (43.)

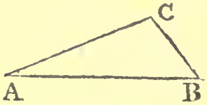
*The truth of this operation is evident; for the ſides are radii of equal circles.*

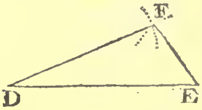


66. By a like operation, an Iſoſceles triangle DEF may be conſtructed on a given baſe DE, with the given equal legs DF, EF, either greater, or leſs, than the baſe DE.

## 67. PROBLEM IX.

*To make a right lined triangle, the ſides of which ſhall be reſpectively equal, either to thoſe of a given triangle* ABC, *or to three given lines, provided any two of them taken together are greater than the third.*





OPERATION. 1ſt. Draw a line DE equal to the line AB (54)

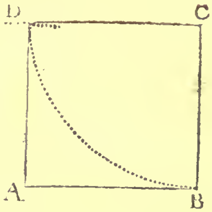
2d. From D, with a radius equal to AC, deſcribe [illegible] (45)

[illegible] with a radius equal to BC, deſcribe [illegible] the former arc in F. (45)

[illegible] nd the triangle DFE will be

## 68. PROBLEM X.

*Upon a given line* AB, *to deſcribe a ſquare.*



OPERATION. 1ſt. Draw BC perpendicular, and equal to AB. (58)

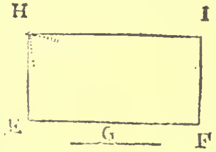
2d. On A and C, with the radius AB, deſcribe arcs cutting in D. (45)

3d. Draw DC, DA; and the figure ABCD is the ſquare required.

*This depends on articles* 101, 104, 95, 30.

## 69. PROBLEM XI.

*To deſcribe a rectangle whoſe length ſhall be equal to a given line* EF, *and breadth equal to another given line* G.



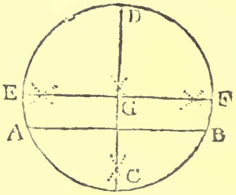
OPERATION. 1ſt. At E and F erect two perpendiculars, EH and FI, each equal to the given line G.

2d. Draw HI, and the figure EFIH will be the rectangle required.

*This depends on articles* 29, 101, 104, 95.

## 70. PROBLEM XII.

*To find the center of a circle.*



OPERATION. 1ſt. Draw any chord AB.

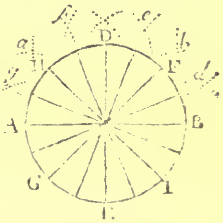
2d. Biſect AB with the chord CD. (55)

3d. Biſect CD with the chord EF, and their interſection G will be the center required.

*This depends on article* 125.

## 71. PROBLEM XIII.

*To divide the circumference of a circle into two, four, eight, ſixteen, thirty-two, &c. equal parts.*



OPERATION. 1ſt. A diameter AB divides the circle into two equal parts. (12)

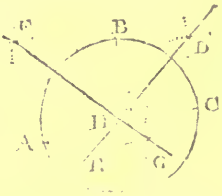
2d. A diameter DE, perpendicular to AB, divides the circumference into four equal parts.

3d. On A, D, B, deſcribe arcs cutting in *a*, *b*; then by the interſections *a*, *b*, and the center, the diameters FG, HI, being drawn, divide the circumference into eight equal parts; and ſo on by continual biſection.

*For at each operation, the intercepted arcs are biſected, and the parts doubled.*

## 72. PROBLEM XIV.

*To deſcribe a circle, the circumference of which ſhall paſs through three given points* A, B, C, *provided they do not lie in one right line.*



OPERATION. 1ſt. Biſect the diſtance CB with the line DE. (55)

2d. Biſect the diſtance AB with the line FG.

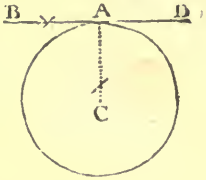
3d. On H, the interſection of theſe lines, with the diſtance to either of the given points, deſcribe the circle required.

*This depends on article* 125.

## 73. PROBLEM XV.

*To draw a tangent to a given circle, that ſhall paſs through a given point* A.

CASE I. *When* A *is in the circumference of the circle.*

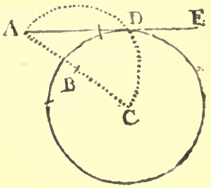


OPERATION. 1ſt. From the center C, draw the radius CA. (43)

2d. Through A draw BD perpendicular to CA (58), and BD is the tangent required.

*This depends on article* 126.

74. CASE II. *When the given point* A, *is without the given circle.*



OPERATION. 1ſt. From the center C, draw CA, which biſect in B. (55)

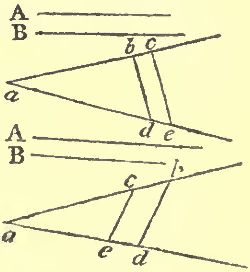
2d. On B, with the radius BA, cut the given circumference in D.

3d. Through D, the line AE being drawn, will be the tangent required.

*This depends on articles* 130, 126.

## 75. PROBLEM XVI.

*To two given right lines* A, B, *to find a third proportional.*



OPERATION. 1ſt. Draw two right lines making any angle, and meeting in *a*.

2d. In theſe lines, take *ab*=firſt term, and *ac*, *ad*, each equal to the ſecond term.

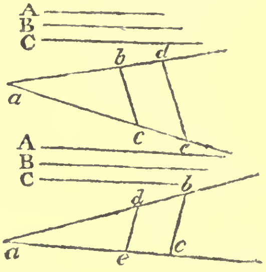
3d. Draw *bd*, and through *c*, draw *ce* parallel to *bd*; then *ae* is the third proportional ſought.

And *ab* : *ac* : : *ad* : *ae**.

*This depends on article* 165.

## 76. PROBLEM XVII.

*To three given right lines* A, B, C, *to find a fourth proportional.*



OPERATION. 1ſt. Draw two right lines making any angle, and meeting in *a*.

2d. In theſe lines take *ab*=firſt term, *ac*=ſecond term, and *ad*=third term.

3d. Draw *bc*, and parallel to it, through *d*, draw *de*; then *ae* is the fourth proportional required.

And *ab* : *ac* : : *ad* : *ae*.

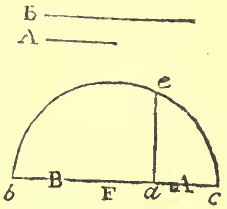
*This depends on article* 165.

---

* And is thus read: *ab* is to *ac*, as *ad* is to *ae*.
The character ( : ) ſtanding for *is to*, and the character ( : : ) for *as*.

## 77. PROBLEM XVIII.

*Between two given right lines* A, B, *to find a mean proportional.*



Operation. 1ſt. Draw a right line, in which take $ac=$ A, $ab=$ B.

2. Biſect $bc$ in F (55); and on F, with the radius F$b$, deſcribe a ſemicircle $bec$.

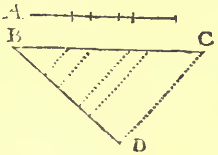
3d. From $a$ draw $ae$ perpendicular to $bc$ (57); then $ae$ is the mean proportional required.

And $ac : ae :: ae : ab$.

*This depends on article* 171.

## 78. PROBLEM XIX.

*To divide a given line* BC *in the ſame proportion as a given line* A *is divided.*



Operation. 1ſt. From one end B of BC draw BD, making any angle with BC.

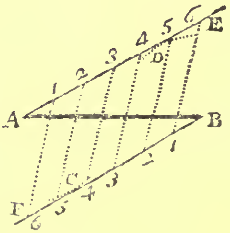
2d. In BD apply from B the ſeveral diviſions of A; ſo BD will be equal to A, and alike divided.

3d. Draw CD; then lines drawn parallel to CD through the ſeveral diviſions of BD, will divide the line BC in the manner required.

*This depends on article* 165.

## 79. PROBLEM XX.

*To divide a given right line* AB *into a propoſed number of equal parts (ſuppoſe 7).*



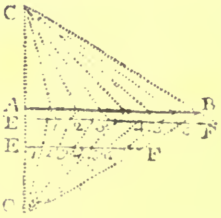
Operation. 1ſt. From A, one end of AB, draw AE to make any angle with AB; and from B, the other end, draw BF, making the angle ABF equal to the angle BAE. (62)

2d. In each of the lines AE, BF, beginning at A and B, take, of any length, as many equal parts, leſs one, as AB is to be divided into, *viz.* 1, 2, 3, 4, 5, 6.

3d. Lines drawn from 1 to 6, 2 to 5, 3 to 4, *&c.* will divide AB as was required.

*This depends on article* 165.

### 80. *Another Method.*



Operation. 1ſt. Through one end A, draw a line CC nearly perpendicular to AB.

2d. Draw EF parallel to AB, at any convenient diſtance.

3d. In EF take, of any length, as many equal parts as AB is to be divided into; as 1, 2, 3, 4, 5, 6, to F.

4th. Through B and F, where the diviſions terminate, draw BC, meeting CC in C.

5th. Lines drawn from C through the ſeveral diviſions of EF, will cut AB into the equal parts required.

Note, If the ſum of the diviſions from E to F chance to be leſs than AB, the point C, and the line EF, will be on the ſame ſide of AB; but if greater, C falls on the contrary ſide.

## 81. PROBLEM XXI.

*To make Scales of equal parts.*

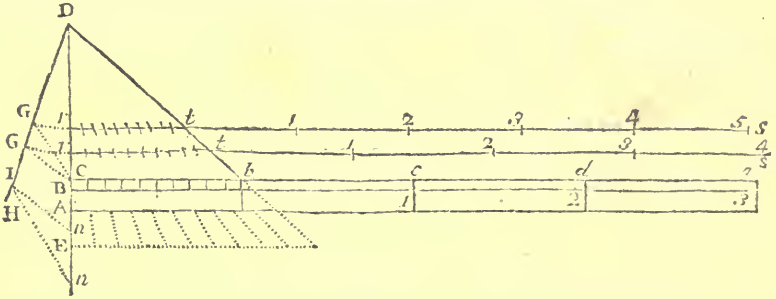
Operation. 1ſt. Draw three lines, A, B, C, parallel to one another, and at convenient diſtances, ſuch as are here expreſſed.

2d. In the line C, take the equal parts *cb*, *bc*, *cd*, *da*, &c. each equal to ſome propoſed length.

3d. Through C, draw the line DE perpendicular to C*a*; and parallel to DE, through the ſeveral points *b*, *c*, *d*, *a*, &c. draw lines acroſs the three parallels A, B, C; then are the ſpaces or diſtances C*b*, *bc*, *cd*, &c. called the primary diviſions.

4th. Divide the left-hand ſpace into 10 equal parts (80), and through theſe points let lines, parallel to DE, be drawn acroſs the parallels BC; and the primary diviſion C*b* will be parted into 10 equal ſpaces, called ſubdiviſions.

5th. Number the primary diviſions from the left towards the right, beginning at *c*, and the ſcale is conſtructed.



Such Scales are uſeful for conſtructing figures, the ſides of which are expreſſed in meaſures of length; as feet, yards, miles, &c. Or to find the meaſures of the length of lines in a given figure.

Thus a line of 36, or 360, feet, yards, &c. is taken by ſetting one point of the compaſſes on the third primary diviſion, and extending the other point to the ſixth ſubdiviſion: and ſo of others.

In theſe Scales it is uſually ſuppoſed, that an inch is divided into ſome number of ſuch parts, as are expreſſed by the ſubdiviſions.

82. *To find the diviſions of a Scale of equal parts, when any given number of them are to make an inch.*

Operation. 1ſt. Make a Scale C*a*, where the primary diviſions ſhall be each one inch; let DC be at right angles to C*a*, and the left-hand ſpace C*b* be divided into 10 equal parts, and draw D*b* from any point D.

2d. Draw DH, making with DC any angle; and make DI = C*b*.

3d. Take the number of parts propoſed in an inch, from the Scale C*a*, and apply them from D to *n*, in the line DC, continued if neceſſary.

4th. Draw *n*I, and CG parallel to *n*I; and make D*r* = DG.

5th. Through *r* draw *rs* parallel to C*a*, cutting D*b* in *t*; then *rt* will be one of the primary diviſions, containing 10 of the parts the inch was to be divided into.

6th. Lines drawn from D to the diviſions of C*b*, divide *rt* into 10 equal parts.

## 83. PROBLEM XXII.

*To divide the circumference of a circle into degrees; and thence to make a Scale of chords.*

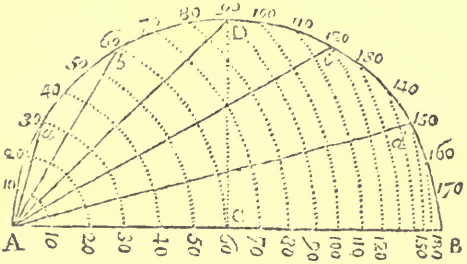
Operation. 1ſt. Deſcribe the ſemicircle ABD, the center of which is C, and draw CD at right angles to AB.

2d. Triſect the angles ACD, DCB, in the points *a*, *b*; *c*, *d* (61): then by trials, divide the arcs A*a*, *ab*, *b*D, D*c*, *cd*, *d*B, each into three equal parts, and the ſemicircumference will be divided into 18 equal parts, of 10 degrees each.

3d. Theſe arcs being biſected, will give arcs of 5 degrees.

4th. And theſe arcs being divided into 5 equal parts, by trials, will give arcs of 1 degree each.

5th. The chords of the arcs A*d*, A*c*, AD, A*b*, A*a*, and of every other arc, being applied from A on the diameter AB, will divide AB into a Scale of chords; which are to be numbered from A towards B.



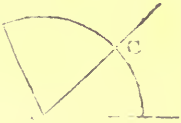
A Scale of chords is uſeful for conſtructing an angle of a given number of degrees: And for finding the meaſure of a given angle.

## 84. PROBLEM XXIII.

*To make an angle of a propoſed number of degrees.*

Operation. 1ſt. Take the firſt 60 deg. from the Scale of chords, and with this radius deſcribe an arc BC, the center of which is A.

2d. Take the chord of the propoſed number of degrees from the Scale, reckoning from its beginning, and apply this diſtance to the arc BC, from B to C.



3d. Lines drawn from A, through the points B and C, will form an angle BAC, the meaſure of which is the degrees propoſed.

With Scales of chords not exceeding 90 degrees, ſuch as are generally put on rulers, angles greater than 90 degrees, ſuppoſe of 138 deg. are to be taken at twice, *viz.* 69 and 69 ($\frac{1}{2}$ of 138), or 90 and 48.

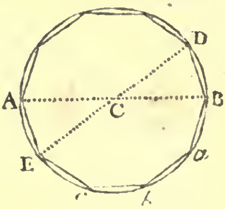
*When a given angle,* BAC, *is to be meaſured.*

From the angular point A, with the chord of 60 degrees, deſcribe the arc BC, cutting the legs in the points B and C.

Then the diſtance BC, applied to the chords, from the beginning, will ſhew the degrees which meaſure the angle BAC.

## 85. PROBLEM XXIV.

*In a given circle to inſcribe a regular polygon, the number of ſides being given.*



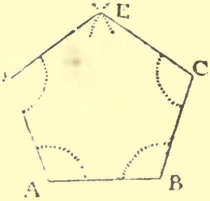
OPERATION. 1ſt. Divide 360 degrees by the number of ſides, and the Quotient will be the degrees which meaſure the angle at the center of the circle, ſubtended by a ſide of the *polygon**.

2d. Draw the radius CB, make an angle BCD equal to thoſe degrees (84), and draw the chord DB; then will DB be the ſide of the polygon required; which applied to the circumference from B to *a*, *a* to *b*, *b* to *c*, &c. will give the points to which the ſides of the polygon are to be drawn.

If the polygon has an even number of ſides, draw the diameter AB; and divide half the circumference, as before; then lines drawn from theſe points through the center, as ED, will give the remaining points, in the other ſemicircumference.

## 86. PROBLEM XXV.

*On a given right line* AB, *to conſtruct a regular polygon of any aſſigned number of ſides.*

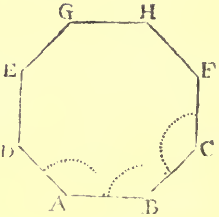


OPERATION. 1ſt. Divide 360 degrees by the number of ſides; ſubtract the Quotient from 180 degrees, the remainder will be the degrees which meaſure the angle made by any two adjoining ſides of that polygon, and is called the angle of the polygon†.

2d. At the ends A, B, of the line AB, make angles ABC, BAD, equal to the angle of the polygon.

3d. Make AD, BC, each equal to AB.

4th. At the points C, D, make angles equal to that of the polygon as before; and let the ſides including thoſe angles be each equal to AB; and thus proceed until the polygon is conſtructed.



In figures of any number of ſides, the two laſt ſides DE, CE; or EG, HG; are readieſt found by deſcribing arcs from C and D, or from E and H, with the radius AB, interſected in E, or in G.

In figures of an even number of ſides, having drawn half the number, AD, AB, BC, CF, by means of the angles; the remaining ſides may be found, by drawing through the points D and F the lines DE, FH, parallel and equal to their oppoſite ſides CF, AD; and ſo of the reſt.

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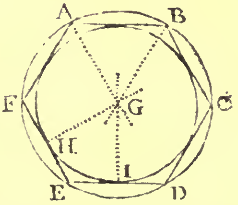
* *For there will be as many equal angles at the center as there are equal ſides. And the whole cirumference meaſures the ſum of all the angles at the center.*

† *For each ſide of the polygon with radii drawn to its ends form an Iſoſceles triangle.*

*Then the angle of the polygon is derived from articles* 85, 96, 104.

## 87. PROBLEM XXVI.

*About a given regular Polygon, to circumſcribe a circle: or within that Polygon to inſcribe a circle.*



OPERATION. 1ſt. Biſect any two angles FAB, CBA, with the lines AG, BG, (56) and the point G, where they interſect one another, will be the center of the polygon.

2d. A circle deſcribed from G, with the radius GA, will circumſcribe the given polygon.

*This depends on article* 85, 96, 104.

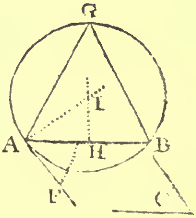
88. Again, 1ſt. Biſect any two ſides FE, ED, in the points H and I (55); and draw HG, IG, at right angles to FE, ED (57); then the point G, where they interſect each other, will be the center of the Polygon.

2d. A circle deſcribed from G, with the radius GH, will be inſcribed in the given Polygon.

*This depends on article* 126.

## 89. PROBLEM XXVII.

*On a given right line* AB, *to deſcribe a ſegment of a circle, that ſhall contain an angle equal to a given right lined angle* C.



OPERATION. 1ſt. Make an angle BAF equal to the given angle C. (62)

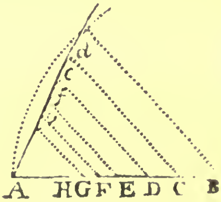
2d. From H, the middle of AB, draw HI at right angles to AB (57), and from A draw AI at right angles to AF (58), cutting HI in I.

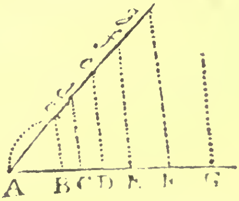
3d. From I, with the radius IA, deſcribe a circle. Then will the ſegment AGB contain an angle AGB equal to the given angle C.

*This depends on articles* 125, 126, 132.

## 90. PROBLEM XXVIII.

*To divide a right line in continued proportion, in the ratio of two given right lines* AB, AC.





OPERATION. 1ſt. From B, with the radius AB (the antecedent) deſcribe an arc A*c*.

2d. In that arc apply (the conſequent) AC from A to *c*; draw A*c*, and apply *c*A from *c* to D.

3d. Apply the following lines in the order directed, *viz.* AD from A to *d*, and from *d* to E; AE from A to *e*, and from *e* to F; AF from A to *f*, and from *f* to G; AG from A to *g*, and from *g* to H, &c. Then will the proportional lines be AB, AC, AD, AE, AF, AG, &c. And

AB : AC : : (A*c* =) AC : (A*d* =) AD.
AB : AC : : (A*d* =) AD : (A*e* =) AE.
AB : AC : : (A*e* =) AE : (A*f* =) AF.
*&c.*

*This depends on articles* 104, 95, 165.

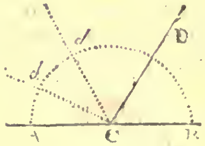
SECTION

## SECTION III.

## *Geometrical Theorems.*

### OF RIGHT LINES AND PLANES.

### 91. THEOREM I.

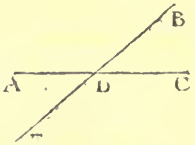


*When one right line* CD *ſtands upon another right line* AB, *they make two angles* BCD, ACD *which together are equal to two right angles.*

DEMONSTRATION. For deſcribing a ſemicircle ADB, on C. (45)
Then the arc DB meaſures the angle BCD. (15)
And the arc DA meaſures the angle ACD. (15)
But the arcs DB and AD together meaſure two right angles. (13, 16)
Therefore BCD and ACD together, are equal to two right angles (50)

92. COROLLARY. Hence if any number of right lines C*d* ſtands on one point C, on the ſame ſide of another right line AB; the ſum of all the angles are equal to two right angles; or are meaſured by 180 degrees.

### 93. THEOREM II.



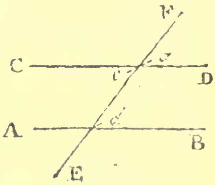
*If two right lines* AC, EB *interſect each other in* D, *the oppoſite angles are equal, viz.*

* ∠ CDB = ∠ ADE, and ∠ CDE = ∠ ADB.

† DEM. For the angles ADE and ADB together make two right angles. (91)
And the angles CDB and ADB together make two right angles. (91)
Therefore the ſum of ADE and ADB = ſum of CDB and ADB. (46)
Conſequently the ∠ CDB is equal to the ∠ ADE. (48)

94. ‡ COROL. Hence if any number of right lines croſs each other in one point, the ſum of all the angles which they make about that point, is equal to four right angles; or is meaſured by 360 degrees.

### 95. THEOREM III.



*If a right line* FE *cut two parallel right lines* AB, CD; *then is the outward angle a* ‖ *equal to the inward and oppoſite angle d; and the alternate angles c, d are equal: and the contrary.*

DEM. Becauſe CD and AB are parallel by ſuppoſition:
Then FE has the ſame inclination to CD and AB. (53)
And this inclination is expreſſed by the ∠ $a$ or ∠ $d$: (14)
Therefore the outward ∠ $a$ is equal to the inward and oppoſite ∠ $d$.
Now the ∠ $a$ is equal to the ∠ $c$. (93)
And ſince the ∠ $a$ is equal to the ∠ $d$;
Therefore the alternate angles $c$ and $d$ are equal. (46)

---

* *The mark* ∠ *ſtands for the word angle.*

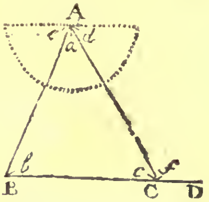
† DEM. *ſtands for Demonſtration.* ‡ COROL. *ſtands for Corollary.*

‖ *Angles are ſometimes marked by a ſingle letter. Thus angle a is uſed for angle* FAD.

96. THEO-

## 96. THEOREM IV.

*In any right lined triangle* ABC, *the ſum of the three angles $a$, $b$, $c$, is equal to two right angles: And if one ſide* BC *be continued, the outward angle $f$ is equal to the ſum of the two inward and oppoſite angles $a$, $b$.*



DEM. Through A, draw a right line parallel to BC, (63) making with AB the $\angle e$, and with AC the $\angle d$.

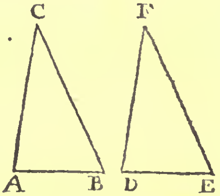
Now $\angle e = \angle b$; and $\angle d = \angle c$. being alternate. (95)
And two right angles meaſure the $\angle c + \angle a + \angle d$. (92)
Therefore $\angle b + \angle a + \angle c = \angle e + \angle a + \angle d$. (47)
Conſequently $\angle b + \angle a + \angle c =$ two right angles. (46)
Moreover $\angle c + \angle f =$ two right angles. (91)
Therefore $\angle b + \angle a = \angle f$. (46)

97. Hence, if one angle is right or obtuſe, each of the other is acute.

98. If two angles of one triangle are equal to two angles of another triangle, the remaining angles are equal. And if one angle in one triangle is equal to one angle in another triangle, then is the ſum of the remaining angles in one, equal to the ſum of the remaining angles in the other.

## 99. THEOREM V.

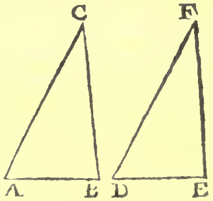
*If two ſides* AB, AC *and the included angle* A *in one triangle* ABC, *are reſpectively equal to two ſides* DE, DF *and the included angle* D *of another* DEF, *each to each; then are thoſe triangles congruous.*



DEM. Apply the point D to the point A, and the line DE to AB. (54)
Now as DE $=$ AB (by ſuppoſition); therefore the point E falls on B.
But $\angle$ D $= \angle$ A (by ſup.); therefore DF will fall on AC.
And ſince DF $=$ AC (by ſup.); therefore the point F falls on C.
Conſequently FE will fall on CB.
Therefore the triangles ACB, DFE, are congruous, ſince every part agrees.

## 100. THEOREM VI.

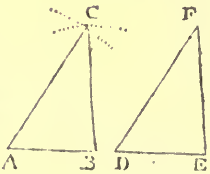
*If two triangles* ABC, DEF *have two angles* A, B *and the included ſide* AB *in one reſpectively equal to two angles* D, E *and the included ſide* DE *in the other, each to each; then are thoſe triangles congruous.*



DEM. Apply the point D to A, and the line DE to AB.
Now as DE $=$ AB (by ſup.); therefore the point E falls on B.
And as $\angle$ D $= \angle$ A (by ſup.); therefore the line DF falls on AC.
Now if the line AC is leſs or greater than the line DF;
Then the line FE not falling on CB, makes the $\angle$ B leſs or greater than $\angle$ E.
But $\angle$ B $= \angle$ E (by ſup.); therefore AC is neither leſs nor greater than DF.
Or the line AC $=$ DF; conſequently FE $=$ CB.
Therefore the triangles are congruous,

101. THEO-

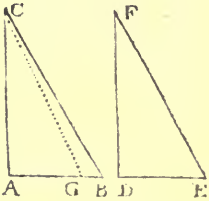
### 101. THEOREM VII.



*Two triangles* ABC, DEF *are congruous, when the three ſides in the one are equal to the three ſides in the other, each to each.*

DEM. Apply the point D to A, and the line DE to AB. (54)
Now as $DE = AB$ (by ſup.); therefore the point E falls on B.
On A, with the radius AC, deſcribe an arc.
Then as $DF = AC$, the point F will fall in that arc.
Alſo on B, with the radius BC, deſcribe another arc, cutting the former in C.
And ſince $EF = BC$, the point F will fall in this arc alſo.
But if the point F can fall in both theſe arcs, it can be only where they interſect, as in C.
Conſequently the triangles are congruous.

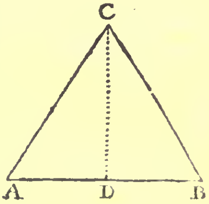
### 102. THEOREM VIII.



*Two triangles* ABC, DEF *are congruous, when two angles* A, B *and a ſide* AC *oppoſite to one of them, in one triangle, are reſpectively equal to two angles* D, E *and a ſide* DF *oppoſite to a like angle in the other triangle, each to each.*

DEM. Apply the point D to A, and the line DF to AC (54)
Now as $DF = AC$ (by ſup.); therefore the point F falls on C.
And as $\angle D = \angle A$ (by ſup.); the line DE will fall on the line AB.
And if the point E does not fall on B, it muſt fall on ſome other point G. Draw CG.
Then the angle AGC is equal to the angle DEF. (99)
And the angle $ABC = (DEF =)\ AGC$, which is not poſſible. (96)
Therefore the point E can fall no where but on the point B.
Conſequently the triangles are congruous.

### 103. THEOREM IX.



*In the Iſoſceles, or equilateral triangle* ACB; *a line drawn from the vertex* C *to the middle of the baſe* AB *is perpendicular to the baſe, and biſects the vertical angle: and the contrary* *.

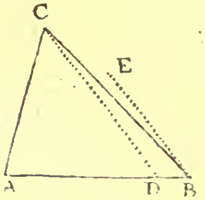
DEM. The triangles ADC, BDC, are congruous.
Since $CA = CB$ (23); $CD = CD$, and $AD = DB$ by ſuppoſition:
Therefore $\angle A = \angle B$, $\angle ACD = \angle BCD$, $\angle ADC = \angle BDC$. (101)
Conſequently CD is at right angles to AB. (17)

104. COROL. Hence in any right lined triangle where there are equal ſides, or angles;

The angles A, B, oppoſite to equal ſides BC, AC are equal.
And the ſides BC, AC, oppoſite to equal angles A, B, are equal.

---

* That is, if a line drawn perpendicular to the baſe of a triangle biſects the vertical angle; then that triangle muſt be Iſoſceles, and the perpendicular is drawn from the middle of the baſe.

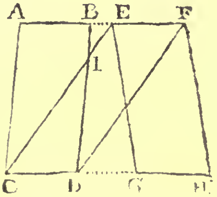


## 105. THEOREM X.

*In every right lined triangle* ABC *the greater angle* C *is oppoſite to the greater ſide* AB.

DEM. In the greater ſide AB, take AD = AC; draw CD, and through B draw BE parellel to CD. (63)
Then the angles ADC, ACD, are equal. (104)
And ∠ADC = ∠ABE. (95)
Therefore ∠ACD = ∠ABE. (46)
That is, a part of the angle ACB is greater than the angle ABC.
Conſequently the ∠C is greater than the ∠B; and in the ſame manner, it may be proved to be greater than the ∠A, if the ſide AB be greater than CB.

106. COROL. Hence in every right lined triangle, the greater ſide is oppoſite to the greater angle.

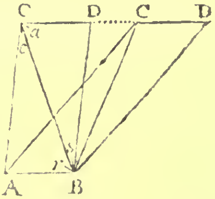


## 107. THEOREM XI.

*Parallelograms* ACDB, ECDF, EGHF *ſtanding on the ſame baſe* CD, *or on equal baſes* CD, GH, *and between the ſame parallels*, CH, AF, *are equal.*

DEM. For AB = EF, being each equal to CD. (28, 46)
To each add BE, and AE = BF. (47)
Now AC = BD, and CE = DF. (28)
The triangles ACE, BDF, are therefore congruous. (101)
Now if from each of the triangles ACE and BDF, be taken the triangle BIE, the remaining trapeziums ABIC and FEID are equal. (48)
Then if to each of the trapeziums ABIC, FEID, be added the triangle CID, their ſum will be the parallelograms AD and DE, which are equal. (47)

And in like manner it may be ſhewn, that the parallelogram EH is equal to the parellelogram ED = AD.



## 108. THEOREM XII.

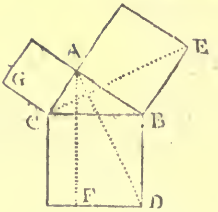
*A triangle* ABC *is the half of a parallelogram* AD, *when they ſtand on the ſame baſe* AB, *and are between the ſame parallels* AB, CD.

DEM. AC is equal to DB, and AB to DC. (28)
Alſo BC is a ſide common to both the triangles ABC and DCB.
Theſe triangles are therefore congruous. (101)
Conſequently the triangle ABC is half the parallelogram AD.

109. COROL. I. Hence every parallelogram is biſected by its diagonal.

110. COROL. II. Alſo, triangles ſtanding on the ſame baſe, or on equal baſes, and between the ſame parallels, are equal.
They being the halves of equal parallelograms under like circumſtances.

## 111. THEOREM XIII.



*In every right angled triangle* BAC *the ſquare on the ſide* BC *oppoſite to the right angle* A *is equal to the ſum of the ſquares of the two ſides* AB, AC *containing the right angle.*

Dem. On the ſides AB, AC, BC, conſtruct the ſquares AG, AE, CD (68): draw AD, CE; and draw AF parallel to BD. (63)
Then the triangles ABD, EBC, are congruous. (99)
For the ∠ABE = ∠CBD, being right angles. (30)
To each add the angle ABC, then ∠EBC and ∠ABD are equal. (47)
Therefore EB, BC, ∠EBC are reſpectively equal to AB, BD, ∠ABD.
Alſo the triangle EBC is half the parallelogram AE (108)
For they ſtand upon the ſame baſe EB, and are between the ſame parallels EB and AC; BA making right angles with BE and CA continued.
Likewiſe the triangle ABD is half the parallelogram BF. (108)
For they ſtand upon the ſame baſe BD, and are between the ſame parallels BD, AF.
Therefore, as the halves of the parallelograms EA and BF are equal, conſequently the parallelogram BF is equal to the ſquare AE. (49)
In the ſame manner may it be ſhewn, that the parallelogram CF is equal to the ſquare AG.
But the parallelograms BF and CF together, make the ſquare CD,
Therefore the ſquare CD is equal to the ſquares EA and AG.

112. Corol. I. Hence if any two ſides of a right angled triangle are known, the other ſide is alſo known.

For BC = ſquare root of the ſum of the ſquares of AC and AB.
AC = ſquare root of the difference of the ſquares of BC and AB.
AB = ſquare root of the difference of the ſquares of BC and AC.

113. Or thus, making the quantities $\overline{BC}^2$, $\overline{AB}^2$, $\overline{AC}^2$, to ſtand for the ſquares made on thoſe lines.

And the mark $\surd$ to ſtand for the ſquare root of ſuch quantities as ſtand under the line joined to the top of this mark.

Then $BC = \surd\overline{\overline{AC}^2 + \overline{AB}^2}$;
$AC = \surd\overline{\overline{BC}^2 - \overline{AB}^2}$;
$AB = \surd\overline{BC^2 - \overline{AC}^2}$.

Scholium. The lines of the lengths 5, 4, 3, (or their doubles, triples, &c.) will form a right angled triangle.

For $5^2 = 4^2 + 3^2$. Or $25 = 16 + 9$.

114. Corol. II. Of all the lines drawn from a given point to a given line, the perpendicular is the ſhorteſt.

115. Corol. III. The ſhorteſt diſtance between two parallel right lines, is a right line drawn from one to the other perpendicular to both.

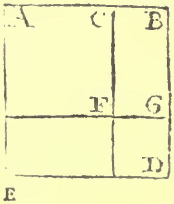
116. Corol. IV. Parallel right lines are equidiſtant; and the contrary. For two oppoſite ſides of a rectangular parallelogram are equal (28); and each is the ſhorteſt diſtance between the other ſides.

## 117. THEOREM XIV.

*If a right line* AB *be divided into any two parts* AC, CB; *then will the ſquare on the whole line be equal to the ſum of the ſquares on the parts, together with two rectangles under the two parts.*

That is $\overline{AB}^2 = \overline{AC}^2 + \overline{CB}^2 + 2 \times AC \times CB$.*

Dem. Let AD, AF, be ſquares on AB, AC. (68)
Then will FG, and GD, be each equal to CB. (48)
Hence FD is a ſquare on a line equal to CB. (30)
Alſo FB and FE are rectangles on lines equal to AC, CB.
But the ſquares AF, FD, and the rectangles FB, FE, fill up the ſquare AD, or are equal to AD.



118. Corol. I. Hence the ſquare of AC the difference between two lines AB, CB, is equal to the ſquare of the greater AB, leſſened by the ſquare of the leſs CB and by two rectangles under the leſſer line CB and the ſaid difference.

That is $\overline{AB-BC}^2 = \overline{AB}^2 - \overline{BC}^2 - 2BC \times AC$.
For $AB - BC = AC$.
Then $\overline{AC}^2 = \overline{AB}^2 - \overline{BC}^2 - 2BC \times AC$. (48)

119. Corol. II. The difference between the ſquares on two lines AB, AC is equal to the rectangle under the ſum AB + BC and difference AB — BC of thoſe lines.

That is $\overline{AB}^2 - \overline{AC}^2 = \overline{AB+BC} \times \overline{AB-AC}$.
For $\overline{AB}^2 - \overline{AC}^2 = \overline{CB}^2 + 2AC \times CB$. (117, 48)
$= \overline{CB+AC+AC} \times CB$.
$= \overline{AB+AC} \times (CB =) \overline{AB-AC}$.

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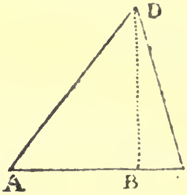
* *The rectangle under two lines is generally expreſſed by 3 letters; the firſt two letters ſtand for one line, and the laſt two for the other line.*
*Thus for* AC × CB, *is written* ACB.
*for* AB × BC, *is written* ABC.
*And for* 2 × AC × CB, *is written* 2ACB.

## 120. THEOREM XV.

*In every triangle,* ADC *the ſquare of a ſide* CD *ſubtending an acute angle* A, *is equal to the ſquares of the ſides* AD, AC *about that angle, leſſened by two rectangles under one of thoſe ſides* AC *and that part* AB *contained between the acute angle and the perpendicular* DB *drawn to that ſide* AC *from its oppoſite angle* D.



That is, $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2CAB$.

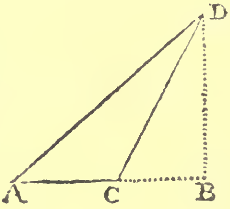
DEM. $\overline{DC}^2 - \overline{BC}^2 = (\overline{DB}^2 =)\, \overline{AD}^2 - \overline{AB}^2$. (111)
And $\overline{AC}^2 = 2ABC + \overline{BC}^2 + \overline{AB}^2$. (117)
Then $\overline{DC}^2 - \overline{BC}^2 - \overline{AC}^2 = \overline{AD}^2 - \overline{BC}^2 - 2\overline{AB}^2 - 2ABC$. (48)

And $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - 2\overline{AB}^2 - 2ABC$, by adding $\overline{BC}^2 + \overline{AC}^2$. (47)
$(-AB \times 2AB - BC \times 2AB)$
$(-\overline{AB+BC} \quad \times 2AB \quad )$
Then $\overline{DC}^2 = \overline{AD}^2 + \overline{AC}^2 - (AC \times 2AB =)\, 2CAB$.

121. COROL. Hence $AB = \dfrac{\overline{AD}^2 + AC^2 - DC^2}{2CA}$.

## 122. THEOREM XVI.

*In an obtuſe angled triangle* ACD, *the ſquare of the ſide* AD *oppoſite to the obtuſe angle* C *is equal to the ſum of the ſquares of the ſides* AC, CD *about the obtuſe angle; together with two rectangles under one ſide* AC, *and the continuation* CB *of that ſide to meet a perpendicular* DB *drawn to it from the oppoſite angle* D.

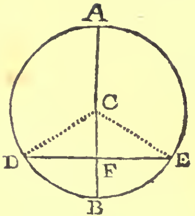


That is, $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2AC \times CB$.

DEM. For $\overline{AD}^2 - \overline{AB}^2 = (\overline{DB}^2 =)\, \overline{CD}^2 - \overline{CB}^2$. (111)
And $\overline{AB}^2 = 2ACB + \overline{AC}^2 + \overline{CB}^2$. (117)
Then $\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 + 2ACB$. (47)

123. COROL. Hence $CB = \dfrac{\overline{AD}^2 - \overline{AC}^2 - \overline{CD}^2}{2AC}$.

## 124. THEOREM XVII.

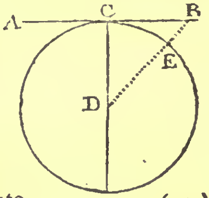


*In any circle, a diameter,* AB, *drawn perpendicular to a chord,* DE, *biſects that chord and its ſubtended arc* DBE.

DEM. From the center C, draw the radii CD, CE, to the extremities of the chord DE.
Then the triangles CFE, CFD, are congruous. (102)
For CF being at right angles to DE, the $\angle CFD = \angle CFE$. (17)
And the triangle CDE being iſoſceles (23), the $\angle D = \angle E$ (104). Alſo CF is common.
Therefore $DF = FE$: And the arc $DB = BE$:
For thoſe arcs meaſure the equal angles FCE, FCD. (15)

125. COROL. Hence, in a circle, a right line drawn through the middle of a chord at right angles to it, paſſes through the center of that circle; and the contrary.

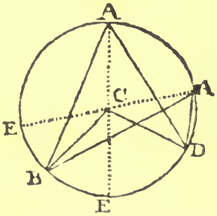
## 126. THEOREM XVIII.



*A tangent,* AB, *to a circle is perpendicular to a diameter,* DC, *drawn to the point of contact,* C.

DEM. If it be denied that DC is perpendicular to AB. Then from the center D, let ſome other line DE, cutting the circle in E, be drawn perpendicular to AB.
Now the angle DBC being right, the angle DCB is acute. (97)
Conſequently DC is greater than DB. (106)
But $DC = DE$ (9). Therefore DE is greater than DB, which is abſurd.
Therefore no other line paſſing through the center can be perpendicular to the tangent, but that which meets it at the point of contact.

## 127. THEOREM XIX.



*An angle,* BCD, *at the center of a circle, is double of the angle,* BAD *at the circumference, when thoſe angles ſtand on the ſame arc,* BD.

DEM. Through the point A draw the diameter AE.
Then the angle $ECD = \angle CAD + \angle CDA$. (96)
But the $\angle CAD = \angle CDA$. (104)
Therefore the $\angle ECD$ is equal to twice the angle CAD.
In the ſame manner it may be ſhewn, that the angle BCE is equal to twice the angle BAE.
Conſequently the angle BCD $(= \angle ECD \underset{\backsim}{+}{}^{*} \angle BCE)$ is equal to twice the angle BAD $(= \angle EAD \underset{\backsim}{+} \angle BAE)$. (47, 48)

128. COROL. I. Hence an angle, BAD, at the circumference is meaſured by half the arc, BD, on which it ſtands.
For the angle at the center BCD is meaſured by the arc BD. (15)
Conſequently the angle $BAD =$ half the angle BCD, is meaſured by half the arc DB.

129. COROL. II. All angles in the circumference, and ſtanding on the ſame arc, are equal.

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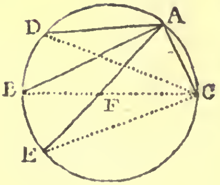
* The mark $\underset{\backsim}{+}$, ſignifies the ſum or difference.

## 130. THEOREM XX.

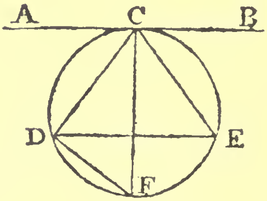


*An angle,* BAC, *in a ſemicircle, is a right one.*
*An angle,* DAC, *in a ſegment leſs than a ſemicircle, is obtuſe.*
*An angle,* EAC, *in a ſegment greater than a ſemicircle, is acute.*

DEM. For the angle BAC is meaſured by half the ſemicircular arc BEC, or is meaſured by half of 180 degrees; that is, by 90 degrees. (128, 16)
And ∠DAC is meaſured by half the arc DEC, greater than 180° *.
Alſo ∠EAC is meaſured by half the arc EC, leſs than 180°.
Therefore theſe angles are reſpectively equal to, greater, or leſs than 90 degrees.

131. COR. Hence in a right angled triangle BAC; the angular point A of the right angle, and the ends, B, C, of the oppoſite ſide, are equally diſtant from F, the middle of that ſide; that is, a circle will always paſs through the right angle, and the ends of its oppoſite ſide taken as a diameter.

## 132. THEOREM XXI.



*The angle* ACD, *formed by a tangent* AB, *to a circle,* CDE, *and a chord,* CD, *drawn from the point of contact,* C, *is equal to an angle,* CED, *in the alternate ſegment; and is meaſured by half the arc* DC *of the included ſegment.*

DEM. Draw the diameter FC, and join DF.
The $\angle^s$ ACF and CDF are both right. (126, 130)
Therefore the $\angle^s$ DCF and DFC are together = a right ∠. (96)
But the $\angle^s$ ACD and DCF are together = a right ∠.
Conſequently the ∠ DCF and DFC = the $\angle^s$ ACD and DCF. (46)
Take the ∠ DCF from each, and the ∠ DFC = the ∠ ACD. (48)
But the $\angle^s$ DFC and DEC are equal. (129)
Conſequently the $\angle^s$ DEC and ACD are equal alſo. (46)

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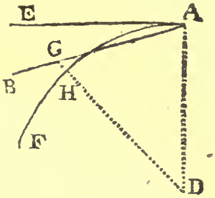
* A ſmall ° put above any figure, ſignifies degrees.

## 133. THEOREM XXII.



*Between a circular arc* AHF, *and its tangent* AE, *no right line can be drawn from the point of contact* A.

DEM. For if any other right line can be drawn, let it be the right line AB.
From D, the center of AHF, draw DG perpendicular to AB, cutting AB in G, and the arc in H.
Now as ∠ DGA is right; therefore DA is greater than DG. (106)
But DA = DH (9). Therefore DH is greater than DG, which is abſurd.
Conſequently no right line can be drawn between the tangent AE and the arc AHF.

134. COROL. I. Hence the angle DAH, contained between the radius DA, and an arc AH, is greater than any right lined acute angle.

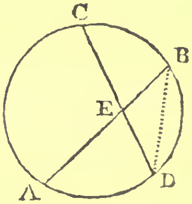
For a right line AB muſt be drawn from A, between the tangent AE and radius AD, to make an acute angle.

But no ſuch right line can be drawn between AE and the arc AH. (133)

135. COROL. II. Hence the angle EAH, between the tangent EA and arc AH, is leſs than any right lined acute angle.

136. COROL. III. Hence it follows, that at the point of contact the arc has the ſame direction as the tangent, and is at right angles to the radius drawn to that point.

## 137. THEOREM XXIII.



*If two right lines*, AB, CD, *interſect any how (in* E) *within a circle, their inclination*, AED, *or* CEB, *is meaſured by half the ſum of the intercepted arcs*, AD, CB.

DEM. For drawing DB; (43)
The ∠ AED = ∠ EDB + ∠ EBD. (96)
But the ∠ EDB is meaſured by $\frac{1}{2}$ arc CB. (128)
And the ∠ EBD is meaſured by $\frac{1}{2}$ arc AD. (128)
Conſequently the ∠ AED is meaſured by half the arc CB, together with half the arc AD. (50)

## SECTION IV.

## *Of Proportion.*

### DEFINITIONS and PRINCIPLES.

138. *One quantity* A, *is ſaid to be meaſured or divided by another quantity* B, *when* A *contains* B *ſome number of times, exactly.*

Thus if A = 20, and B = 5 ; then A contains B four times.

A is called a multiple of B ; and B is ſaid to be part of A.

139. *If a quantity* A (= 20) *contains another* B (= 5) *as many times as a quantity* C (= 24) *contains another* D (= 6) ; *then*
A *and* C *are called like multiples of* B *and* D.
B *and* D *are called like parts of* A *and* C : *And*
A *is ſaid to have the ſame relation to* B, *as* C *has to* D.

Or, like multiples of quantities are produced, by taking their Rectangle, or Product, by the ſame quantity, or by equal quantities.

The Rectangle or Product of quantities, A and B, is expreſſed by writing this mark × between them. Thus, $A \times B$, or $B \times A$, expreſſes the rectangle contained by A and B.

140 *When two quantities of a like kind are compared together, the relation which one of them has to the other, in reſpect to quantity, is called Ratio.*

The firſt term of a ratio, or the quantity compared, is called the Antecedent ; and the ſecond term, or the quantity compared to, is called the Conſequent.

A ratio is uſually denoted by ſetting the antecedent above the conſequent with a line drawn between them.

Thus $\frac{A}{B}$ ſignifies, and is thus to be read, the ratio of A to B.

The multiple of a ratio $\frac{A}{B}$, is the product of each of its terms by the ſame quantity, or by equal quantities. Thus $\frac{A \times C}{B \times C}$ is the ratio $\frac{A}{B}$ taken C times.

The product of two or more ratios, $\frac{A}{B}$, $\frac{C}{D}$, is expreſſed by taking the product of the antecedents for a new antecedent, and the product of the conſequents for a new conſequent. Thus $\frac{A \times C}{B \times D} = \frac{A}{B} \times \frac{C}{D}$.

141. *Equal ratios are thoſe where the antecedents are like multiples or parts of their reſpective conſequents.*

Thus in the quantities A, B, C, D : Or 20, 5, 24, 6.

In the ratio of A to B, or of 20 to 5, the antecedent is a multiple of its conſequent four times.

And in the ratio of C to D, or 24 to 6, the antecedent is a multiple of its conſequent four times.

That is, the ratio of A to B is the ſame as the ratio of C to D.

And this equality of ratios is thus expreſſed, $\frac{A}{B} = \frac{C}{D}$.

142. *Ratio of equality is, when the antecedent is equal to the conſequent.*

Thus when $A = B$, then $\frac{A}{B}$, or $\frac{A}{A}$, or $\frac{B}{B}$, is a ratio of equality.

143. *Four quantities are ſaid to be proportional, which, when compared together by two and two, are found to have equal ratios.*

Thus, let the quantities to be compared be A, B, C, D : Or 20, 5, 24, 6.
Now in the ratio of A to B, or of 20 to 5 ; A contains B four times.
And in the ratio of C to D, or of 24 to 6 ; C contains D four times.

Then the ratios of A to B, and of C to D, are equal : Or $\frac{A}{B} = \frac{C}{D}$. (141)

And their proportionality is thus expreſſed, $A : B :: C : D$. (75)
Alſo in the ratio of A to C, or of 20 to 24 ; C contains A, once and $\frac{1}{5}$.
And in the ratio of B to D, or of 5 to 6 ; D contains B, once and $\frac{1}{5}$.

Where the ratios are likewiſe equal, *viz.* $\frac{A}{C} = \frac{B}{D}$.

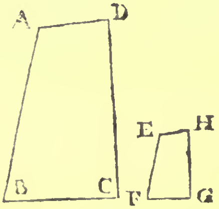
And theſe are alſo proportional, $A : C :: B : D$.

144. *So that when four quantities of the ſame kind are proportional, the ratio between the firſt and ſecond is equal to the ratio between the third and fourth ; and this proportionality is called Direct.*

145. *Alſo the ratio between the firſt and third is equal to the ratio between the ſecond and fourth ; and this proportionality is called Alternate.*

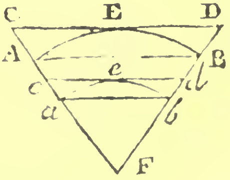
146. *Similar, or like, right lined figures, are thoſe which are equiangular,* (that is, the ſeveral angles of which are equal one to the other ;) *and alſo, the ſides about the equal angles proportional.*

Thus if the figures AC and EG are equiangular,
And $AB : BC :: EF : FG$ ; Or $BC : CD :: FG : GH$ ;
Then are thoſe figures called ſimilar, or like figures.
And the like in triangles, or other figures.



147. *Like arcs, chords, or tangents, in different circles, are thoſe which ſubtend, or are oppoſite to, equal angles at the center.*

Let F be the center of two concentric arcs AEB, *aeb*, terminated by the radii F*a*A, F*b*B, produced ; AB, *ab*, their chords, and CD, *cd*, their tangents. Then as the angle CFD is meaſured either by the arc AEB, or *aeb*, thoſe arcs are ſaid to be alike, or ſimilar ; that is, the arc *aeb* is the ſame part of its whole circumference, as the arc AEB is of its whole circumference.



## 148. THEOREM XXIV.

*Quantities, and their like multiples, have the ſame ratio.*

That is, the ratio of A to B is equal to the ratio of twice A to twice B, or thrice A to thrice B, &c. Or thus $\frac{A}{B}=\frac{2A}{2B}=\frac{3A}{3B}$, &c. $=\frac{C\times A}{C\times B}$; that is, equal to the ratio of C times A to C times B.

DEM. For the ratio of A to B muſt either be equal to the ratio of like multiples of A and B, or to the ratio of unlike multiples of them.
Now ſuppoſe the ratio of A to B is equal to the ratio of their unlike multiples, C times A, D times B; that is, $\frac{A}{B}=\frac{C\times A}{D\times B}$.
Then $A : B :: C\times A : D\times B$ (143). And $A : C\times A :: B : D\times B$. (145)
Therefore $\frac{A}{C\times A}=\frac{B}{D\times B}$ (144). Where the conſequents are unequal multiples of their antecedents, by ſuppoſition.
But $\frac{A}{C\times A}$ is not equal to $\frac{B}{D\times B}$. (141)
Then $A : C\times A :: B : D\times B$ is not true. Alſo $A : B :: C\times A : D\times B$ is not true.
Conſequently $\frac{A}{B}$ is unequal to $\frac{C\times A}{D\times B}$.
Therefore the ratio of unlike multiples of two quantities, is not equal to the ratio of thoſe quantities.
Conſequently the ratio of two quantities, and the ratio of their like multiples, are the ſame. Or $\frac{A}{B}=\frac{C\times A}{C\times B}$.

149. COR. I. In any ratio, if both terms contain the ſame quantity or quantities; the value of the ratio will not be altered by omitting, or taking away thoſe quantities. For $\frac{C\times A}{C\times B}=\frac{A}{B}$, by taking away C.

150. COR. II. Quantities, and their like parts, have equal ratios. For A and B are like parts of $C\times A$ and $C\times B$.

151. COR. III. Quantities, and their like multiples, or like parts, are proportional. For $A:B::C\times A:C\times B$. And $C\times A:C\times B::A:B$ (148)

152. COR. IV. If quantities are equal, their like multiples, or like parts, are alſo equal. For if $A=B$; and $\frac{A}{B}=\frac{C\times A}{C\times B}$;
Then are the antecedents and conſequents in a ratio of equality. (141)

153. COR. V. If the parts of one quantity are proportional to the parts of another quantity, they are like parts of their reſpective quantities.
For only like parts are proportional to their wholes. (151)

154. COR. VI. Ratios, which are equal to the ſame ratio, are equal to one another. For the ratio of $\frac{A}{B}=\frac{C\times A}{C\times B}=\frac{D\times A}{D\times B}$, &c. (148)

155. Cor. VII. Proportions, which are the ſame to the ſame proportion, are the ſame to one another.
If $A:B::C:D$; and $A:B::E:F$; Then $C:D::E:F$.

For $\frac{A}{B}=\frac{D}{C}$; and $\frac{A}{B}=\frac{E}{F}$ (144). Then $\frac{C}{D}=\frac{E}{F}$. (46)

156. Cor. VIII. If two ratios or products are equal, their like multiples, either by the ſame or by equal quantities, or by equal ratios, are alſo equal.

That is, if $\frac{A}{B}=\frac{C}{D}$: Then $\frac{A\times E}{B\times E}=\frac{C\ \times}{D\times E}$.

And if $E=F$: Then $\frac{A\times E}{B\times E}=\frac{C\times F}{D\times F}$.

And if $\frac{E}{F}=\frac{G}{H}$: Then $\frac{A\times E}{B\times F}=\frac{C\times G}{D\times H}$.

For in either caſe, the ratios may be conſidered as quantities.

## 157. THEOREM XXV.

*Equal quantities,* A *and* B, *have the ſame ratio or proportion to another quantity* C. *And any quantity has the ſame ratio to equal quantities.*

That is, if $A=B$: Then $A:C::B:C$. And $C:A::C:B$.

Dem. Since $A=B$; then C is the like multiple, or part of B, as it is of A.
And $A:B::C:C$ (151). Therefore $A:C::B:C$. (145)
Alſo $C:C::A:B$ (151). Therefore $C:A::C:B$. (145)

158. Cor. I. Hence, when the antecedents are equal, the conſequents are equal; and the contrary.

159. Cor. II. Quantities are equal, which have the ſame ratio to another quantity: or to like multiples or parts of another quantity.
Thus, if $A:C::B:C$. Then $A=B$.

160. Cor. III. Since $A:C::B:C$; and $C:A::C:B$. Therefore, when four quantities are in proportion, As antecedent is to conſequent, ſo is antecedent to conſequent: Then ſhall the firſt conſequent be to its antecedent, as the ſecond conſequent to its antecedent: and this is called *the inverſion of ratios.*

## 161. THEOREM XXVI.

*In two, or more, ſets of proportional quantities, the rectangles under the like terms are proportional.*

That is, if $A:B::C:D$; and $E:F::G:H$.
Then $A\times E:B\times F::C\times G:D\times H$.

Dem. Since $\frac{A}{B}=\frac{C}{D}$; and $\frac{E}{F}=\frac{G}{H}$. (144)

Therefore $\frac{A\times E}{B\times F}=\frac{C\times G}{D\times H}$. (156)

Conſequently $A\times E:B\times F::C\times G:D\times H$. (143)

## 162. THEOREM XXVII.

*In four proportional quantities* $A : B :: C : D$. *Then the Rectangle or Product of the two extremes is equal to the Rectangle or Product of the two means.* That is, $A \times D = B \times C$.

DEM. Since $A : B :: C : D$ by ſuppoſition. Therefore $\frac{A}{B} = \frac{C}{D}$. (144)

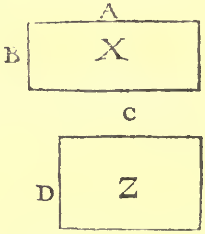
And $\frac{A}{B} = \frac{A \times D}{B \times D}$ (156). Alſo $\frac{C}{D} = \frac{C \times B}{D \times B}$. (156)

Therefore $\frac{A \times D}{B \times D} = \frac{C \times B}{D \times B}$ (46), where the conſequents are equal.

Conſequently $A \times D = C \times B$. (158)

163. Hence, if the Rectangle or Product of two quantities is equal to the Rectangle or Product of other two quantities; thoſe four quantities are proportional.

Thus, ſuppoſe the two Rectangles, X, Z, are equal;
Where A, C, are their lengths, and B, D, their breadths.



Then $A \times B = C \times D$ by ſuppoſition.
Therefore $A : C :: D : B$,
That is, As the length of X is to the length of Z.
So the breadth of Z is to the breadth of X.

In ſuch caſes, the lengths are ſaid to be to one another reciprocally, as their breadths.
Or that proportion $A : C :: D : B$ is reciprocal, when $A \times B = C \times D$.

## 164. THEOREM XXVIII.

*If four quantities are proportional; then will either of the extremes, and the ratio of the product of the means to the other extreme, be in the ratio of equality: And either mean, and the ratio of the product of the extremes to the other mean, will be alſo in a ratio of equality.*

That is, if $A : B :: C : D$. Then $A = \frac{B \times C}{D}$. And $B = \frac{A \times D}{C}$.

DEM. Since $A : B :: C : D$ by ſuppoſition.
Therefore $A \times D = B \times C$. (162)

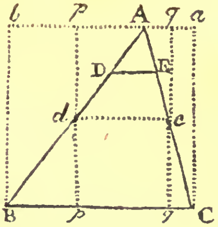
And $\frac{A \times D}{C} = \frac{B \times C}{C}$ (157). Alſo $\frac{B \times C}{D} = \frac{A \times D}{D}$. (157)

But $B = \frac{B \times C}{C}$ (149). And $A = \frac{A \times D}{D}$. (149)

Therefore $\frac{A \times D}{C} = B$, And $\frac{B \times C}{D} = A$. (46)

165. THEO-

## 165. THEOREM XXIX.



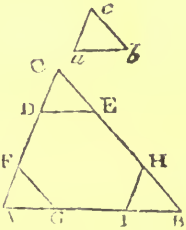
*In any plane triangle,* ABC, *any two adjoining ſides,* AB, AC, *are cut proportionally by a line* DE, *drawn parallel to the other ſide* BC, *viz.*
AD : DB : : AE : EC.

DEM. Through B and C draw B*b*, C*a*, at right angles to BC, meeting *ba*, drawn through A, parallel to BC: Through *p*, *q*, the middles of A*b*, A*a*, draw *pp*, *qq*, parallel to B*b* or C*a*, meeting AB, AC, in the points *d*, *c*; and join *dc*.
Now the triangles A*dp*, B*dp*, and A*cq*, C*cq*, are congruous. (95,100)
Therefore A*d*=B*d*, A*c*=C*c*, *pd*=*pd*, *qc*=*qc*.
But *pp*=*qq* (116): Therefore *pd*=*qc*. (49)
And *dc* is parallel to BC. (116)

In the ſame manner it may be ſhewn, that lines parallel to B*b*, drawn through the middles of A*p*, *pb*; A*q*, *qa*; will alſo biſect A*d*, B*d*; A*c*, C*c*; and that lines joining theſe points of biſection will alſo be parallel to BC: And the ſame may be proved at any other biſections of the ſegments of the lines AB, AC: Alſo the like may be readily inferred at any other diviſions of the lines A*b*, A*a*.
Therefore lines parallel to BC, cut off like parts from the lines AB, AC.
Then AB : AC : : AD : AE. And AB : AC : : BD : CE. (151)
Therefore AD : AE : : BD : CE. (155)
And by Alternation AD : BD : : AE : CE. (145)

166. COR. Hence, when the ſides AB, AC, of a triangle are cut proportionally, in D, E, the ſegments AD, AE; DB, EC; of thoſe ſides are proportional to the ſides: And the line DE, drawn to thoſe ſections, is parallel to the other ſide BC.

## 167. THEOREM XXX.



*In equiangular triangles,* ABC, *abc, the ſides about the equal angles are proportional; and the ſides oppoſite to equal angles are alſo proportional.*

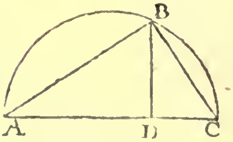
DEM. In CA, CB, take CD=*ca*, CE=*cb*; and draw DE.
Then the triangles CDE, *cab*, being congruous. (99)
The ∠CDE=(∠*a*=) ∠A. Therefore DE is parallel to AB. (95)
In the ſame manner, taking AF=*ac*, AG=*ab*; alſo BH=*bc*, BI=*ba*; and drawing FG, HI, the triangles AGF, IBH, *abc*, are congruous; therefore FG is parallel to CB, and HI is parallel to CA.

$$\text{Then}\left.\begin{array}{l}(\mathrm{CD}=)\ ca : \mathrm{CA} : : (\mathrm{CE}=)\ cb : \mathrm{CB}.\\ (\mathrm{AF}=)\ ca : \mathrm{CA} : : (\mathrm{AG}=)\ ab : \mathrm{AB}.\\ (\mathrm{BH}=)\ bc : \mathrm{BC} : : (\mathrm{BI}=)\ ab : \mathrm{AB}.\end{array}\right\}\quad(165)$$

168. COR. Hence, Triangles have one angle in each equal, and the ſides about thoſe equal angles proportional, thoſe triangles are equiangular and ſimilar.

169. THEO-

## 169. THEOREM XXXI.



*In a right angled triangle*, ABC, *if a line*, BD, *be drawn from the right angle* B, *perpendicular to the oppoſite ſide*, AC; *then will the triangles* ABD, BCD, *on each ſide the perpendicular, be ſimilar to the whole* ABC, *and to one another.*

DEM. For in the triangles ABC, ADB, the $\angle A$ is common;
And the right angle ABC = right angle ADB.
Therefore the remaining $\angle C = \angle ABD$. (98)
In the ſame manner it will appear, that the triangles ABC, BDC, are like.
Therefore the triangles ABD, BCD, are alſo ſimilar.

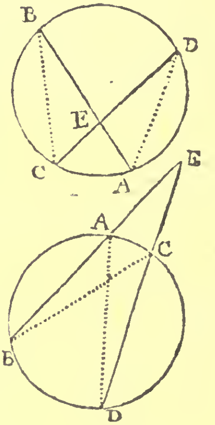
170. COR. I. Hence,

$$\left.\begin{array}{l} AC : AB :: AB : AD. \\ AC : BC :: BC : DC. \\ AD : DB :: DB : DC. \end{array}\right\} \quad (167)$$

171. COR. II. Hence a right line BD, drawn from a circumference of a circle perpendicular to the diameter AC, is a mean proportional between the ſegments AD, DC, of the diameter.
And $AD \times DC = DB^2$. (162)
For a circle, the diameter of which is AC, will paſs through A, B, C. (131)

Scholium. This corollary includes what is uſually called one of the chief properties of the circle, namely;
*The ſquare of the Ordinate is equal to the rectangle under the two Abſciſſas.*
Here, the ordinate is the perpendicular BD; and the two Abſciſſas are the two ſegments AD, DC, of the diameter AC.

## 172 THEOREM XXXII.

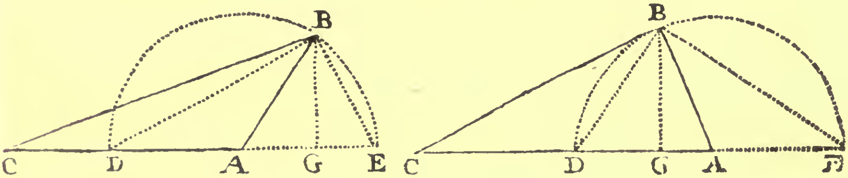


*In a circle, if two chords*, AB, CD, *interſect each other in* E, *either within the circle, or without, by prolonging them; then the rectangle under the ſegments, terminated by the circumference and their interſection, will be equal.*

That is, $AE \times EB = CE \times ED$.
DEM. Draw the lines BC, DA.
Then the triangles DEA, BEC, are ſimilar.
For the angle at E is equal (93), or common.
And the $\angle D = \angle B$, as ſtanding on the ſame arc AC (129). Then the other angles are equal. (98)
Therefore AE : CE :: ED : EB. (167)
Conſequently $AE \times EB = CE \times ED$. (162)

## 173. THEOREM XXXIII.

*If with the leaſt ſide* AB *of a given triangle* ABC, *a ſemicircle be deſcribed from the angular point* A; *meeting the ſide* AC, *produced in the points* D, E; *and from* B, *the lines* BE, BD, *be drawn, and alſo* BG *perpendicular to* DE: *Then the values of the ſeveral lines* AG, CG, GE, GD, BE, BD, BG, *may be expreſſed in terms of the ſides of the triangle* ABC, *as follow.*



174. $$AG = \frac{\overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (123)$$

Or $$AG = \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2AC}. \quad (121)$$

175. $$CG = \frac{\overline{AC}^2 - \overline{AB}^2 + \overline{BC}^2}{2AC}.$$ For $CG = AC + AG$, or to $AC - AG$.

And $$AC + AG = AC \pm \frac{\overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (174)$$

$$= \frac{2\overline{AC}^2 + \overline{BC}^2 - \overline{AC}^2 - \overline{AB}^2}{2AC}. \quad (149)$$

176. $$GE = \frac{2H \times \overline{H - CB}}{2AC}:$$ Here $2H = AC + AB + BC$.

For $$GE = AB(AE) \pm AG = AB + \frac{\overline{AC}^2 + \overline{AB}^2 - \overline{CB}^2}{2AC}. \quad (174)$$

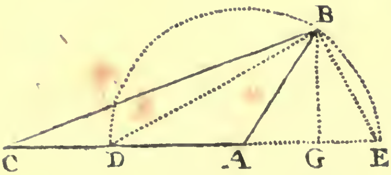
$$= \frac{2AC \times AB + \overline{AC}^2 + \overline{AB}^2 - \overline{BC}^2}{2AC}. \quad (149)$$

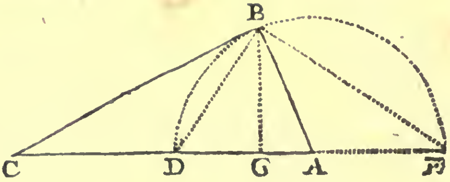
But $2AC \times AB + \overline{AC}^2 + \overline{AB}^2 = (\overline{AC + AB}^2 =) \overline{CE}^2$. (117)

Then $$GE \left( = \frac{\overline{CE}^2 - \overline{BC}^2}{2AC} \right) = \frac{\overline{CE + BC} \times \overline{CE - BC}}{2AC}. \quad (119)$$

$$= \frac{\overline{CA + AB + BC} \times \overline{CA + AB - BC}}{2AC}.$$

$$= \frac{2H \times \overline{2H - 2BC}}{2AC} = \frac{2H \times \overline{H - BC}}{AC}.$$





177. $GD = \frac{\overline{H - AC} \times \overline{2H - 2AB}}{AC}$. Here $2H = AC + AB + BC$.

For $GD = AD \mp AG = AB \mp AG = AB - \frac{\overline{AC}^2 - \overline{AB}^2 + \overline{BC}^2}{2AC}$. (174)

$$= \frac{2AC \times AB - \overline{AC}^2 - \overline{AB}^2 + \overline{BC}^2}{2AC}.$$

$$= \frac{\overline{AC - AB}^2 + \overline{BC}^2}{2AC} = \frac{\overline{BC}^2 - \overline{CD}^2}{2AC}. \quad (118)$$

$$= \frac{\overline{BC - CD} \times \overline{BC + CD}}{2AC}. \quad (119)$$

$$= \frac{\overline{BC + AB - AC} \times \overline{BC + AC - AB}}{2AC}.$$

$$= \frac{\overline{2H - 2AC} \times \overline{2H - 2AB}}{2AC}.$$

178. $BE = \sqrt{\frac{AB}{AC} \times 2H \times \overline{H - CB}}$.

For $\overline{BE}^2 = DE \times GE$. (170)

$$= 2AB \times \frac{\overline{CE}^2 - \overline{BC}^2}{2AC}. \quad (176)$$

Therefore $BE = \sqrt{\frac{2AB}{2AC} \times \overline{\overline{CE}^2 - \overline{BC}^2}} = \sqrt{\frac{AB}{AC} \times 2H \times \overline{H - CB}}$. (176)

179. $BD = \sqrt{\frac{AB}{AC} \times \overline{H - AC} \times \overline{2H - 2AC}}$.

For $\overline{BD}^2 = DE \times GD$. (170)

$$= 2AB \times \frac{\overline{BC}^2 - \overline{CD}^2}{2AC}. \quad (177)$$

Therefore $BD = \sqrt{\frac{2AB}{2AC} \times \overline{\overline{BC}^2 - \overline{CD}^2}}$. (177)

180. $BG = \frac{2}{AC} \times \sqrt{H \times \overline{H - CB} \times \overline{H - AC} \times \overline{H - AB}}$.

For $\overline{BG}^2 = GE \times GD$. Therefore $BG = \sqrt{GE} \times \sqrt{GD}$. (170)

And $GE = \frac{2}{AC} \times H \times \overline{H - CB}$. (176) $GD = \frac{2}{AC} \times \overline{H - AC} \times \overline{H - AB}$. (177)

181. Corol. Hence is derived the Rule uſually given for finding the area, or ſuperficial content, of a Triangle, the three ſides being known.

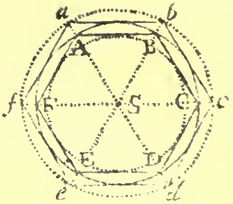
Rule. 1. From half the ſum of the three ſides, ſubtract each ſide ſeverally, noting the three remainders.

2d. Multiply the ſaid half ſum, and the three noted remainders continually.

3d. The ſquare root of the product is the area of the Triangle.

182. THEO-

## 182. THEOREM XXXIV.



*If a regular polygon,* ABCDEF, *be inſcribed in a circle; and parallel to theſe ſides if tangents to the circle be drawn, meeting one another in the points a, b, c, d, e, f; then ſhall the figure formed by theſe tangents circumſcribe the circle, and be ſimilar to the inſcribed figure.*

DEM. Since the circle touches every ſide of the figure *abcdef*, by conſtruction; therefore the circle is circumſcribed by that figure. (41)
Through A and B, draw the radii SA, SB, prolonged till they meet the tangent *ab*, in *a*, *b*.
Then the triangles ASB, *asb*, are equiangular.
For the ∠ at S is common; and the other angles are equal, becauſe AE and *ab* are parallel, by ſuppoſition.
Alſo $Sa = Sb$: For the triangles ASB, *asb*, are iſoſceles. (104)
And the ſame may be proved of the other triangles; and alſo, that they are equal to one another.
Therefore the figure *abcdef* has equal ſides, and is equiangular to the figure ABCDEF.
Now $SA : Sa :: AB : ab$; and $SA : Sa :: AF : af$. (167)
Therefore $AB : ab :: AF : af$. And the like of the other ſides. (155)
Conſequently the figures ABCDEF, *abcdef*, are ſimilar. (145)

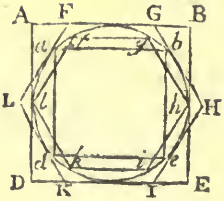
183. COR. I. If two figures are compoſed of like ſets of ſimilar triangles, thoſe figures are ſimilar.

184. COR. II. Hence, if from the angles *a*, *b*, of a regular polygon circumſcribed a circle, lines *a*S, *b*S, be drawn to the center S; the chords AB of the intercepted arcs will be the ſides of a ſimilar polygon, inſcribed in the circle: and the ſides AB, *ab*, of the inſcribed and circumſcribing polygons will be parallel.

185. COR. III. The chords or tangents of like arcs in different circles, are in the ſame proportion as the radii of thoſe circles.
For if a circle circumſcribe the polygon *abcdef*; then the ſides of the polygons *abcdef*, ABCDEF, are chords of like arcs in their reſpective circumſcribing circles.
And if a circle be inſcribed in the polygon ABCDEF, the ſides AB, *ab*, *&c.* are tangents of like arcs alſo: And theſe have been ſhewn to be proportional to their radii SA, S*a*.

186. COR. IV. The Perimeters of like polygons (or the ſum of their ſides) are to one another as the radii of their inſcribed or circumſcribed circles.
For $SA : Sa :: AB : ab$. (182)
And AB, *ab*, are like parts of the perimeters of their polygons.
Therefore $SA : Sa ::$ perimeter ABCDEF : perimeter *abcdef*. (151)

## 187. THEOREM XXXV.



*If there be two regular and like polygons applied to the ſame circle, the one inſcribed and the other circumſcribed: Then will the circcumference of that circle, and half the ſum of the perimeters of thoſe polygons, approach nearer to equality, as the number of ſides in the polygons increaſe.*

Dem. It is evident at ſight, that the circumſcribing hexagon FGHIKL is leſs than the circumſcribing ſquare ABED.

And alſo that the inſcribed hexagon *fghikl* is greater than the inſcribed ſquare *abed.*

And in both caſes, the difference between the hexagon and the circle is leſs than the difference between the circle and the ſquare.

Therefore the polygon, whether inſcribed or circumſcribed, differs leſs from the circle, as the number of its ſides is increaſed.

And when the number of ſides in both is very great, the perimeters of the polygons will nearly coincide with the circumference of the circle; for then the difference of the polygonal perimeters becomes ſo very ſmall, that they may be eſteemed as equal.

And yet ſo long as there is any difference between theſe polygons, though ever ſo ſmall, the circle is greater than the inſcribed, and leſs than the circumſcribed polygons: Therefore half their ſums may be taken for the circumference of the circle, when the number of thoſe ſides is very great.

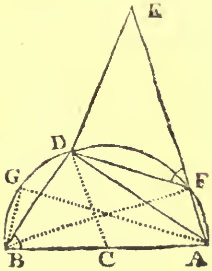
Hence, the circumferences of circles are in proportion to one another, as the radii of thoſe circles, or as their diameters.

For the perimeters of the inſcribed and circumſcribing polygons are to one another, as the radii of the circles. (186)

And theſe perimeters and circumferences continually approach to equality.

189. THEO-

## 189. THEOREM XXXVI.



*In a circle* AFB, *if lines,* BA, DA, FA, *be drawn from the extremities of two equal arcs,* BD, DF, *to meet in that point* A *of the circumference determined by one of them,* BA, *paſſing through the center; then ſhall the middle line* AD, *be a mean proportional between the ſum* AB + AF *of the extreme lines, and the radius* BC *of that circle.*

DEM. On D, with the diſtance DA, cut AF produced in E.
Then drawing DE, DF, DB, the triangles ADB, EDF, are congruous. (102)
For $\angle EFD = (\angle FDA + \angle FAD\ (96) =)\ \angle DBA$ (128). Becauſe the arc $DFA = DF + FA$.
And $\angle E = \angle FAD$ (104) $= \angle DAB$, by conſtruction; and $DE = DA$.
Therefore $EF = AB$; and $AE = AB + AF$.
Draw CD; then the triangles ACD, ADE, are ſimilar.
For they are Iſoſceles and equiangular.
Therefore $AC : AD :: AD : (AE =)\ AB + AF$. (167)

190. Hence, whence the radius of the circle is expreſſed by 1, and one of the extreme lines, or chords, paſſes through the center; then if the number 2 be added to the other extreme chord, the ſquare root of that ſum will be equal to the length of the mean chord.
For ſince $AC : AD :: AD : AB + AF$ (189.) Th. $\overline{AD}^2 = AC \times \overline{AB \times AF}$. (162)
Now if $AC = 1$, then $AB = 2$; And $\overline{AD}^2 = 2 + AF$; becauſe multiplying by 1 is uſeleſs here. Therefore $AD = \sqrt{2 + AF}$.

As the arcs BD and DFA make a ſemicircle, they are called the ſupplements of one another: Therefore if the arc BD is any part (as, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, &c.) of the ſemicircumference; then is the line DA called the ſupplemental chord of that part.

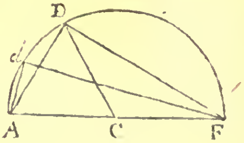
191. REMARK. In the poſthumous works of the Marquis *de le Hoſpital*, (page 319, Engliſh edition) this principle is applied to the doctrine of angular ſections; that is, to the dividing of a given arc into any propoſed number of equal parts: Or the finding of the chord of any propoſed arc.

For if BF was any aſſumed arc, the chord of which had a known ratio to the given radius BC; then as BFA is a right angled triangle (130), the ſide $AF = \sqrt{\overline{AB}^2 - \overline{BF}^2}$ (113) will alſo be known. And by this Theorem the mean chord AD will be known; and alſo DB $(= \sqrt{\overline{AB}^2 - \overline{AD}^2})$ the chord of half the arc BF will alſo be known.

And by biſecting the arc DB in G, and drawing AG, GB, the mean chord AG is known (189); and GB $(= \sqrt{\overline{AB}^2 - \overline{AG}^2})$ is alſo given.

And in this manner, by a continual biſection, the chord of a very ſmall arc may be obtained; the practice of which is facilitated by article (190) deduced from page 330 of the ſaid work.

192. EXAMPLE. *Required the chord of the* $\frac{1}{3072}$ *part of the circumference of a circle, the radius of which is* 1. *Or, required the ſide of a regular polygon of* 3072 *ſides, inſcribed in a circle, the diameter of which is* 2.



Let ADF be a ſemicircle, the diameter AF$=$2, and center C. Take the arc AD$=\frac{1}{3}$ of the ſemicircumference, or equal to 60 degrees; and draw DC, DA, DF.

Let *d* repreſent the point where the arc is biſected; *d*F the ſupplemental chord to that biſection: and let the marks $d$, $d''$, $d'''$, $d^{\text{iv}}$, &c. expreſs the biſected points agreeable to the number of biſections.

193. Now ſince $\angle$ACD$=60°$.
Therefore $\angle \text{CAD} + \angle \text{ADC} = (180° - 60° =) 120°$. (98)

But $\angle$ CAD$=\angle$ADC (104); then $\angle$ CAD$= (\frac{120}{2} =) 60°$.
Therefore DA$=$(DC$=$AC$=$) 1.

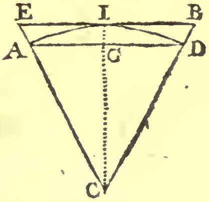
And as the triangle ADF is right angled at D. (130)
Then $\text{DF} = (\sqrt{\overline{\text{AF}}^2 - \overline{\text{AD}}^2}\ (113) = \sqrt{4 - 1} =) \sqrt{3} = 1{,}73205080756888773$
Therefore the ſupplemental chord of the arc AD, or of

| | | | |
|---|---|---|---|
| $\frac{1}{3}$ of the ſemicircumference is | FD | $=\sqrt{3}$ | $=1{,}73205080756888773$ |
| $\frac{1}{6}$ of the ſame (190) is | F$d'$ | $=\sqrt{2+\text{FD}}$ | $=1{,}93185165257813 66$ |
| $\frac{1}{12}$ | F$d''$ | $=\sqrt{2+\text{F}d'}$ | $=1{,}98288972274762 08$ |
| $\frac{1}{24}$ | F$d'''$ | $=\sqrt{2+\text{F}d''}$ | $=1{,}99571784647720 70$ |
| $\frac{1}{48}$ | F$d^{\text{iv}}$ | $=\sqrt{2+\text{F}d'''}$ | $=1{,}99892917495273 13$ |
| $\frac{1}{96}$ | F$d^{\text{v}}$ | $=\sqrt{2+\text{F}d^{\text{iv}}}$ | $=1{,}99973227581912 36$ |
| $\frac{1}{192}$ | F$d^{\text{vi}}$ | $=\sqrt{2+\text{F}d^{\text{v}}}$ | $=1{,}99993306783480 22$ |
| $\frac{1}{384}$ | F$d^{\text{vii}}$ | $=\sqrt{2+\text{F}d^{\text{vi}}}$ | $=1{,}99998326688870 13$ |
| $\frac{1}{768}$ | F$d^{\text{viii}}$ | $=\sqrt{2+\text{F}d^{\text{vii}}}$ | $=1{,}99999581671780 04$ |
| $\frac{1}{1536}$ | F$d^{\text{ix}}$ | $=\sqrt{2+\text{F}d^{\text{viii}}}$ | $=1{,}99999895417917 67$ |

Now F$d^{\text{ix}}$ the ſupplemental chord of $\frac{1}{1535}$ being known, the chord A$d^{\text{ix}}$ of $\frac{1}{1536}$, part of the ſemicircumference, or of $\frac{1}{3072}$, part of the whole circumference, is alſo known.

That is $\text{A}d^{\text{ix}} = (\sqrt{\overline{\text{AF}}^2 - \overline{\text{F}d^{\text{ix}}}^2} =)\ \sqrt{4 - \overline{2 + \text{F}d^{\text{viii}}}} = 0{,}00204530736067 64$

194. Conſequently, the ſide of a regular polygon of 3072 ſides, inſcribed in a circle whoſe diameter is 2, is 0,0020453073606764



195. *The ſide of a ſimilar polygon circumſcribing the ſame circle, the center of which is* C, *may be thus found.*

Let BE be the ſide of the circumſcribed polygon; and draw BC, EC, cutting the circle in D and A.
Draw DA, and it will be the ſide of the inſcribed polygon; and is parallel to BE. (184)

Draw CI biſecting the angle BCE, and it will biſect BE and DA at right angles (103). And $DG = (\frac{1}{2}DA =)\ \frac{1}{2}Ad^{ix}$.
Then $CG = \sqrt{\overline{CD}^2 - \overline{DG}^2} = \sqrt{1 - \frac{1}{2}\overline{Ad^{ix}}^2}$.
But $\frac{1}{2}Ad^{ix} = 0,0010226536803 38$. And its ſquare is 0,00000104582055
Which ſubtracted from 1 leaves 0,99999895417945
Whoſe ſquare root, or CG, is equal to 0,99999947708959
Now the triangles CBI, CDG, are ſimilar.
Then CG : CI : : 2DG : 2BI. (167, 151)

$$\text{Therefore } (2BI =)\ BE = \left(\frac{2DG \times CI}{CG}(164) =\right) \frac{DA}{CG};\ \text{For } IC = 1.$$

$$\text{Or } BE = \frac{0,0020453073606764}{0,9999994770895883} = 0,0020453084301895$$

which is the ſide of a regular polygon of 3072 ſides, circumſcribing a circle the diameter of which is 2.

196. Scholium. The ſide of a regular polygon of 3072 ſides, inſcribed in a circle, the diameter of which is 2, is 0,0020453073606764. (194). Which multiplied by 3072, will give the perimeter of that polygon, which is 6,2831842119979622.
The ſide of a ſimilar polygon, circumſcribing the ſame circle is 0,0020453084301895. (195)
Which multiplied by 3072, will give for the perimeter of that polygon 6,2831874973420925.
The ſum of theſe perimeters is 12,5663717093400547.
The half ſum is 6,28318585, &c.
Which is very nearly equal to the circumference of a circle, the diameter of which is 2 (187), the difference between it
and the inſcribed polygon being only 0,00000164, &c.
and the circumſcribed polygon being only 0,00000164, &c.

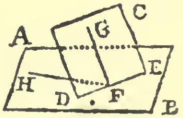
197. Now the circumferences of circles being in the ſame proportion, as their diameters. (188)
Therefore the diameter of a circle being 1,
The circumference will be 3,141592, &c. which agrees with the circumference as found by other methods.

## SECTION V.

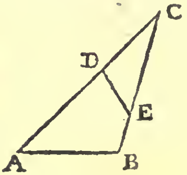
## *Of Planes and Solids.*

### DEFINITIONS and PRINCIPLES.

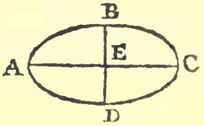
198. A line is ſaid to be in a plane, when it paſſes through two or more points in that plane; and the common ſection of two planes is a line which is in both of them.



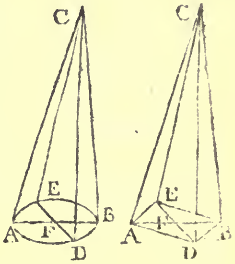
199. The inclination of two meeting planes AB, CD, is meaſured by an acute angle GFH, made by two right lines FG, FH, one in each plane, and both drawn perpendicular to the common ſection DE, of theſe planes from F, ſome point in it.



200. A right line DE interſecting two ſides AC, BC, of a triangle ABC, ſo as to make angles CDE, CED, within the figure, equal to the angles CBA, CAB, at the baſe AB, but with contrary ſides of the triangle, is ſaid to be in a ſubcontrary poſition to the baſe.



201. If a circle in an oblique poſition be viewed, it will appear of an oval form, as ABCD; that is, it will ſeem to be longer one way, as AC, than another, as DB; nevertheleſs the radii EA, EB, are to be eſteemed as equal. And the ſame muſt be underſtood in viewing any regular figure, when placed obliquely to the eye.



202. If a line be fixed to any point C above the plane of a circle ADBE, and this line while ſtretched be moved round the circle, ſo as always to touch it; then a ſolid which would fill the ſpace paſſed over by the line, between the circle and the point C, is called a CONE.

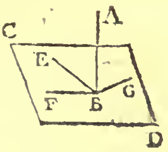
203. If the figure ADBE had been a polygon, and the ſtretched line had moved along its ſides, the figure which would then have been deſcribed, is called a PYRAMID.

So that Cones and Pyramids are ſolids which regularly taper from a circle, or polygon, to a point.

The circle or polygon is called the *Baſe*; and the point C the *Vertex*.

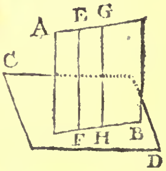
When the vertex is perpendicularly over the middle or center of the baſe, then the ſolid is called a RIGHT CONE, or a RIGHT PYRAMID; otherwiſe an OBLIQUE CONE, or OBLIQUE PYRAMID.

204. If a Cone or Pyramid be cut by a plane paſſing through the vertex C, and center of the baſe F, the ſection ABC, or EDC, is a triangle.



205. A right line AB, is perpendicular to a plane CD, when it makes right angles ABE, ABF, ABG, with all the right lines BE, BF, BG, drawn in that plane to touch the ſaid right line AB.

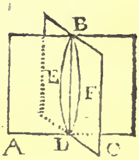
206. So that from the ſame point B, in a plane, only one perpendicular can be drawn to that plane on the ſame ſide.



207. A plane AB, is perpendicular to a plane CD, when the right lines EF, GH, drawn in one plane AB, at right angles to FB, the common ſection of the two planes, are alſo at right angles to the other plane CD.

208. So that a line EF, perpendicular to a plane CD, is in another plane AB, and at right angles to FB, the common ſection of the two planes.

## 209. THEOREM XXXVII.



*If two planes* AB, CD, *cut each other, their common ſection* BD, *will be a right line.*

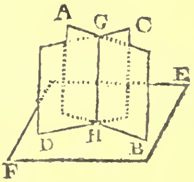
DEM. For if it be not, draw a right line DEB in the plane AB, from the point D to the point B; alſo draw a right line DFB in the plane BC.

Then two right lines DEB, DFB, have the ſame terms, and include a ſpace or figure, which is abſurd. (7)

Therefore DEB and DFB are not right lines: Neither can any other lines drawn from D to B, beſides BD, be right lines.

Conſequently the line DB, the common ſection of the planes, is a right line.

## 210. THEOREM XXXVIII.



*If two planes* AB, CD, *which are both perpendicular to a third plane* EF, *cut one another; their interſection* HG *is at right angles to that third plane* EF.

DEM. For the common ſection of AB and CD is a right line GH. (209) Alſo HB, HD, are the common ſections of AB, CD, with the plane EF.

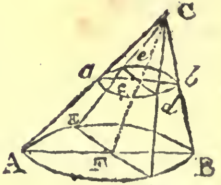
Now from the point H, a line HG drawn perpendicular to the plane EF, muſt be at right angles to HB, HD. (205)

But HG muſt be in both planes AB, CD. (208)

Therefore it muſt be in the common ſection of thoſe planes.

Conſequently the ſection HG of the planes AB, CD, is at right angles to the plane EF.

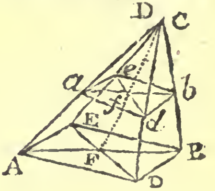
## 211. THEOREM XXXIX.



*The ſections, aebd, of a Cone or Pyramid* CAEBD, *which are parallel to the baſe* AEBD, *are ſimilar to that baſe.*

DEM. For let AFBC, DFEC, be ſections through the vertex C, and center F of the baſe.

Then theſe ſections will cut one another in the right line FC (209), and the tranſverſe ſection *abde*, in the right lines *ab*, and *ed*, interſecting in *f*.



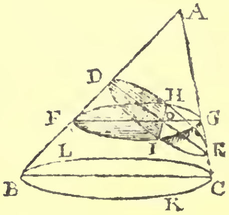
Then are the following ſets of triangles ſimilar; namely, AFC, *afc*; BFC, *bfc*; DFC, *dfc*; EFC, *efc*.

$$\text{Wherefore FC} : fc \left.\begin{array}{l} :: \text{FA} : fa \\ :: \text{FB} : fb \\ :: \text{FD} : fd \\ :: \text{FE} : fe \end{array}\right\}$$

And the like in any other ſections through C and F. (165)

Now in the Cone, $\text{FA}=\text{FB}=\text{FD}=\text{FE}$; therefore $fa=fb=fd=fe$. (152)
So that all the right lines drawn from *f* to the circumference of the figure *adbe* are equal to one another.
Conſequently the figure *adbc* is a circle. (9)
And in the Pyramid, FC : *fc* :: DC : *dc* :: DB : *db* :: DA : *da*.
FC : *fc* :: EC : *ec* :: EA : *ea* :: EB : *eb*.
Therefore in each pair of correſponding triangles in the baſe and tranſverſe ſection, the ſides are reſpectively proportional.
Conſequently, as the baſe and tranſverſe ſection are compoſed of like ſets of ſimilar triangles; therefore they are alſo ſimilar. (183)

## 212. THEOREM XL.



*If a Cone* ABLCK, *the baſe of which is a circle* CBLCK, *be cut by a plane in a ſubcontrary poſition to the baſe, the ſection* DIEH *will be a circle.*

DEM. Through the vertex A, and center of the baſe, let the triangular ſection ABC be taken, ſo as to be at right angles to the planes of the baſe BKCL, of the ſubcontrary ſection DIEH, and of the ſection FIGH, taken parallel to the baſe, and cutting the ſubcontrary ſection in the line IOH.
Therefore IOH is perpendicular to DE and FG (210) cutting one another in O.
Now the ſection FIGH is a circle (211). Therefore $\text{FO}\times\text{OG}=\text{OI}^2$. (171)
Again the triangles GOE, FOD, are ſimilar.
For $\angle\text{GEO}=\angle\text{DFO}=\angle\text{ABC}$ by conſtr. And $\angle\text{GOE}=\angle\text{DOF}$. (93)
Therefore EO : OG :: FO : DO (167.) And $\text{EO}\times\text{DO}=\text{FO}\times\text{OG}$ (162) $=\text{OI}^2$.
So that OI is a mean proportional, either between FO and OG, or DO and EO. But as the ſame would happen wherever FG cuts DE; therefore all the lines OI, both in the ſections FIGH and DIEH, are lines in a circle.
Conſequently the ſection DIEH is a circle.

213. If the ſection cut both ſides of the cone not in a ſubcontrary poſition to BC, the diameter of the baſe, then the ſection (ſuppoſe it ſtill) DIEH, is called an ELLIPTIC SECTION, which though not a circle, will be a bounded curve, longer one way than the other; and, like a circle, return into itſelf.

The curve DIEH is called an Ellipſis.

214. The line DE, the TRANSVERSE DIAMETER or AXIS.

The line OH or OI, is called an ORDINATE.

215. The ordinate through the middle of DE, is called the CONJUGATE AXIS.

The interſection of the Tranſverſe and Conjugate Axes, is called the CENTER of the Ellipſis.

216. If a circular arc be deſcribed, with a radius equal to half the Tranſverſe Axis, from one end of the Conjugate Axis, its interſections with the Tranſverſe Axis, are called FOCI, one on each ſide of the center of the Ellipſis.

217. Every right line paſſing through the center of the Ellipſis, and terminated at each end by the curve, is called a DIAMETER.

218. The radius that would deſcribe a circular arc of the ſame curvature with the ellipſis at any point of it, is called the RADIUS OF CURVATURE.

A TANGENT to any point in the Ellipſis, is a right line perpendicular to the radius of curvature at that point.

219. Two Diameters being ſo drawn, that one is parallel to a tangent, and the other paſſes through the point of contact; thoſe two Diameters are ſaid to be CONJUGATE DIAMETERS; and have certain relations to their Ordinates, Tangents, Radii of Curvature, and other lines belonging to the Ellipſis.

220. A third proportional to any two Conjugate Diameters, is called the PARAMETER.

## SECTION VI.

## *Of the Spiral.*

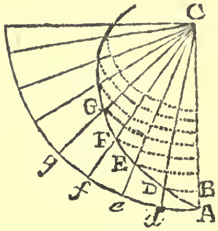
221. *Suppoſe the radius of a circle to revolve with an uniform motion round its center, and while it is ſo revolving, let a point move along the radius; then will the ſucceſſive places of that point be in a curve, which is called a ſpiral.*

This will be readily conceived by imagining a fly to move along the ſpoke of a wheel, while the wheel is turning round.

If while the radius revolves once, the point has moved the length of the radius; then the ſpiral will have revolved but once round the center, or pole; conſequently the motion in the circumference is to the motion in the radius, as the circumference is to the radius: And if the wheel revolves twice, thrice, or in any proportion to the motion in the radius; then the ſpiral will make ſo many turns, or parts of a turn, round the center.

222. *Now ſuppoſe, while the radius revolves equably, a point from the circumference moves towards the center, with a motion decreaſing in a geometric progreſſion; then will a ſpiral be generated, which is called a proportional ſpiral.*

Let the radius CA be divided in any continued decreaſing geometric progreſſion (90), as of 10 to 8; then the ſeries of terms will be 10; 8; 6,4; 5,12; 4,096; 3,2768; 2,62144, &c. Alſo let the circumference be divided into any number of equal parts, in the points *d*, *e*, *f*, *g*, &c. Then if the ſeveral diviſions of the radius CA be ſucceſſively transferred from the center C, cutting the other radii in the points D, E, F, G, &c. and a curved line be evenly drawn through thoſe points, it will be a ſpiral of the kind propoſed.



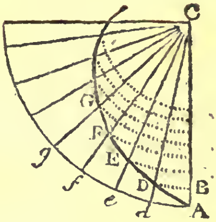
223. From the nature of a decreaſing geometric progreſſion, it is eaſy to conceive that the radius CA may be continually divided; and although each ſucceſſive diviſion becomes ſhorter than the next preceding one, yet if ever ſo great a number of diviſions, or terms, be taken, there will ſtill remain a finite magnitude.

224. Hence it follows, that this ſpiral winds continually round the center, and does not fall into it till after an infinite number of revolutions.

Alſo, that the number of revolutions decreaſe, as the number of the equal parts, into which the circumference is divided, increaſes.

## 225. THEOREM XLI.

*Any proportional ſpiral cuts the intercepted radii at equal angles.*



Dem. If the diviſions A*d*, *de*, *ef*, *fg*, &c. of the circumference were very ſmall, then would the ſeveral radii be ſo cloſe to one another, that the intercepted parts AD, DE, EF, FG, &c. of the ſpiral, might be taken as right lines.

And the triangles CAD, CDE, CEF, &c. would be ſimilar, having equal angles at the point C, and the ſides about thoſe angles proportional. (168) Therefore the angles at A, D, E, F, &c. being equal, the ſpiral muſt neceſſarily cut the radii at equal angles.

## 226. THEOREM XLII.

*If the radii of any proportional ſpiral be taken as numbers, then will the correſponding arcs of the circle, reckoned from their commencement, be as the logarithms of thoſe numbers.*

Dem. As the lines CA, CD, CE, CF, CG, &c. are a ſeries of terms in geometric progreſſion; and the arcs A*d*, A*e*, A*f*, A*g*, &c. are a ſeries of terms in arithmetic progreſſion; therefore theſe arcs may ſerve (I. 66) as the indices to the geometric terms, and be thus placed;

*Radii of the ſpiral* CA, CD, CE, CF, CG, &c. Geometric terms.
*Correſponding arcs* 0, A*d*, A*e*, A*f*, A*g*, &c. Arithm. terms, or indices.

In this diſpoſition, the firſt term CA is not diſtant from itſelf, therefore its index is repreſented by 0.

Then if the diſtance of the ſecond term CD from the firſt term CA be expreſſed by the arc A*d*; the diſtance of the third term CE, from CA, will be expreſſed by the arc A*e*; and ſo of the reſt.

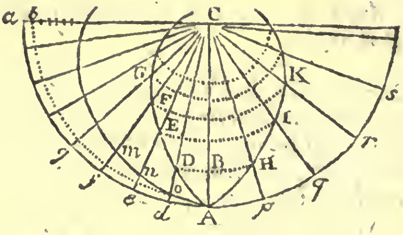
Conſequently, if the terms in the geometric ſeries be repreſented by numbers, taken as parts of the radius, then the numbers of the ſame kind, expreſſing the meaſures of the arcs, or indices, will be as the logarithms of the geometric terms. (I. 73)

227. Corol. If the difference between CA and CB was indefinitely ſmall, or CA and CB were nearly in a ratio of equality; then might the number of proportional lines into which CA could be divided, be ſo many, that any propoſed number might be found among the terms of this ſeries; and if the number of parts in the circumference was increaſed in like manner, then would every term of the proportional diviſion of the radius CA have its correſponding index among the equal diviſions of the circumference; and conſequently would exhibit the logarithms of all numbers.

228.

## THEOREM XLIII.

*There may be almoſt an infinite variety of proportional ſpirals, and as many different kinds of logarithms.*



DEM. For with the ſame equal diviſions A*d*, *de*, *ef*, &c. of the circumference, every variation in the ratio of CA to CB, as of C*a* to C*b*, will produce a different ſpiral, A*onm*.

And with the ſame diviſions of the radius CA, and different ſets of equal parts, A*d*, *de*, *ef*, &c. and A*p*, *pq*, *qr*, &c. of the circumference, may be formed different ſpirals ADEF, AHIK.

Alſo, varying at the ſame time both the diviſions of the radius and circumference, different ſpirals will be produced.

But the variations in theſe three caſes may be almoſt infinite: Therefore the number of ſuch ſpirals are almoſt infinite.

Now it is evident, that there is a peculiar relation between the rays of any ſpiral, and the correſponding arcs of the circle; that is, between the terms of a geometric progreſſion, and its indices: Therefore there may be as many kinds of logarithms, as there are proportional ſpirals.

229.

## THEOREM XLIV.

*That proportional ſpiral which interſects equidiſtant rays at an angle of* 45 *degrees, produces logarithms that are of* Nepier's *kind.*

DEM. Suppoſe AB, the difference between CA and CB, the firſt and ſecond terms of the geometric progreſſion, to be indefinitely ſmall, and take A*p*, the logarithm of CB, equal to AB; then may the figure ABH*p* be taken as a ſquare, whoſe diagonal AH would be part of the ſpiral AHIK, and the angle BAH would be half a right one, or 45 degrees.

Therefore that ſpiral which cuts its rays CA, CH, &c. at angles of 45 degrees, has a kind of logarithms belonging to it, ſo related to their correſponding numbers, that the ſmalleſt variation between the firſt and ſecond numbers is equal to the logarithm of the ſecond number.

But of this kind were the firſt logarithms made by Lord *Nepier*.

Therefore the logarithms to the ſpiral which cuts its equidiſtant rays at an angle of 45 degrees, are of the *Nepierian* kind.

## END OF BOOK II.

THE

# THE ELEMENTS OF NAVIGATION.

## BOOK III.

## OF PLANE TRIGONOMETRY.

### SECTION I.

### *Definitions* and *Principles.*

1. PLANE TRIGONOMETRY is an art which ſhews how to find the meaſures of the ſides and angles of plane Triangles, ſome of them being already known.

It will be proper for the learner, before he reads the following Articles, to turn to the definitions relative to a circle and angle, contained in the Articles 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 36, of Book II.

2. A Triangle conſiſts of ſix parts; namely, three ſides and three angles.

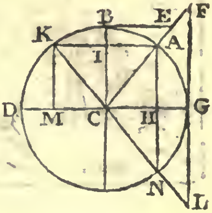
The ſides of plane triangles are denoted, or eſtimated by meaſures of length; ſuch as Feet, Yards, Fathoms, Furlongs, Miles, Leagues, *&c.*

The angles of triangles are eſtimated by circular meaſures, that is, by arcs containing Degrees, Minutes, Seconds, *&c.* (II. 15); and for convenience theſe circular meaſures are repreſented by right lines, called right ſines, tangents, ſecants, and verſed ſines.

3. The Right Sine of an arc, is a right line drawn from one end of the arc perpendicular to a radius drawn to the other end: Or it is half the chord of the double of that arc.

Thus AH is the right ſine of the arc AG; and alſo of the arc DBA.

4. The Tangent of an arc, is a right line touching one end of the arc, and continued till it meets a right line drawn from the center through the other end of that arc.
Thus GF is the tangent of the arc GA.



5. The Secant of an arc, is a right line drawn through the center and one end of the arc, and produced till it meets the tangent drawn from the other end.
Thus CF is the ſecant of the arc AG.

6. The Versed Sine of an arc, is that part of the radius intercepted between the arc and its right ſine.
Thus HG is the verſed ſine of the arc AG.

7. The Complement of an arc, is what that arc wants of 90 degrees.

Thus if the arc GB = 90°. Then AB is called the complement of AG; and AG is the complement of AB.

8. The Supplement of an arc, is what that arc wants of 180 degrees.
Thus the arc ABD is the ſupplement of AG; and AG of ABD.

9. The Co-sine of an arc, is the right ſine of the complement of that arc.

The Co-tangent of an arc, is the tangent of that arc's complement.

The Co-secant of an arc, is the ſecant of its complement.

The Co-versed Sine of an arc, is the verſed ſine of its complement.

Thus AI, BE, CE, BI, being reſpectively the ſine, tangent, ſecant, and verſed ſine of the arc AB, which is the complement of AG; therefore AI is called the co-ſine, BE the co-tangent, CE the co-ſecant, BI the co-verſed ſine, of the arc AG.

The right lines, called ſines, tangents, ſecants, and verſed ſines, are uſed as well for the meaſures of angles, as for the arcs which meaſure theſe angles: And it is as common to ſay the ſine, tangent, *&c.* of an angle, as the ſine, tangent, *&c.* of an arc.

10. The greateſt right ſine, is the ſine of 90°; and the ſines to arcs leſs than 90°, ſerve equally for arcs as much greater than 90°.

Thus the ſines of 80° and 100°; of 60° and 120°; of 40° and 140°, &c. are reſpectively equal.

11. The ſame tangent and ſecant will ſerve to arcs equally diſtant from 90 degrees; that is, to any arc and its ſupplement.

Thus if the arc BAG=90°, and BK=BA; then the arcs GN, GA, DK, are equal; and the arcs GAK and GN, or DK, are ſupplements to one another: Then the ſine KM, the tangent GL, the ſecant CL, of the arc GBK, are reſpectively equal to the ſine AH, the tangent GF, the ſecant CF of the arc GA.

12. When an arc is greater than 90°, the ſine, tangent, ſecant, of the ſupplement is to be uſed.

13. The chord of an arc is equal to twice the co-ſine of half the ſupplemental arc.

Thus AN, the chord of the arc AGN,=2CI, the co-ſine of the arc AB, and AB is half of the arc ABK, the ſupplement of AGN.

14. The verſed ſine and co-ſine together, HG+CH of any arc AG, is equal to the radius; CH being equal to AI.

15. The ſines, tangents, ſecants, or verſed ſines of ſimilar arcs in different circles, are in the ſame proportion to one another, as the radii of thoſe circles. (II. 185)

16. The angles of two triangles may be reſpectively equal, although their ſides may be unequal.

Therefore in a triangle among the things given, in order to find the reſt, one of them muſt be a ſide.

In Trigonometry, the three things given in a triangle muſt be either,
1ſt. Two ſides and an angle oppoſite to one of them.
2d. Two angles and a ſide oppoſite to one of them.
3d. Two ſides and the included angle.
4th. The three ſides.

In either caſe, the other three things may be found by the help of a few Theorems, and a *Triangular Canon*, which is a table where is orderly inſerted every degree and minute in a quadrant or arc of 90 degrees; and againſt them, the meaſures of the lengths of their correſponding ſines, tangents, and ſecants, eſtimated in parts of the radius, which is uſually ſuppoſed to be divided into a number of equal parts, as 10, 100, 1000, 10000, 100000, &c.

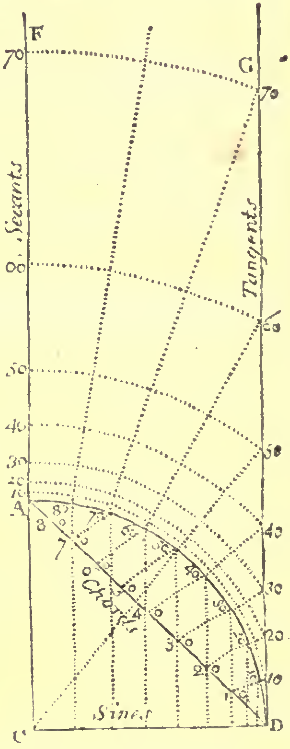
SECTION

## SECTION II.

## *Of the Triangular Canon.*

### PROPOSITION I.

17. *To find the lengths of the Chords, Sines, Tangents, and Secants to arcs of a circle of a given radius.*



Construction. Through each end of the given radius CD, and at right angles to it (II. 60) draw the lines CF, DG: On C, with the radius CD, deſcribe the quadrantal arc DA, and draw the chord DA.

18. For the Chords. Triſect the arc AD (II. 61.), and (by trials) triſect each part; then the arc AD will be divided into 9 equal parts of 10 degrees each; if theſe arcs are divided each into 10 equal parts, the quadrant will be divided into 90 degrees: But, in this ſmall figure, the diviſions to every 10 degrees only are retained, as in (II. 83).

From D, as a center, with the radius to each diviſion, cut the right line DA; and it will contain the chords of the ſeveral arcs into which the quadrantal arc AD was divided.

For the diſtances from D to the ſeveral diviſions of the right line DA, are thus made reſpectively equal to the diſtances or chords of the ſeveral arcs reckoned from D.

19. For the Sines. Through each of the diviſions of the arc AD, draw right lines parallel to the radius AC; theſe parallel lines will be the right ſines of their reſpective arcs, and CD will be divided into a line of ſines, which are to be numbered from C to D, for the right ſines; and from D to C for the verſed ſines.

For the diſtance from C to the ſeveral diviſions of the right line CD, are reſpectively equal to the ſines of the ſeveral arcs beginning from A.

20. For the Tangents. A ruler on C, and the ſeveral diviſions of the arc AD, will interſect the line DG; and the diſtances from D to the ſeveral diviſions of DG, will be the lengths of the ſeveral tangents.

21. For the Secants. From the center C, with radii to the diviſions of the tangents DG, cut the line CF; and the diſtances from C to the ſeveral diviſions of CF, will be the lengths of the ſecants to the ſeveral arcs.

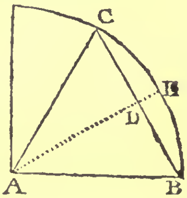
For theſe lengths are made reſpectively equal to the ſecants reckoned from C to the ſeveral diviſions of the tangent DG.

22. If the figure was ſo large, that the quadrantal arc could contain every degree and minute of the quadrant, or 5400 equal parts; then the chord, ſine, tangent, and ſecant to each of them could be drawn. Now a ſcale of equal parts being conſtructed (II. 81), 1000 of which parts are equal to the radius CD; then the lengths of the ſeveral ſines, tangents, and ſecants may be meaſured from that ſcale, and entered in a table called the triangular canon, or the table of ſines, tangents, and ſecants.

But as theſe meaſures cannot be taken with ſufficient accuracy to ſerve for the computation to which ſuch tables are applicable; therefore the ſeveral lengths have been calculated for a radius divided into a much greater number of equal parts; as is ſhewn in the following articles.

## 23. PROP. II.

*In any circle the chord of* 60 *degrees, is equal to the radius: and the ſine of* 30 *degrees is equal to half the radius.*



DEM. Let the arc CB, or ∠ CAB = 60 degrees; and draw the chord CB.
Now ſince the radii AC and AB are equal; (II. 9)
Therefore ∠C = ∠B. (II. 104)
And the ∠C + ∠B = (180° — (∠A =) 60° =) 120° (II. 96)
Therefore ∠ C, or ∠ B = (half 120°; or =) 60° = ∠A
Conſequently CB = AB = AC.
From A, draw the radius AE perpendicular to CB.
Then AE biſects the arc CB, and its chord. (II. 124)
And CD = ſine of (the arc CE = half of 60° =) 30°. (3)
Conſequently CD is equal to half the radius AB.

24. Hence, *Twice the co-ſine of* 60 *degrees is equal to the radius.* For 30° is the complement of 60°, and twice the ſine of 30° is equal to the radius.

## 25. PROP. III.

*To find the ſine of one minute of a degree.*

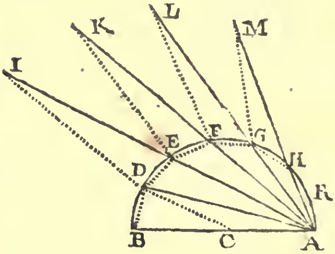
It is evident (II. 187), that the leſs the arc is, the leſs is the difference between the arc and its ſine, or half chord; ſo that a very ſmall arc, ſuch as that of one minute, may be reckoned to differ from its ſine, by ſo ſmall a quantity, that they may be eſteemed as equal; and conſequently may be expreſſed by the ſame number of ſuch equal parts of which the radius is ſuppoſed to contain 1,00000, &c. which is readily found by the following proportion.

| | |
|---|---|
| As the circumference of the circle in minutes | 21600 |
| To the circumf. in equal parts of the radius (II. 196) | 6,283185 |
| So is the arc of one minute | 1, |
| To the correſponding parts of the radius | 0,0002908882 |
| So that for the ſine of one minute, may be taken | 0,0002908882 |

26. PROP.

## 26. PROP. IV.

*In a ſeries of arcs in arithmetic progreſſion, the ſine of any one of them, taken as a mean, and the ſum of the ſines of any other two, taken as equidiſtant extremes, are ever in a conſtant ratio, of radius to twice the co-ſine of the common difference of thoſe arcs.*



DEM. For in a circumference, the center of which is C, and diameter AB, let there be taken a ſeries of arcs, ARB, ARD, ARE, ARF, ARG, ARH, &c. the common difference of which is the arc BD.

Then drawing the chords AB, AD, AE, AF, AG, AH, &c. their halves will be the ſines of half the arcs ARB, ARD, &c. (3)

Alſo half the arc BD, is the common difference of half the arcs ARB, ARD, ARE, &c. (II. 150)

And the chord AD is twice the co-ſine of half the ſupplemental arc BD. (13)

From the points, D, E, F, G, &c. with the radii DA, EA, FA, GA, &c. cut AE, AF, AG, AH, &c. produced in I, K, L, M, &c. draw ID, KE, LF, MG, &c. and BD, DE, EF, FG, GH, &c.

Then by the firſt part of the demonſtration (II. 189), the following triangles are congruous, namely,

ABD, IED; ADE, KFE; AEF, LGF; AFG, MHG, &c.

Therefore $IE = AB$; $KF = AD$; $LG = AE$; $MH = AF$, &c.

Alſo the triangles IDA, KEA, LFA, MGA, &c. being each of them iſoſceles, and their angles reſpectively equal, are ſimilar to DCA. (II. 167)

$$\begin{array}{llll} \text{Therefore } CA : AD & :: (AD : (AI =) AB + AE & :: ) \tfrac{1}{2}AD : \tfrac{1}{2}AB + \tfrac{1}{2}AE. \\ & :: (AE : (AK =) AD + AF & :: ) \tfrac{1}{2}AE : \tfrac{1}{2}AD + \tfrac{1}{2}AF. \\ & :: (AF : (AL =) AE + AG & :: ) \tfrac{1}{2}AF : \tfrac{1}{2}AE + \tfrac{1}{2}AG. \\ & \quad \&c. & & \quad \&c. \end{array}$$

The halves being in the ſame ratio as the wholes. (II. 150)

27. Conſequently, in a ſeries of arcs in arithmetic progreſſion, *viz.* $\tfrac{1}{2}$ARB, $\tfrac{1}{2}$ARD, $\tfrac{1}{2}$ARE, &c. the common difference of which is half the arc BD, it will be, (II. 164)

As (AC) radius.

To (AD) twice the co-ſine of the common difference;

So is the ſine of either arc taken as a mean,

To the ſum of the ſines of two equidiſtant extremes.

28. Hence, *The ſine of either extreme, ſubtracted from the product of the ſine of the mean by twice the co-ſine of the common difference, will give the ſine of the other extreme.* (II. 164)

29. When the common difference of three arcs is 60 degrees; then twice the co-ſine of that difference is equal to the radius. (24)

And with any ſuch three arcs, as 30, 90, 150; or 25, 85, 145; or 20, 80, 140, &c. it will be (27).

As R : (2 coſ. 60 =) R : : s, 90° : (s, 30° + s, 150° =) s, 30° + s, 30°.
: : s, 85 : (s, 25° + s, 145° =) s, 25° + s, 35°.
: : s, 80 : (s, 20° + s, 140° =) s, 20° + s, 40°.
: : s, 75 : (s, 15° + s, 135° =) s, 15° + s, 45°.
&c. &c. (10)

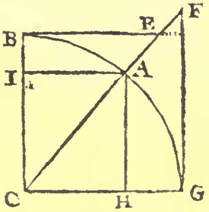
Here the firſt and ſecond terms in the proportions being equal, the third and fourth terms are alſo equal.

30. Hence, *The ſine of an arc greater than* 60 *degrees, is equal to the ſine of an arc as much leſs than* 60 *degrees, added to the ſine of its difference from* 60 *degrees.*

Therefore the ſines of arcs above 60 degrees are readily obtained from thoſe under 60 degrees.

## 31. PROP. V.

*The right ſine of an arc being known, to find its co-ſine; and from theſe to find the tangent, ſecant, verſed ſine; and alſo the co-tangent, co-ſecant, and co-verſed ſine.*



Let AG be any arc, and let AH be its ſine, AI its co-ſine; GF the tangent, BE the co-tangent; CF the ſecant, CE the co-ſecant; HG the verſed ſine, BI the co-verſed ſine.

Now if the ſine AH be given, then the co-ſine AI or CH, will be known (II. 113): For $CA^2 - AH^2 = CH^2$. Therefore *the ſquare root of the difference between the ſquares of the radius and ſine, will be the co-ſine.*

32. Then the verſed ſine HG = CG — CH; and co-verſed ſine IB = CB — CI.
And ſince the triangles CHA, CGF, CBE, are ſimilar,

33. Therefore (II. 167) CH : HA : : CG : GF, the tangent.
That is, *As the co-ſine to the ſine, ſo is radius to the tangent.*

34. And CH : CA : : CG : CF, the ſecant.
That is, *As the co-ſine to the radius, ſo is radius to the ſecant.*

35. And CI : CA : : CB : CE, the co-ſecant.
That is, *As the ſine to radius, ſo is radius to the co-ſecant.*

36. Alſo CI : IA : : CB : BE, the co-tangent.
That is, *As the ſine to the co-ſine, ſo is radius to the co-tangent.*
Or GF : CG : : CB : BE; that is, *As tangent* : *rad.* : : *rad.* : *co-tangent.*

37. Hence it is evident, that *the tangent and co-tangent of an arc of* 45° *are equal to one another, and to the radius, or ſine of* 90 *degrees.*

And as the ſquare of radius is equal to the rectangle of any tangent and its co-tangent,

Therefore tan. × cot. = *tan.* × *cot.* Therefore tan. : *tan.* : : *cot.* : cot. (II. 163)

Or the tangents of different arcs are reciprocally as their co-tangents.

38. The principles by which the lengths of the ſines, tangents, ſecants, *&c.* may be conſtructed, being delivered, the following examples are annexed to illuſtrate this doctrine.

*Required the co-ſine of one minute.*

| | |
|---|---|
| The ſine of 1 minute being | 0,0002908882 (25) |
| Its ſquare is | 0,00000008461594 |
| Which ſubtracted from the ſquare of radius | 1, |
| Leaves | 0,99999991538406 |
| Whoſe ſquare root | 0,9999999577 is the |

co-ſine of 1 minute; or the ſine of 89° 59′.

Now having the ſine and co-ſine of 1 minute, the other ſines may be found in the following manner. (28)

twice the coſ. 1 min. × ſine of 1 m. = ſum of the ſines of 0′ & 2′.
twice the coſ. 1 min. × ſine of 2 m. = ſum of the ſines of 1′ & 3′.
twice the coſ. 1 min. × ſine of 3 m. = ſum of the ſines of 2′ & 4′.
twice the coſ. 1 min. × ſine of 4 m. = ſum of the ſines of 3′ & 5′.
twice the coſ. 1 min. × ſine of 5 m. = ſum of the ſines of 4′ & 6′.

Proceeding thus in a progreſſive order from each ſine to its next, all the ſines may be found.

| Multipliers. | Products. |
|---|---|
| 1 | 1,9999999154 |
| 2 | 3,9999998308 |
| 3 | 5,9999997462 |
| 4 | 7,9999996616 |
| 5 | 9,9999995770 |
| 6 | 11,9999994924 |
| 7 | 13,9999994078 |
| 8 | 15,9999993232 |
| 9 | 17,9999992386 |

But as twice the co-ſine of 1 minute, *viz.* 1,9999999154 is concerned in each operation, therefore if a table be made of the products of this number by the nine digits, as here annexed, the computations of the ſines may be performed by addition only.

For the products by the digits in the given multiplier, being taken from the table, and written in their proper order, will prevent the trouble of multiplication.

And even this operation may be very much ſhortened, by ſetting under the right hand place (*viz.* 4.) of the double co-ſine of one minute, the unit place of the ſine uſed as a multiplier, and reverſing or placing in a contrary order, all its other figures; then the right-hand figure of each line ariſing by the multiplication, is to be ſet under one another; and in theſe lines, the firſt figure to be ſet down, is what ariſes from the figure ſtanding over the preſent multiplying one; obſerving to add what would be carried from the places omitted.

Now if the products of the figures in the multiplier, thus inverted, be taken from the above table of products, it is neceſſary to remark what number of places will ariſe from each digit uſed in the multiplier; then in the products of thoſe digits in the table, take only the like number of places, obſerving to add 1 to the right-hand place, if the next of the omitted figures exceed 5.

*Required the ſine of two minutes.*

The ſine of 1 min. placed in an inverted order under the double coſ. of 1 min. as in the margin; the right-hand figure 2 ſtands under the 9 in the 6th decimal place, therefore the firſt 6 decimal places of the product againſt 2 in the table, are to be uſed; but 1 being added, becauſe the 7th place 8, exceeds 5, makes the product 4000000: Alſo for 9 the next figure in the multiplier, ſtanding under the 5th decimal place, take 17,99999 from the table of products, and 1 being added to the 5th place, becauſe the 6th exceeds 5, make it 18,00000: In like manner the product by 8, adding 1, is 16000, &c. and the ſum of theſe products 0,0005817764 is the ſine of 2 minutes, as required.

| |
|---:|
| 1,9999999154 |
| 2888092000,0 |
| 4000000 |
| 1800000 |
| 16000 |
| 1600 |
| 160 |
| 4 |
| 0,0005817764 |

This kind of operation will be very eaſily conceived without farther illuſtration, by comparing the proceſs in this and the following operations, with what has been already ſaid.

*Required the ſines of* 3′, 4′, 5′, *and* 6 *minutes.*

| For 3 min. | For 4 min. | For 5 min. | For 6 min. |
|---:|---:|---:|---:|
| 1,9999999154 | 1,9999999154 | 1,9999999154 | 1,9999999154 |
| 4677185000,0 | 5466278000,0 | 6255361100,0 | 5044454100,0 |
| 10000000 | 16000000 | 20000000 | 20000000 |
| 1600000 | 1400000 | 2000000 | 8000000 |
| 20000 | 40000 | 1200000 | 1000000 |
| 14000 | 12000 | 60000 | 80000 |
| 1400 | 1200 | 10000 | 8000 |
| 120 | 80 | 1000 | 800 |
| 8 | 10 | 40 | 10 |
| | | 12 | |
| 0,0011635528 | 0,0017453290 | 0,0023271052 | 0,0029088810 |
| 0,0002908882 | 0,0005817764 | 0,0008726645 | 0,0011635526 |
| 0,0008726646 | 0,0011635526 | 0,0014544407 | 0,0017453284 |

In each example, the ſine of an arc which is 2 minutes leſs than that required, (28) is ſubtracted.

The ſines being made, the tangents, ſecants, &c. are to be conſtructed as before ſhewn. (33, 34)

39. There are many methods by which the triangular canon may be made; but that which is here delivered was choſen as the moſt eaſy, the beſt adapted to this work, and what would give the learner a ſufficient notion how theſe numbers are to be found: For at this time there is no occaſion to conſtruct new tables of ſines, and rarely to examine thoſe already extant; they having paſſed through the hands of a great many careful examiners, and for a long time have been received by the learned as a work ſufficiently correct.

Theſe lines were firſt introduced into mathematical computations by *Hipparchus* and *Menelaus*, whoſe methods of performance were contracted by *Ptolemy*, and afterwards perfected by *Regiomontanus*; and ſince his time *Rheticus*, *Clavius*, *Petiſcus*, and many other eminent men, have treated largely on this ſubject, and greatly exemplified the uſe of this triangular Canon, or Tables; which are now, by way of diſtinction, called Tables of *natural* ſines, tangents, &c. But the greateſt improvement ever made in this kind of mathematical learning, was by the Lord *Nepier*, Baron of *Merchiſton* in *Scotland*; who, being very fond of ſuch ſtudies, where calculations by the ſines, tangents, &c. did frequently occur, judged it would be of vaſt advantage if theſe long multiplications and diviſions could be avoided; and this he effected by his happy invention of computing by certain numbers, conſidered as the indices of others (I. 63), which he called logarithms; this was about the year 1614.

The tables now chiefly uſed in Trigonometrical computations, are the logarithms of thoſe numbers which expreſs the lengths of the ſines, tangents, &c. and therefore to diſtinguiſh them from the *natural* ones, they are called *Logarithmic* ſines, tangents, &c. (or by ſome artificial ſines, &c.) Only thoſe of the logarithmic ſines and tangents are annexed to this treatiſe, becauſe the buſineſs of Navigation may be performed by them; neither are theſe tables carried to more than five places beſide the index, that being ſufficiently exact for all nautical purpoſes: But it muſt be allowed that, for general uſe, ſuch tables are the moſt eſteemed, as conſiſt of moſt places.

40. Theſe tables are at the end of Book IX. and are ſo diſpoſed, that each opening of the book contains eight degrees; four of which are numbered at the top, and four at the bottom of the page; and thoſe at the top proceed from left to right, or forwards, from 0 degrees to 45; and thoſe at the bottom, from right to left, or backwards, from 45 to 90 degrees: To each degree there are four columns, titled ſines, co-ſines, tangents, co-tangents; and the minutes are in the marginal column of each page, ſigned with M; thoſe on the left ſide of the page belong to the degrees which are at the top, and thoſe on the right-hand ſide, to the degrees which are at the bottom of the page.

41. A ſine, tangent, co-ſine, co-tangent, to a given number of degrees, is found as follows:

For an arc leſs than 45 degrees,

Seek the degree at the top, and the minutes in the column ſigned M at the top; againſt which, in the column ſigned at the top with the propoſed name, ſtands the ſine, or tangent, &c. required.

But when the arc is greater than 45 degrees,

Seek the degrees at the bottom, the minutes in the column with M at the bottom, and the propoſed name at the bottom.

EXAMPLE I. *Required the logarithmic ſine of* 28° 37′.

Find 28 deg. at the top of the page; and in the ſide column, marked with M at the top, find 37; againſt which, in the column ſigned at the top with the word ſine, ſtands 9,68029, the log. ſine of 28° 37′, as required.

EXAMPLE II. *Required the logarithmic tangent of* 67° 45′.

Find 67 deg. at the bottom of the page; and in the ſide column, titled M at the bottom, find 45; then againſt this, in the column marked tangent at the bottom, ſtands 10,38816, which is the log. tangent required.

42. But when a logarithmic ſine or tangent is propoſed, to find the degrees and minutes belonging to it, then,

Seek in the table, among the proper columns, for the neareſt logarithm to the given one; and the correſponding degrees and minutes will be found; obſerving to reckon them from the top or bottom, according as the column is titled, where the neareſt logarithm to the given one is found.

43. It may ſometimes happen that a log. ſine or log. tang. may be wanted to degrees, minutes, and parts of minutes; which may be thus found.

Take the difference between the logs. of the degrees and minutes next leſs, and thoſe next greater than the given number.

Then for $\frac{1}{4}$, take a quarter of this difference; for $\frac{1}{3}$, take a third; for $\frac{1}{2}$, take a half; for $\frac{2}{3}$ take two thirds; for $\frac{3}{4}$, take three quarters, &c.

Add the parts taken of this difference to the right-hand figures of the log. belonging to the deg. and min. next leſs, and the ſum will be the log. to the deg. min. and parts propoſed.

EXAMPLE I. *Required the log. tang. to* 60° 56′$\frac{1}{2}$.

| | |
|---|---|
| Log. tang. 60° 57′ is | 10,25535 |
| Log. tang. 60 56 is | 10,25506 |
| The diff. is | 29 |
| Its half is | 14 |
| Add it to tang. 60° 56′ | 10,25506 |
| Gives tang. 60 56$\frac{1}{2}$ | 10,25520 |

EXAMPLE II. *Required the log. ſine to* 32° 15′$\frac{3}{4}$.

| | |
|---|---|
| Log. ſine 32° 16′ is | 9,72743 |
| Log. ſine 32 15 is | 9,72723 |
| The diff. is | 20 |
| Its three fourths is | 15 |
| Add it to ſine 32° 15′ | 9,72723 |
| Gives ſine 32 15$\frac{3}{4}$ | 9,72738 |

In moſt moſt caſes the work may be done by inſpection.

44. And if a given log. ſine or log. tangent falls between thoſe in the tables: then the degrees and minutes anſwering may be reckoned $\frac{1}{4}$, or $\frac{1}{3}$, or $\frac{1}{2}$, &c. minutes more than thoſe belonging to the neareſt leſs log. in the tables, according as its difference from the given one is $\frac{1}{4}$, or $\frac{1}{3}$, or $\frac{1}{2}$, &c. of the difference between the logarithm next greater and next leſs than the given log.

## SECTION III.

## *Of the Solution of Plain Triangles.*

### 45. PROBLEM I.

In any plain triangle, ABC, if among the things given there be a ſide and its oppoſite angle, to find the reſt.

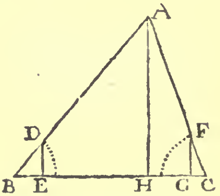
Then ſay, *As a given ſide,* (AB)
*To the ſine of its oppoſite angle* ; (C)
*So is another given ſide,* (AC)
*To the ſine of its oppoſite angle.* (B)

Therefore, to find an angle, begin with a ſide oppoſite to a known angle.

Alſo, *As the ſine of a given angle,* (B)
*To its oppoſite ſide* ; (AC)
*So is the ſine of another given angle,* (C)
*To its oppoſite ſide.* (AB)

Therefore, to find a ſide, begin with an angle oppoſite to a known ſide.

DEM. Take BD = CF = radius of the tables.
Draw DE, AH, FG, each perpendicular to BC. (II. 59)
Then DE and FG are the ſines of the angles B and C. (3)
Now the triangles BDE, BAH, are ſimilar, and ſo are the triangles CFG, CAH.
Therefore BD : DE : : BA : AH. (II. 167)
And (CF =)BD : FG : : CA : AH. (II. 167)
But (BD × AH =) DE × BA = FG × CA. (II. 162)
Therefore DE : CA : : FG : BA. Or, s, ∠B : AC : : s, ∠C : BA. (II. 163)



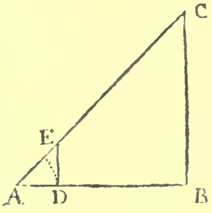
SCHOL. Or, by circumſcribing the triangle with a circle, it will readily appear, that the half of each ſide is the ſign of its oppoſite angle. And halves have the ſame proportion as the wholes.

### 46. PROBLEM II.

In a right-angled plane triangle, ABC, if the two ſides containing the right angle B are known, to find the reſt.

Then, *As one of the known ſides,* (AB)
*To the radius of the tables (or tangent of* 45°*)* ; (AD)
*So is the other known ſide,* (BC)
*To the tangent of its oppoſite angle.* (DE)

DEM. Take AD = radius of the tables.
Then DE, perpendicular to AD, is the tangent of the angle A. (4)
And the triangles ADE, ABC, are ſimilar.
Therefore AB : AD : : BC : DE. (II. 167)



47. PRO-

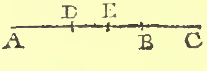
## 47. PROBLEM III.

In any two quantities, *their half difference added to their half ſum, gives the greater.*

*The half diff. ſubtracted from the half ſum, gives the leſs.*

*And if half the ſum be taken from the greater, the remainder will be the half difference of thoſe quantities.*

DEM. Let AB be the greater, and BC the leſs, of two quantities.
Take $AD = BC$; then BD is their difference.
Biſect DB in E; then $DE = EB$, is the half diff.
And $AD + DE = BC + BE$ (II. 47); therefore AE is the half ſum.
Now $AE + EB = AB$, is the greater.
And $AE - ED = (AD =) BC$, is the leſs.
Alſo $AB - AE = BE$, is the half diff.



## 48. PROBLEM IV.

In any plane triangle, ABC; if the three things known, be two ſides, AC, CB, and their contained angle C, to find the reſt.

*Find the ſum and difference of the given ſides.*

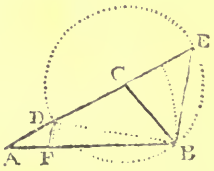
*Take half the given angle from 90 degrees, and there remains half the ſum of the unknown angles.* Then ſay,

*As the ſum of the given ſides,* $AC + CB$
*To the difference of thoſe ſides;* $AC - CB$
*So is the tangent of half the ſum of the unknown angles,* $t.\ \frac{1}{2}\overline{B+A}$
*To the tangent of half the difference of thoſe angles.* $t.\ \frac{1}{2}\overline{B-A}$

*Add the half difference of the angles to the half ſum, and it will give the greater angle* $= B$.

*Subtract the half difference of the angles from the half ſum, and it will give leſſer angle* $= A$.

DEM. On C, with the radius CB, deſcribe a circle, cutting AC, produced, in E and D; draw EB, and BD; alſo draw DF parallel to EB.
Then $AE = AC + CB$, is the ſum of the ſides.
And $AD = AC - CB$, is the difference of the ſides.
Now $\angle CDB = \angle CBD$. (II. 104)
And $(CDB + CBD =) 2CDB = \angle CBA + \angle A$. (II. 98)



Therefore $\frac{1}{2}\overline{\angle CBA + \angle A} = \angle CDB$, is half the ſum of the unknown angles.
And BE (II. 123) is the tangent of CDB, to the radius DB. (4)
Alſo $(CBA - CBD =) DBA = \frac{1}{2}\angle CBA - \frac{1}{2}\angle A$ (47)
Therefore $\frac{1}{2}\overline{\angle CBA - \angle A} = \angle DBA$, is half the difference of the unknown angles.
And DF is the tangent of DBA, to the radius DB. (4)
Now the triangles AEB, ADF, are ſimilar, DF being parallel to EB.
Therefore $AE : AD :: BE : DF$. (II. 167)
Or $AC + CB : AC - CB :: t.\ \frac{1}{2}\overline{\angle CBA + \angle A} : t.\ \frac{1}{2}\overline{\angle CBA - \angle A}$.

## 49 PROBLEM V.

In a plane triangle, ABC, if the three ſides are known, and the angles required.

*From the greateſt angle*, B, *ſuppoſe a line* BD *drawn perpendicular to its oppoſite ſide, or baſe, dividing it into two ſegments*, AD, CD, *and the given triangle into two right-angled triangles*, ADB, CDB: Then ſay,

| | |
|---|---|
| *As the baſe, or ſum of the ſegments,* | AC |
| *Is to the ſum of the other two ſides;* | AB + BC |
| *So is the difference of thoſe ſides,* | AB — BC |
| *To the difference of the ſegments of the baſe.* | AD — DC |

*Add half the difference of the ſegments to half the baſe, gives the greater ſegment* AD. (47)

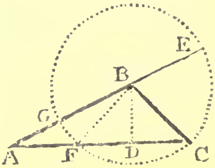
*Subtract half the difference of the ſegments from half the baſe, there remains the leſſer ſegment* DC. (47)

*Then, in each of the triangles*, ADB, CDB, *there will be known two ſides, and a right angle oppoſite to one of them; therefore the angles will be found by Problem* I. (45)

When two of the given ſides are equal; *then a line drawn from the included angle, perpendicular to the other ſide, biſects the ſide.* (II. 103)

*And the angles being found in one of the right-angled triangles, will alſo give the angles of the other.*

DEM. of the foregoing proportion.
In the triangle ABC, the line BD, perpendicular to AC, divides AC into the ſegments AD, DC.
On B with the radius BC, deſcribe a circle GCE, cutting AB, continued, in G, E; and AC in F; draw BF.

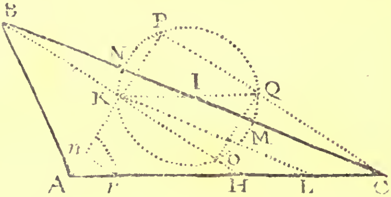


Then DC $=$ DF. (II. 103)
Now AC ($=$ AD $+$ DC) is the ſum of the ſegments.
And AF ($=$ AD — FD) is the difference of the ſegments.
Alſo AE ($=$ AB $+$ BC) is the ſum of the other ſides.
And AG ($=$ AB — BC) is the difference of thoſe ſides.
But AC $\times$ AF $=$ AE $\times$ AG. (II. 172)
Therefore AC : AE : : AG : AF. (II. 163)
Or AC : AB $+$ BC : : AB — BC : AD — DC; or $=$ AD — DF.

## 50. PROBLEM VI.

In any plane triangle, ABC, the three ſides being known, to find either of the angles.

*Put* E *and* F *for the ſides including the angle ſought.*
G *for the ſide oppoſite to that angle.*
D *for the difference between the ſides* E *and* F.
*Find half the ſum of* G *and* D.
*And half the difference of* G *and* D.
*Then write theſe four logarithms under one another*, namely,
*The Arithmetical complement of the logarithm of* E; (I. 83)
*The Arithmetical complement of the logarithm of* F;
*The logarithm of the aforeſaid half ſum of* G *and* D;
*The logarithm of the aforeſaid half difference of* G *and* D.
*Add them together, take half their ſum; which ſeek among the log. ſines. And the degrees and minutes anſwering, being doubled, will give the meaſure of the angle ſought.*



DEM. In the triangle, ABC, let A be the angle ſought.

Take AH = AB, draw BH; and through K, the middle of BH, draw AP, which biſects the angle A, and is perpendicular to BH. (II. 103)
Through K draw KL, KQ, parallel, to BC, AC; which will biſect HC, BC, in L and I; then KL = IC, KI = LC. (II. 28, 163)
And the difference between AC and AB is HC = D; then KI = $\frac{1}{2}$D.

From I, with the radius IK, deſcribe a circle cutting AP, BH, KQ, BC, in P, O, Q, M, N, and join CQ; now IQ = IK = LC = LH; therefore KQ = HC, and CQ = KH, as the triangles CQI, KHL, are congruous. (II. 99)

Therefore CQ parallel to KH (II. 28.) being produced, will meet AP at right angles (II. 53), in the point P, by the reverſe of (II. 130).

Then PQ = KO, as the triangles KQP, QOK, are congruous. (II. 95, 100)
Now BM = (BI + IM = $\frac{1}{2}$BC + $\frac{1}{2}$HC =) $\frac{1}{2}\overline{G+D}$: And BN = $\frac{1}{2}\overline{G-D}$.
Alſo BO = CP: For BK = (KH =) CQ, and KO = PQ.
Let A$r$ = radius of the tables; then $rn$ (parallel to BH) = ſine of $\frac{1}{2}\angle$A. (3)
Then the triangles A$nr$, AKB, APC, are ſimilar.
And A$r$ : $rn$ : : AB : BK; alſo A$r$ : $rn$ : : AC : (CP =) BO. (II. 167)
Therefore A$r^2$ : $rn^2$ : : AC × AB : (BK × BO =) BM × BN. (II. 161, 172)
Or (ſq. rad. = R$^2$ : ſq. ſine $\frac{1}{2}\angle$A : : AC × AB : $\frac{1}{2}\overline{G+D} \times \frac{1}{2}\overline{G-D}$.

Therefore the ſquare of the ſine $\frac{1}{2}\angle A = \frac{\frac{1}{2}\overline{G+D} \times \frac{1}{2}\overline{G-D}}{AC \times AB} \times R^2$. (II. 164)

Therefore ſine $\frac{1}{2}\angle A = \sqrt{\frac{\frac{1}{2}\overline{G+D} \times \frac{1}{2}\overline{G-D}}{E \times F}}$; (II. 113)

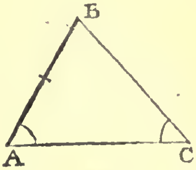
And R, the radius of the tables, being 1,

$$\left.\begin{aligned}\text{Or Log. s,}\tfrac{1}{2}\angle \text{A} &= \frac{\text{Log.}\,\tfrac{1}{2}\overline{\text{G}+\text{D}}+\text{Log.}\,\tfrac{1}{2}\overline{\text{G}-\text{D}}-\text{Log. E}-\text{Log. F}}{2}\\ \text{Or Log. s,}\tfrac{1}{2}\angle \text{A} &= \frac{\text{l. E}+\text{l. F}+\text{L.}\tfrac{1}{2}\overline{\text{G}+\text{D}}+\text{L.}\tfrac{1}{2}\overline{\text{G}-\text{D}}}{2}\end{aligned}\right\}\left.\begin{matrix}85\\86\\91\end{matrix}\right\}\text{I.}$$

Where L. ſtands for logarithm, and l. for Arith. comp. of the logarithm.

51. Every poſſible caſe in plane Trigonometry may be readily ſolved by the preceding Problems, obſerving the following Precepts.

I. Make a rough draught of the triangle, and put the letters A, B, C, at the angles.



II. Let ſuch parts of this triangle be marked, as repreſent the things which are given in the queſtion. Thus, mark a given ſide with a ſcratch acroſs it; and a given angle by a little crooked line; as in the figure; where the ſide AB, and the angles A and C, are marked as given.

III. If two angles are known, the third is always known.
For if one angle is 90 degrees, the other given angle (which (II. 97) will be acute) taken from 90 degrees, leaves the third angle.
And if both the given angles are oblique; their ſum taken from 180 degrees, gives the other angle. (II. 96)

IV. Compare the given things together, and determine to which Problem the queſtion propoſed belongs.

V. Then according as the Problem directs, perform the preparatory work; and write down, under one another, in four lines, (or more if neceſſary), the *literal ſtating*; expreſſing each angle by a letter, or by three; each line by two letters; and the ſums, or differences, of lines, by proper marks.

VI. Againſt ſuch terms as are known, write their numeral value, as given in the queſtion, or as found in the preparatory work; and againſt theſe numbers write their logarithms; thoſe for the lines being found (by I. 81) in the table of the logarithms of numbers; and thoſe for the angles, found (by 41) in the table of logarithmic ſines and tangents: Obſerving that an Arithmetical complement (ſee I. 88) is always uſed in the firſt term: And that when an angle is greater than 90 degrees, its ſupplement is uſed.

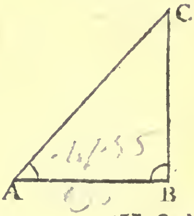
VII. Add theſe logarithms together, and ſeek the ſum (I. 81) in the log. numbers, when a line is wanted; or (42) in the log. ſines or tangents, when an angle is wanted. Then the number or degree, anſwering to that logarithm which is the neareſt to the ſaid ſum, will be the thing required.

## A SYNOPSIS.

52. *Of the Rules in Plane Trigonometry.*

| PROB. | *Given.* | *Required.* | SOLUTION. |
|---|---|---|---|
| | All the angles and one ſide. | Either of the other ſides. | Since two angles are known, the third is known.<br>And, As ſin. of ∠ opp. to ſide given, is to that opp. ſide;<br>So ſin. of another angle, to its opp. ſide. |
| I. ſee art, 45 | Two ſides and an ∠ oppoſ. to one ſide. | The angle oppoſ. to the other given ſide. | As one given ſide is to the ſine of its opp. angle;<br>So is the other given ſide, to the ſide of its opp. angle.<br>Then two angles being known, the third is known.<br>And the other ſide is found as before. |
| II. art. 46 | Two ſides and the included right ∠. | Either of the other angles. | As one of the given ſides, is to the Radius;<br>So is the other given ſide, to the tangent of its opp. ∠<br>Then two ∠s being known, the third is known.<br>The other ſide is found by opp. ſides and ∠s. |
| III. art. 48 | Two ſides and the included oblique angle. | The other angles. | Find the ſum and diff. of the given ſides.<br>Take $\frac{1}{2}$ given ∠ from 90° leaves $\frac{1}{2}$ ſum of the other ∠s.<br>Then, As ſum ſides, is to diff. of ſides;<br>So tan. $\frac{1}{2}$ ſum other ∠s, to tan. $\frac{1}{2}$ diff. thoſe ∠s.<br>The $\frac{1}{2}$ ſum ∠s $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ $\frac{1}{2}$ diff. ∠s, gives $\left\{\begin{matrix}\text{greater } \angle.\\\text{leſſer } \angle.\end{matrix}\right.$<br>Find the other ſide by opp. ſides and angles. |
| IV. art. 49 | The three ſides. | All the angles. | Draw a line perpend. to the greateſt ſide, from the opp. ∠, dividing that ſide into two parts.<br>Then, As the longeſt ſide is to ſum other two ſides;<br>So is the diff. thoſe ſides, to the diff. $p^{ts}$ of longeſt.<br>Then $\frac{1}{2}$ long. ſide $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ $\frac{1}{2}$ dif. $p^{ts}$ gives the $\left\{\begin{matrix}\text{great.}\\\text{leſſer}\end{matrix}\right\}$ part.<br>Now the ſaid perp. cuts the triangle into 2 right ∠d ones.<br>In both, are known the Hyp. a Leg. and the right ∠.<br>The angles are found by Problem I. |
| V. art. 50 | The three ſides. | Either Angle. | Having choſe which angle to find; call the ſides including that angle E and F.<br>The ſide opp. that ∠, call G.<br>Put D for the difference between E and F.<br>Find the half ſum, and half diff of G and D.<br>Then write theſe four Logs. under one another;<br>*Viz.* $\left\{\begin{matrix}\text{The Ar. Co. Log. of E, The Ar. Co. Log. of F,}\\\text{The Log. of } \frac{1}{2} \text{ ſum, And the Log. of } \frac{1}{2} \text{ difference.}\end{matrix}\right.$<br>Add the four logs. together, take half their ſum.<br>Seek it among the log ſines; and the correſponding deg. and min. doubled, is the angle ſought. |

53. EXAMPLE I. *In the plane Triangle* ABC.



*Given* AB = 195 Poles.
∠B = 90° 00′
∠A = 47 55.
*Required the other parts.*

### FOR THE LINEAR SOLUTION.

1ſt. Draw AB equal to 195 poles, taken from a ſcale of equal parts.
2d. From B, draw BC, making with AD an angle of 90°. (II. 84)
3d. From A, draw AC, making with AB an angle of 47° 55′; and meeting BC in the point C.
Then is the triangle ABC ſuch, the parts of which correſpond with the things given; and the ſides CA, CB, being applied to the ſcale that AB was taken from, their meaſures will be found, *viz.* AC = 291; and BC = 216.

### FOR THE NUMERAL SOLUTION, OR COMPUTATION.

Since two angles are known; Therefore,

From 90° 00′
Take 47 55 = ∠A
Remains 42 05 = ∠C

Now in this triangle, there are known all the angles and one ſide; therefore among the known things, there is a ſide and its oppoſite angle; which belongs to the firſt problem.
Then to find the ſide AC, begin with the angle C oppoſite AB.

| | | |
|---|---|---|
| As the ſine of ∠ C, | Or thus, As s, ∠C = 42° 05′ | 0,17379 Ar. Co. |
| To the oppoſite ſide AB; | To AB = 195 po. | 2,29003 |
| So the ſine of the ∠ B, | So s, ∠B = 90° 00′ | 10,00000 |
| To the oppoſite ſide AC. | | |
| | To AC = 291 po. | 2,46382 |

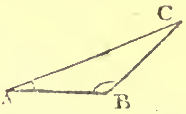
And to find the ſide BC, begin with the angle C oppoſite AB.

| | | |
|---|---|---|
| As the ſine of the ∠ C, | Or thus, As s, ∠C = 42° 05′ | 0,17379 Ar. Co. |
| To the oppoſite ſide AB; | To AB = 195 po. | 2,29003 |
| So the ſine of the ∠ A, | So s, ∠A = 47° 55′ | 9,87050 |
| To the oppoſite ſide BC. | | |
| | To BC = 216 po. | 2,33432 |

So that AC is 291 poles, and BC is 216 poles.
The letters Ar. Co. ſtanding on the right of the firſt line, ſignify the arithmetical complement of the log. ſine of 42° 05′. (I. 88)

### 54. EXAMPLE II. *In the plane Triangle* ABC.

*Given* AB = 117 miles.
∠B = 134° 46′
∠A = 22 37.
*Required the other parts.*



### FOR THE LINEAR SOLUTION, OR CONSTRUCTION.

Make AB = 117 equal parts; at A make an angle = 22° 37′ (II. 84); and at B make an angle of 134° 46′; then the lines which make with AB thoſe angles, will meet in C, and form the triangle propoſed. And the meaſure of BC will be 117, and of AC 216.

### BY COMPUTATION. See art. 45.

Since two angles are known,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Namely, ∠B = | 134° | 46′ | Now from | 180° | 00′ | And from | 180° | 00′ |
| ∠A = | 22 | 37 | Take | 157 | 23 | Take | 134 | 46 |
| Their ſum = | 157 | 23 | Leaves ∠C = | 22 | 37 | The ſupᵗ ∠B = | 45 | 14 |

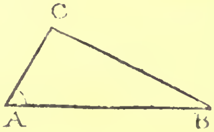
Since the angle C = ∠A
Therefore BC = AB (II. 104)

*To find the ſide* AC.

| | | | |
|---|---|---|---|
| As s, ∠C = 22° 37′ | 0,41503 | Ar. Co. |
| To AB = 117 M. | 2,06819 | |
| So s, ∠B = 134° 46′ | 9,85125 | ſup. |
| To AC = 216 M. | 2,33447 | |

### 55. EXAMPLE III. *In the plane Triangle* ABC.

*Given* AB = 408 yards.
∠B = 22° 37′
∠A = 58 07.
*Required the other parts.*



### CONSTRUCTION.

Make AB = 408 yards, or equal parts; make the angle A = 58° 07′; and the ∠B = 22° 37′; then the lines forming theſe angles will meet in C; and the meaſure of AC is 159 yards, and of BC is 351.

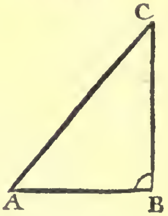
### COMPUTATION. See art. 45.

| | | | | | |
|---|---|---|---|---|---|
| Two angles being known, *viz* ∠A = | 58° | 07′ | From | 180° | 00′ |
| ∠B = | 22 | 37 | Take | 80 | 44 |
| Their ſum = | 80 | 44 | Leaves | 99 | 16 = ∠C. |

| | | | |
|---|---|---|---|
| *To find the ſide* AC. | | *To find the ſide* BC. | |
| As s, ∠C = 99° 16′ | 0,00570 Ar. Co. | As s, ∠C = 99° 16′ | 0,00570 Ar. Co. |
| To AB = 408 Y. | 2,61066 | To AB = 408 Y. | 2,61066 |
| So, s, ∠B = 22° 37′ | 9,58497 | So s, ∠A = 58° 07′ | 9,92897 |
| To AC = 159 Y. | 2,20133 | To BC = 351 Y. | 2,54533 |

In theſe operations the ſupplement of the angle C is uſed.

56. EXAMPLE IV. *In the plane Triangle* ABC.



*Given* AB=195 } Furlongs.
AC=291 }
∠B=90° 00′.
*Required the reſt.*

### CONSTRUCTION.

Make AB=195 equal parts; draw BC, making an angle at B=90° 00′. From A with 291 equal parts cut BC in C, and draw AC.

Then the ∠A meaſured on the ſcale of chords will be about 48 degrees, and ∠C about 42°: Alſo BC, on the equal parts, meaſures about 216.

### COMPUTATION.

Here being two ſides, and an angle oppoſite to one of them, the ſolution falls under problem the firſt. See art. 45.

*To find the angle* C.

| | | |
|---|---|---|
| As | AC=291 F. | 7,53611 Ar. Co. |
| To s,∠B=90° 00′ | | 10,00000 |
| So | AB=195 F. | 2,29003 |
| To s,∠C=42° 05′ | | 9.82614 |

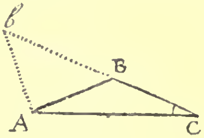
| | | |
|---|---|---|
| From | 90° 00′ | |
| Take | 42 05 | =∠C, |
| Leaves | 47 55 | =∠A. |

*To find the ſide* BC.

| | |
|---|---|
| As s,∠B=90° 00′ | 10,00000 |
| To AC=291 F. | 2,46389 |
| So s,∠A=47° 55′ | 9,87050 |
| To BC=216 F. | 2,33439 |

Here the ſine of 90° 00′ or radius being the firſt term, its Arith. Comp. being 0, is not taken.

57. EXAMPLE V. *In the plane Triangle* ABC.



*Given* AC=216 } Yards.
AB=117 }
∠C=22° 37′.
*Required the reſt.*

### CONSTRUCTION.

Make AC=216 yards; the ∠C=22° 37′; and draw CB: Then from A, with 117 yards, cut CB in *b* or in B; and either of the triangles AC*b* or ACB will anſwer the conditions propoſed: But the triangle to be uſed is generally determined by ſome circumſtances in the queſtion it belongs to. Thus if the angle oppoſite to AC is to be obtuſe, the triangle is ABC.

### COMPUTATION.

The ſolution belongs to problem the firſt. See art. 45.

*To find the angle* B

| | | |
|---|---|---|
| As | AB=117 Y. | 7,93181 |
| To s,∠C=22° 37′ | | 9,58497 |
| So | AC=216 Y. | 2,33445 |
| To s,∠B=134° 46′ | | 9,85123 |
| ∠C+∠B=157 23 | | |

| | | |
|---|---|---|
| From | 180° 00′ | |
| Take | 157 23 | =∠C+∠B, |
| Leaves | 22 37 | ∠A, |

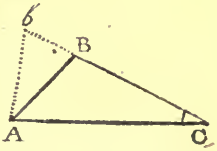
And as ∠A=∠C,
Therefore BC=AB, (II. 104)

If the angle required be obtuſe, ſubtract the deg. and min. correſponding to the fourth log. from 180; the remainder is the ∠B For the fourth log. gives the ∠*b*, which is the ſupplement to the angle B. (II. 104, 96).

58. Ex-

58. EXAMPLE VI. *In the plane Triangle* ABC.

*Given* $AC = 408$, $AB = 159$ } Fathoms.
$\angle C = 22° 37'$.
*Required the reſt.*



### CONSTRUCTION.

Make $AC = 408$ fathoms; the $\angle C = 22° 37'$; and draw $Cb$; from A, with 159 fathoms, cut $Cb$ in $b$, or in B, and draw $Ab$ or AB:
Then if the angle oppoſite to AC is to be acute, the triangle $ACb$ is that which is required; but if the angle is to be obtuſe, ACB is the triangle ſought.

### COMPUTATION. See art. 45.

Here being a ſide and its oppoſite angle known, the ſolution falls under problem the firſt; the $\angle B$ is to be obtuſe.

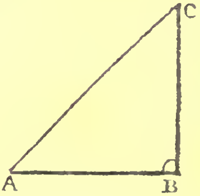
| *To find the angle* B *obtuſe.* | |
|---|---|
| As $AB = 159$ F. | 7,79860 |
| To s, $\angle C = 22° 37'$ | 9,58497 |
| So $AC = 408$ F. | 2,61066 |
| To s, $\angle B = 99° 19'$ | 9,99423 |

$\angle C + \angle B = 121\ 56$
Taken from $180\ 00$
Leaves $\angle A = 58\ 04$

| *To find* BC. | |
|---|---|
| As s, $\angle C = 22° 37'$ | 0,41503 |
| To $AB = 159$ F. | 2,20140 |
| So s, $\angle A = 58° 04'$ | 9,92874 |
| To $BC = 350,9$ F. | 2,54517 |

59. EXAMPLE VII. *In the plane Triangle* ABC.

*Given* $AB = 195$, $BC = 216$ } Furlongs.
$\angle B = 90° 00'$.
*Required the reſt.*



### CONSTRUCTION.

Make the angle $ABC = 90°$; take $BA = 195$ equal parts, and $BC = 216$; and draw AC; then ABC is the triangle propoſed; where the parts required may be meaſured by the proper ſcales.

### COMPUTATION. See art. 46.

As two ſides and the contained right angle are known, the ſolution belongs to problem the ſecond.

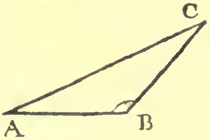
| *To find the angle* A | |
|---|---|
| As $AB = 195$ F. | 7,70997 |
| To Rad. or tang. $45° 00'$ | 10,00000 |
| So $BC = 216$ F. | 2,33445 |
| To t. $\angle A = 47° 55'$ | 10,04442 |

$90\ 00$
$\angle C = 42\ 05$

| *To find* AC. | |
|---|---|
| As s, $\angle A = 47° 55'$ | 0,12950 |
| To $BC = 216$ F. | 2,33445 |
| So s, $\angle B = 90° 00'$ | 10,00000 |
| To $AC = 291$ F. | 2,46395 |

60. EXAMPLE VIII. *In the plane Triangle* ABC.

*Given* AB=117 } Yards.
BC=117 }
∠B=134° 46′.
*Required the reſt.*



### CONSTRUCTION.

Make the ∠ABC=134° 46′; take BA and BC, each equal to 117 equal parts, from the ſame ſcale, and draw AC; then is the triangle ABC equal to that propoſed; and the parts required, meaſured on their proper ſcales, will give their values.

### COMPUTATION. See art. 48.

Now as AB and BC are equal; therefore the angles A and C are alſo equal.

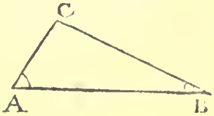
From 180° 00′
Take 134 46=∠B.
Leaves 45 14=∠A+∠C.
The half 22 37=∠A=∠C.

*To find* AC.

| | | |
|---|---|---|
| As | s,∠A=22° 37′ | 0,41503 |
| To | BC=117 Y. | 2,06818 |
| So | s,∠B=134° 46′ | 9,85125 |
| To | AC=216 Y. | 2,33446 |

61. EXAMPLE IX. *In the plane Triangle* ABC.

*Given* AB=408 } Yards.
AC=159 }
∠A=58° 07′.
*Required the reſt.*



### CONSTRUCTION.

Make an angle CAB=58° 07′; take AC=159, AB=408, from the ſame ſcale of equal parts; and draw CB; then will the triangle ACB be equal to that which was propoſed.

### COMPUTATION.

Here, there being two ſides and their contained angle known, the ſolution belongs to art. 48.

AB=408
AC=159
AB+AC=567=ſum of ſides.
AB—AC=249=diff. of ſides.

The half of 58° 07′
Is 29 03½, which
Taken from 90 00
Leaves 60 56½=½∠C+½∠B.

*To find the angles.*

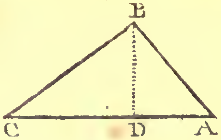
| | | |
|---|---|---|
| As | AB+AC=567 | 7,24642 |
| To | AB—AC=249 | 2,39620 |
| So | t.½∠C+∠B=60° 56½′ (See 43) | 10,25520 |
| To | t.½∠C—∠B=38 19½ | 9,89781 |
| Then (47) | 99 16=∠C. | (II. 105) |
| And | 22 37=∠B. | |

*To find* BC.

| | | |
|---|---|---|
| As | s,∠C=99° 16′ | 0,00570 |
| To | AB=408 Y. | 2,61066 |
| So | s,∠A=58° 07′ | 9,92897 |
| To | BC=351 Y. | 2,54533 |

62. EXAMPLE X. *In the plane Triangle* ABC.

*Given* AB = 195
BC = 216
AC = 291
*Required the angles.*



### CONSTRUCTION.

Make CA = 291 equal parts; from C, with 216, deſcribe an arc B; from A with 195 cut the arc B in B; draw BC, BA, and the triangle is conſtructed, then the angles may be meaſured by the help of a ſcale of chords.

### COMPUTATION.

The three ſides being given, the ſolution falls under either Problem V. or Problem VI. But that the uſe of theſe Problems may be ſufficiently illuſtrated, the ſolutions according to both of them are here annexed.

*Solution by Problem* V. (49)

From the angle B, draw BD perpendicular to CA, which will be divided into the ſegments CD, DA, the ſum of which AC is known.

Now BC = 216
And AB = 195

BC + AB = 411 = ſum of ſides.

BC — AB = 21 = diff. of the ſides.

*To find the diff. of the ſegments.*

| | | | |
|---|---|---|---|
| As AC | = 291 | — | 7,53611 |
| To BC + AB | = 411 | — | 2,61384 |
| So BC — AB | = 21 | — | 1,32222 |
| To CD — AD | = 29,66 | — | 1,47217 |

Now the half of 291 is 145,5
And the half of 29,66 is 14,83

Therefore (47) the ſum 160,33 = CD; or CD = 160,3

the difference 130,67 = AD; or AD = 130,7.

*Then in the triangle* CDB.

| | | | |
|---|---|---|---|
| As BC | = 216 | — | 7,66555 |
| To s, ∠CDB | = 90° 00′ | — | 10,00000 |
| So CD | = 160,3 | — | 2,20493 |
| To s, ∠CBD | = 47° 55′ | — | 9,87048 |

Wh. taken from 90 00

Leaves ∠C = 42 05

*And in the triangle* ADB.

| | | |
|---|---|---|
| As AB | = 195 | 7,70997 |
| To s, ∠ADB | = 90° 00′ | 10,00000 |
| So AD | = 130,7 | 2,11628 |
| To s, ∠ABD | = 42 05 | 9,82625 |

Wh. taken from 90 00

Leaves ∠A = 47 55, & ∠B = 90° 0′.

*Solution by Problem* VI. (50)

*To find the angle* C.

Put E = 291 = AC
F = 216 = BC

D = 75 = E — F
G = 195 = AB

2)270(135 = $\frac{1}{2}\overline{G+D}$

2)120(60 = $\frac{1}{2}\overline{G-D}$.

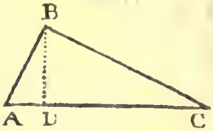
| | | | |
|---|---|---|---|
| Then To Ar. Co. log. E. | = 291 | - - - | 7,53611 |
| Add Ar. Co. log. F. | = 216 | - - - | 7,66555 |
| And the log. $\frac{1}{2}\overline{G+D}$ | = 135 | - - - | 2,13033 |
| Alſo the log. $\frac{1}{2}\overline{G-D}$ | = 60 | - - - | 1,77815 |
| The half of this ſum | | | 2)19,11014 |
| Is the log. ſine of | 21° 02′½ (43) | - - - | 9,55507 |

Which doubled, gives 42° 05′ = ∠C.

The angle C being known, the other angles may be found by Prob. I.

63. EXAMPLE XI. *In the plane Triangle* ABC.

*Given* AC=408
BC=351
AB=159
*Required the angles.*



### CONSTRUCTION.

The conſtruction and menſuration is performed as in the laſt EXAMPLE.

### COMPUTATION BY PROB. V. art. 49.

From the ∠ B draw the perpendicular BD; and find the ſegments AD, DC; which may be done without logarithms.

| | |
|---|---|
| Thus BC=351<br>AB=159 | Then (I. 46), As 408 : 510 : : 192 : 240=DC—DA.<br>For 192×510=97920; which divided by 408 gives 240. |
| BC + AB=510 | Now half of 408=204; and half of 240 is 120. |
| BC — AB=192 | Then 204+120=324=DC; and 204—120=84=DA. |

| *In the triangle* ADB. | | | *In the triangle* BDC. | | |
|---|---|---|---|---|---|
| As | AB=159 | 7,79860 | As | BC=351 | 7,45469 |
| To | s, ∠D=90° 00′ | 10,00000 | To | s, ∠D=90° 00′ | 10,00000 |
| So | AD=84 | 1,92428 | So | DC=324 | 2,51054 |
| To s, | ∠ABD=31° 53′ | 9,72288 | To s, | ∠CBD=67° 23′ | 9,96523 |
| And | ∠A =58 07 | | And | ∠C=22 37 | |

Then ∠ABD+∠CBD= ∠ABC=99° 16′

### COMPUTATION BY PROB. VI. art. 50.

*To find the angle* C.
Put E=408=AC
F=351=BC
D= 57=E—F.
G=159=AB
2)216(108=$\frac{1}{2}\overline{G+D}$
2)102(51 =$\frac{1}{2}\overline{G-D}$

| | | |
|---|---|---|
| Then, to Ar. Co. log. E | =408 | 7,38934 |
| Add Ar. Co. log. F | =351 | 7,45469 |
| And the log. | $\frac{1}{2}\overline{G+D}$=108 | 2,03342 |
| Alſo the log. | $\frac{1}{2}\overline{G-D}$= 51 | 1,70757 |
| The half of this ſum | | 2)18,58502 |
| Is the log. ſine of 11° 18′½ | | 9,29251 |
| Which doubled, is 22° 37′=∠C. | | |

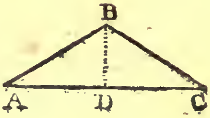
Now the angle C being known, the other angles may be found by Prob. I. But for a farther illuſtration of Prob. VI. the work for another angle is here repeated.

*To find the angle* B.
Put E=351=BC
F=159=AB
D=192=E—F.
G=408=AC
2)600(300=$\frac{1}{2}\overline{G+D}$
2)216(108=$\frac{1}{2}\overline{G-D}$

| | | |
|---|---|---|
| Then, to Ar. Co. log. E | =351 | 7,45469 |
| Add Ar. Co. log. F | =159 | 7,79860 |
| And the log. | $\frac{1}{2}\overline{G+D}$=300 | 2,47712 |
| Alſo the log. | $\frac{1}{2}\overline{G-D}$=108 | 2,03342 |
| The half of this ſum | | 2).9,76383 |
| Is the log. ſine of 49° 38′ | | 9,88191 |
| Which doubled, is 99° 16′=∠B. | | |

64. EXAMPLE XII. *In the plane Triangle* ABC.

*Given* AB=117 ⎫
BC=117 ⎬ Miles.
AC=216 ⎭
*Required the angles.*



## CONSTRUCTION.

The conſtruction of this triangle, and the meaſuring of the angles, is performed as in the Xth and XIth Examples.

## COMPUTATION.

In the triangle ABC, as AB=BC; therefore the angles A and C are equal (II. 104); and the perpendicular BD biſects the ſide AC; ſo that the right angled triangles ADB, CDB, are congruous; conſequently, the angles being found in one triangle, will give thoſe of the other.

Now in the triangle ADB, the ſide AB=117; the ſide AD, $=\frac{1}{2}$AC, is 108; and the ∠D is 90° 00′: Here, therefore, being a ſide and its oppoſite angle given, the ſolution belongs to Prob. I.

*To find the angle* ABD.

| | | | |
|---|---|---|---|
| As | AB | =117 | 7,93181 |
| To s, ∠ | D | =90° 00′ | 10,00000 |
| So | AD | =108 | 2,03342 |
| To s, | ∠ABD | =67° 23′ | 9,96523 |

Wh. taken from 90° 00′

Leaves 22° 37′=∠A=∠C.

And 67° 23′ doubled

Gives 134° 46′=∠ABC.

A like proceſs is to be uſed in every triangle, in which are two equal ſides.

The foregoing examples contain all the variety that can poſſibly happen in the ſolutions of plane triangles, conſidered only with regard to their ſides and angles; but beſides the methods ſhewn of reſolving ſuch triangles by conſtruction and computation, there is another way to find theſe ſolutions, called Inſtrumental; and this is of two kinds, *viz.* either by a ruler called a Sector, or by one called the Gunter's ſcale: The method by the ſector, the curious reader may ſee in many books, particularly in a treatiſe on Mathematical Inſtruments publiſhed in the year 1775, 3d edition *: But the other method by the Gunter's ſcale being in great uſe at ſea, it will be proper in this place to treat of it.

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* By the author of theſe Elements.

## SECTION IV.

## *Deſcription and Conſtruction of the Gunter's Scale.*

65. Mr. *Edmund Gunter*, Profeſſor of Aſtronomy at *Greſham* College, ſometime about the year 1624, applied the Logarithms of Numbers to a flat ruler: This he effected by taking the lengths expreſſed by the figures in thoſe logarithms from a ſcale of equal parts, and transferring them to a line, or ſcale, drawn on ſuch a ruler; and this is the line which, from his name, is called the *Gunter's line:* He alſo, in like manner, conſtructed lines containing the logarithms of the ſines and tangents; and ſince his time there have been contrived other logarithmic ſcales adapted to various purpoſes.

The Gunter's ſcale is a ruler, commonly two feet long; having on one of its flat ſides ſeveral lines or logarithmic ſcales; and on the other ſide various other ſcales; which, to diſtinguiſh them from the former, may be called natural ſcales.

While the reader is peruſing what follows, it is proper he ſhould have a Gunter's ſcale before him.

### 66. *Of the Natural Scales.*

The half of one ſide is filled with different ſcales of equal parts, for the convenience of conſtructing a larger, or ſmaller figure: The other half contains ſcales of Rhumbs, marked Rhu; Chords, marked Ch; Sines, marked Sin; Tangents, marked Tan; Secants, marked Sec; Semitangents, denoted by S. T. and Longitude diſtinguiſhed by M. L. The deſcriptions and uſes of theſe ſcales will be conſidered hereafter, in the places where they will be wanted.

### 67. *Of the Logarithmic Scales.*

On the other ſide of the ſcale are the following lines.

I. A line marked S. R. (ſine rhumbs), which contains the logarithmic ſines of the degrees to each point and quarter point of the compaſs.

II. A line ſigned T. R. (tan. rhumbs), the diviſions of which correſpond to the logarithmic tangents of the ſaid points and quarters.

III. A line marked Num. (numbers), where the logarithms of numbers are laid down.

IV. A line marked Sin. containing the log. ſines.

V. A line of log. verſed ſines, marked V. S.

VI. A line of log. tangents, marked Tan.

VII. A meridional line ſigned Mer.

VIII. A line of equal parts, marked E. P.

68. I. Of

## 68. I. *Of the Line of Numbers.*

The whole length of this line, or ſcale, is divided into two equal ſpaces, or intervals: the beginning, or left-hand end of the firſt, is marked 1; the end of the firſt interval, and beginning of the ſecond, is alſo marked 1; and the end of the ſecond interval, or end of the ſcale, is marked with 10; Both theſe diſtances are alike divided, beginning at the left-hand ends, by laying down in each the lengths of the logarithms of the numbers 20, 30, 40, 50, 60, 70, 80, 90; taken from a ſcale of equal parts, ſuch that 10 of its primary diviſions make the length of one interval: And the intermediate diviſions are found, by taking the logarithms of like intermediate numbers.

From this conſtruction it is evident, that when the firſt 1 ſtands for 1, the ſecond 1 ſtands for 10, and the end 10 denotes 100;

| And if the firſt 1 is called | the 2d 1 ſtands for | And the 10 at the end ſtands for |
|---|---|---|
| 10, | 100, | 1000; |
| 100, | 1000, | 10000; |
| &c. | &c. | &c.; |
| $\frac{1}{10}$, | 1, | 10; |
| $\frac{1}{100}$, | $\frac{1}{10}$, | 1; |
| &c. | &c. | &c.; |

And the primary and intermediate diviſions in each interval muſt be eſtimated according to the values ſet on their extremities, *viz.* at the beginning, middle, and end of the ſcale.

Now the examples moſt proper to be worked by this ſcale, are ſuch where the numbers concerned do not exceed 1000, and then the firſt 1 ſtands for 10, the middle 1 for 100, and the 10 at the end for 1000: The primary diviſions in the firſt interval, *viz.* 2, 3, 4, 5, 6, 7, 8, 9, ſtands for 20, 30, 40, 50, 60, 70, 80, 90, and the intermediate diviſions ſtand for units. In the ſecond interval, the primary diviſions ſigned 2, 3, 4, 5, 6, 7, 8, 9, ſtand for 200, 300, 400, 500, 600, 700, 800, 900; each of theſe diviſions are alſo divided into ten parts, which repreſent the intermediate tens. Between 100 and 200 the diviſions for tens are each ſubdivided into five parts; ſo that each of theſe leſſer diviſions ſtand for two units. The tens between 200 and 500 are divided into two parts, each ſtanding for five units: The units between the tens from 500 to 1000 are to be eſtimated by the eye; which by a little practice is readily done.

From this deſcription it will be eaſy to find the diviſion repreſenting a given number not exceeding 1000: Thus the number 62 is the ſecond ſmall diviſion from the 6, between the 6 and 7 in the firſt interval: The number 435 is thus reckoned; from the 4 in the ſecond interval, count towards the 5 on the right, three of the larger diviſions, and one of the ſmaller; and that will be the diviſion expreſſing 435. And the like of other numbers.

## 69. II. *For the Line of Sines.*

This ſcale terminates at 90 degrees, juſt againſt the 10 at the end of the line of numbers; and from this termination the degrees are laid backwards, or from thence towards the left: Now ſeeking in a table of logarithmi ſines, for the numbers expreſſing their arithmetic complements, witho the index, take thoſe numbers from the ſcale of equal parts the logs. the numbers were taken from, and apply them to the ſcale of ſines fr 90°, and they will give the ſeveral diviſions of this ſcale.

Thus the arith. comp. of the log. ſines (or the co-ſecants) abating t index, of 10°, 20°, 30°, 40°, *&c.* are the numbers 76033, 46595, 3010 19193, *&c.* then the equal parts to thoſe numbers, laid from 90°, wi give the diviſions for 10°, 20°, 30°, 40°, *&c.* and the like for the intermediate degrees.

Proceeding in this manner, the arith. comp. of the ſine of 5° 45′ will be about equal to 10 of the primary diviſions of the ſcale of equal parts, or to one interval in the log. ſcale; ſo that a decreaſe of the index by unity, anſwers to one interval; then a decreaſe of the index by 2 anſwers to two intervals, or the whole length of the log. ſcale; and this happens about the ſine of 35 min. and the diviſions anſwering to the ſine of a little above 3 min. *viz.* 3′ 26″. will be equal to 3 intervals; and the ſine of about 26″ will be 4 intervals, &c. ſo that the ſine of 90° being fixed, the beginning of the ſcale is vaſtly diſtant from it.

It is uſual to inſert the diviſions to every 5 minutes, as far as 10 degrees; from 10° to 30°, the ſmall diviſions are of 15 minutes each; from 30° to 50°, contains every half degree; from 50° to 70°, are only whole degrees; the reſt are eaſily reckoned.

## 70. III. *For the Line of Tangents.*

As the tangent of 45 degrees is equal to the radius, or ſine of 90°; therefore 45° on this ſcale, is terminated directly oppoſite to 90 on the ſines; and the ſeveral diviſions of this ſcale of log. tangents are conſtructed in the ſame manner as thoſe of the ſines, by applying their arith. comp. backwards from 45°, or towards the left-hand.

The degrees above 45, are to be counted backwards on the ſcale: Thus the diviſion at 40° repreſents both 40° and 50°; the diviſion 30 ſerves for 30° and 60°; and the like of the other diviſions, and their intermediates.

## 71. IV. *For the Line of Verſed Sines.*

This line begins at the termination of the numbers, ſines, and tangents: But as the numbers on thoſe lines deſcend from the right to the left, ſo theſe aſcend in the ſame direction: Now having a table of logarithmic verſed ſines to 180 degrees, let each log. verſed ſine be ſubtracted from that of 180 degrees; then the remainders being ſucceſſively taken from the ſaid ſcale of equal parts, and laid on the ruler backwards from the common termination, the ſeveral diviſions of this ſcale will be obtained.

The numbers for each 10 deg. are in the following table.

| D. | Numb. | D. | Numb. | D. | Numb. | Deg. | Numb. | Deg. | Numb. | Deg. | Numb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0,0033 | 40 | 0,0540 | 70 | 0,1733 | 100 | 0,3839 | 130 | 0,7481 | 160 | 1,5207 |
| 20 | 0,0133 | 50 | 0,0854 | 80 | 0,2315 | 110 | 0 4828 | 140 | 0,9319 | 170 | 2,2194 |
| 30 | 0,0301 | 60 | 0,1219 | 90 | 0,3010 | 120 | 0,6021 | 150 | 1,1740 | 180 | 10,3010 |

The other ſcales will be deſcribed in their proper places.

### 72. *Demonſtration of the foregoing conſtructions.*

That of the log. numbers, is evident from the nature of logarithms.

### *For the Sines and Tangents.*

Now co-ſine : rad. : : rad. : ſecant (34). Then co-ſ. × ſecant = 1
ſine : rad. : : rad. : co-ſec. (35). ſine × co-ſec. = 1
tan. : rad. : : rad. : co-tan. (36). tan. × co-tan. = 1 II. 162

The radius of the tables being ſuppoſed equal to 1.

Hence it is evident, that in either caſe, one of the quantities will be equal to the quotient of unity divided by the other.

But diviſion is performed by ſubtraction with logarithms.

And to ſubtract a log. is the ſame as to add its arith. comp.

Conſequently, the logarithmic co-ſine and ſecant of the ſame degrees are the arithmetical complements of one another.

And ſo are the logarithmic ſines and co-ſecants: Alſo the logarithmic tangents and co-tangents are the arith. comp. of one another.

73. Now as the arith. comp. of any number is what that number wants of unity in the next ſuperior place;

Therefore every natural ſine and its arith. comp. together make the radius.

And the ſines begin at one end of a radius, and end in 90° at the other end.

Therefore in a ſcale of ſines, the arith. comp. of any ſine, or its co-ſecant, laid backwards from 90°, gives the diviſion for that ſine. And the like muſt happen in a ſcale of log. ſines.

74. Alſo, as the logarithmic tangents and co-tangents are the arith. comp. of one another; therefore in a ſcale of log. tangents, the diviſions to the degrees both under and above 45, are equally diſtant from the diviſion of 45°.

Conſequently the diviſions ſerving to the degrees under 45, will ſerve, by reckoning backwards, for thoſe above 45.

### 75. *For the Verſed Sines.*

Although the numbers in the line of verſed ſines aſcend from right to left, yet they are only the ſupplements of the real verſed ſines, which are numbered in the ſame order as the ſines, that is, from left to right: But as the beginning of the verſed ſines falls without the *ruler*, therefore it is moſt convenient to lay down the diviſions from the point where the verſed ſines terminate at 180 degrees, that is, againſt 90° on the ſines.

Now it is evident that the diviſions laid off from this termination muſt be the differences between the log. verſed ſines of the ſeveral degrees, &c. and that of 180 degrees,

## SECTION V.

### *The uſe of the Gunter's Scale in Plane Trigonometry.*

76. When a Trigonometrical Queſtion is to be ſolved by the Gunter's ſcale, it muſt firſt be ſtated by the precepts to that problem under which the queſtion falls, whether it be by oppoſite ſides and angles, or by two ſides and their included angle, or by three ſides.

77. In all proportions wrought by the Gunter's ſcale, when the firſt and ſecond terms are of the ſame kind, then

*The extent from the firſt term to the ſecond will reach from the third term to the fourth.*

Or, when the firſt and third terms are of the ſame kind.

*The extent from the firſt term to the third will reach from the ſecond term to the fourth.*

That is, ſet one point of the compaſſes on the diviſion expreſſing the firſt term, and extend the other point to the diviſion expreſſing the ſecond (or third) term; then, without altering the opening of the compaſſes, ſet one point on the diviſion repreſenting the third term (or ſecond term), and the other point will fall on the diviſion ſhewing the fourth term or anſwer.

In working by theſe directions, it is proper to obſerve,

78. Firſt. The extent from one ſide to another ſide, is to be taken from the ſcale of numbers; and the extent from one angle to another is to be taken from the ſcale of ſines, in working by oppoſite ſides and angles; or from the ſcale of tangents, in working by two ſides and the included angle.

Secondly. When the extent from the firſt term to the ſecond (or third) is decreaſing, or is from the right to the left, then the extent from the third term (or ſecond) muſt be alſo decreaſing; that is, applied from the right towards the left: And the like caution is neceſſary when the extent is from the left towards the right.

Theſe precepts being carefully attended to, what follows will be readily underſtood.

79. In Example I. See article 53.

As s,∠c : ab : : s,∠b : ac. Or s,42° 05′ : 195 : : s, 90° 00′ : Q, where Q ſtands for the number ſought.

Now the extent from 42° 05′ to 90° 00′, taken on a ſcale of ſines, and applied to the ſcale of numbers, will reach from 195 to 291. See art 69.

Alſo. As s,∠c : ab : : s,∠a : bc. Or s, 42° 05′ : 195 : : s,47° 55′ : Q.

Then the extent from 42° 05′ to 47° 55′ on the ſines, being applied to the numbers, will reach from 195 to 216. See art. 68.

In each of theſe operations, the firſt extent was from the left to the right, or increaſing; therefore the ſecond extent muſt be from left to right alſo.

80. In Example IV. See art. 56.

As ac : s,∠b : : ab : s,∠c. Or 291 : s,90° 00′ : : 195 : Q.

Here the extent from 291 to 195, taken on the numbers, and applied to the ſines, will reach from 90° 00′ to 42° 05′.

The firſt extent being from the right towards the left, or decreaſing; therefore the ſecond extent muſt be alſo from the right to the left.

In Example VII. See art. 59.

As ab : Rad. : : bc : t,∠a. Or 195 : t, 45° 00′ : : 216 : Q.

Then the extent from 195 to 216 on the numbers, will reach from 45° 00′ to 47° 55′ on the tangents.

Here the firſt extent being from left to right, or increaſing, therefore the ſecond extent muſt alſo be increaſing: Now on the tangents, this increaſe above 45° does not proceed from left to right, but from right to left, the ſame way that the decreaſe proceeds (70); conſequently the diviſion which the point falls on for the fourth term, muſt be eſtimated according as the firſt extent is increaſing or decreaſing.

Thus had the proportion been,

As bc : Rad. : : ab : t,∠c. Or, 216 : t,45° 00′ : : 195 : Q.

Then the extent from 216 to 195 on the numbers, will reach from 45° 00′ to 42° 05′, eſtimated as decreaſing.

81. When two ſides and the included angle are given, and the tangent of half the difference of the unknown angles is required.

Then, on the line of numbers take the extent from the ſum of the given ſides to their difference; and on the line of tangents apply this extent from 45° downwards, or to the left; let the point of the compaſſes reſt where it falls, and bring the other point (from 45°) to the diviſion anſwering to the half ſum of the unknown angles; then this extent applied from 45° downwards, will give the half difference of the unknown angles: Whence the angles may be found. (47)

In EXAMPLE IX. See the art. 61.

$$\text{As } AB+AC : AB-AC :: t,\tfrac{1}{2}\overline{\angle C+\angle B} : t,\tfrac{1}{2}\overline{\angle C-\angle B}.$$
$$\text{Or } 567 : 249 :: t,\ 60^\circ\, 56' : Q.$$

Now the extent from 567 to 249 on the numbers, being applied to the tangents, will reach from 45° to about 23° 40′: Let one point of the compaſſes reſt on this diviſion, and bring the other to 60° 56′; then this extent will reach from 45° to 38° 19′, the half difference ſought.

And this method will always give the half difference, whether the half ſum of the angles is greater or leſs than 45°.

82. But when the half ſum and half difference are greater than 45°; then the extent from the ſum of the ſides to their difference on the ſcale of numbers, will (on the tangents) reach from the half ſum of the angles to their half difference, reckoning from left to right.

And when the half ſum and half difference are both leſs than 45; then the extent from the ſum of the ſides to their difference, taken from the numbers and applied to the tangents, will reach from the half ſum of the angles downwards to their half difference.

83. When the three ſides are given to find an angle, and a perpendicular is drawn from an angle to its oppoſite ſide. See Ex. X. art. 62.

$$\text{As } AC : BC+AB :: BC-AB : CD-AD.$$
$$\text{Or } 291 : 411 :: 21 : Q.$$

Now the extent from 291 to 411 on the ſcale of numbers, will reach from 21 to 29,6 on the numbers alſo.

Then the extent for the angles is performed in the ſame manner as ſhewn in Ex. I. (78)

84. Or an angle may be found by Problem VI. as follows.

In the ſcale of numbers, take the extent from the half ſum (of G and D) to either of the containing ſides (as E); apply this extent from the other containing ſide (as F), to a fourth term: Let one point of the compaſſes reſt on this fourth term, and extend the other to the half difference (of G and D); then this extent applied to the verſed ſines from the beginning, will give the ſupplement of the angle ſought.

In EXAMPLE X. See art. 62.

E = 291; F = 216; half ſum = 135; half diff. = 60.

Then on the numbers, the extent from 135 to 291, will reach from 216 to 465; let the point reſt there, and extend the other to 60; then this extent applied to the verſed ſines, will reach from the beginning to 137° 56′; which taken from 180°, leaves 42° 04′ for the angle ſought.

85. In Example XI. See art. 63.

Here E=408; F=351; half ſum of G and D=108; half diff.=51. Then on the numbers, the extent from 108 to 408, will reach from 351 in the ſecond interval, to a fourth number: But as the point of the compaſſes falls beyond the end of the ſcale, therefore let the extent from 108 to 408 be applied in the firſt interval, which will reach from 35,1 to 132,6; let one point reſt on 132,6, and extend the other point of the compaſſes to 51. Now as this extent of the compaſſes is leſs than it ought to be, by one interval, or half the length of the ſcale of numbers; therefore the laſt extent, when applied to the verſed ſines, muſt be from that diviſion, on the verſed ſines, oppoſite to the middle of the ſcale of numbers, which is nearly at 143°; and it will reach from thence to the verſed ſine of 157° 23′; which taken from 180°, leaves 22° 37′ for the angle ſought.

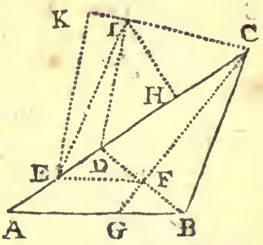
86. Moſt of the writers on Plane Trigonometry treat of right angled, and of oblique angled triangles ſeparately; making ſeven caſes in the former, and ſix caſes in the latter: But as every one of theſe thirteen caſes fall under one or other of the foregoing Problems, therefore ſuch diſtinctions are here avoided, it being conceived, that they rather tend to perplex than inſtruct a learner: Alſo in the generality of the treatiſes on this ſubject, it is uſually ſhewn how the ſolutions of right angled triangles are performed, by making (as it is called) each ſide radius; that is, by comparing each ſide of the triangle with the radius of the tables: And although theſe conſiderations are here omitted, yet the inquiſitive reader will find them in Book VII. near the beginning.

87. Beſide the demonſtration of Problem IV. at art. 48. it has been thought proper to give another demonſtration; becauſe there ariſes from it a Theorem uſeful on ſome occaſions: Moreover, there is alſo added methods of deriving other rules for the ſolution of the caſe where the three ſides are given to find an angle; which, if they ſhould be found of no other uſe, will perhaps be agreeable exerciſes of Geometry to thoſe who are delighted with theſe ſtudies.

SECTION

## SECTION VI.

### *Properties of Plane Triangles.*



88. In any plane triangle ABC,
Given CA, CB, and $\angle C$.
Required the angles B and A.

SOLUTION. Take $CD=CB$, and draw DB. Biſect DB in F, DA in E, draw CFG, and EF, which is parallel to AB. (II. 165)
Now DE or AE is equal to half the difference of CA and CB.
And CE $(=CA-AE)$ is equal to the half ſum of CA and CB. (47)

The ſum of the equal angles CBD, CDB (II. 104) is equal to the ſum of the unknown angles CBA, CAB. (II. 98)

Then the angle CBD is the half ſum, and the angle ABD is the half difference of the unknown angles CBA, CAB. (47)
And as CFG is at right angles to DB (II. 103); CF is the tangent of $\angle CBD$, and GF is the tangent of $\angle ABF$ to the rad. BF. (4)
Then $CE : EA :: CF : GF$ (II. 165) : Or $2CE : 2EA :: CF : FG$. (II. 151)

That is, $CA+CB : CA-CB :: \tan.\frac{1}{2}\angle\overline{CBA+CAB} : \tan.\frac{1}{2}\angle\overline{CBA-CAB}$.

89. AGAIN. From H, the middle of CD, draw HI at right angles, and equal to CH; draw DI, and EK parallel to DI, meeting CI produced in K; and join IE.
Now $HE=(\frac{1}{2}CD+\frac{1}{2}DA)\frac{1}{2}CA$; and $CH=\frac{1}{2}CB$.
And HE = tangent of the angle HIE to the radius, $HI=HC$.
Then $CB : CA :: (2HC : 2HE :: HC : HE ::)$ Radius : tan. $\angle HIE$.
And $\angle HIE-(\angle HID=)\,45^\circ=\angle DIE=\angle KEI$ (II. 95) is known.

Then rad. : tan. $\angle KEI :: EK : KI :: CK : KI$; becauſe $\angle KCE=\angle KEC$.
But $:: CK : KI :: CE : ED$. (II. 166)
$:: 2CE : 2ED$. (I. 151)
$:: CA+CB : CA-CB :: t,\frac{1}{2}$ ſum $\angle s : t,\frac{1}{2}$ diff. $\angle s$ (48)

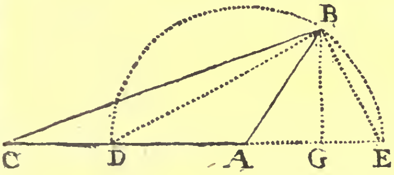
Conſequently rad. : tan. $\angle KEI :: \tan.\frac{1}{2}\angle\overline{CBA+CAB} : \tan.\frac{1}{2}\angle\overline{CBA-CAB}$.

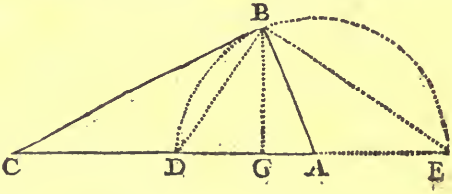
This rule is often uſeful in Aſtronomical calculations, when the logarithms of the ſides AB, BC are only known, and the angles BAC, BCA, are required, without finding, from thoſe logarithms, the ſides themſelves.

For the difference between the logarithms of the ſides, increaſed by radius, gives the tangent of an arc; which arc leſſened by 45° leaves a ſecond arc.
Then as rad. : tan. ſecond arc :: tan. $\frac{1}{2}$ ſum $\angle s$ : tan. $\frac{1}{2}$ diff. $\angle s$.

90. In any plane triangle ABC, where the three ſides are known, the meaſure of either angle (as the angle A, included between the ſides AB, AC) may be found ſeveral ways, as ſhewn in the following articles.

The letters $s$, $t$, $v$, ſtand for ſine, tangent, verſed ſine.
$s^{\backprime}$, $t^{\backprime}$, ſtand for co-ſine, co-tangent.
$v^{\backprime}$, ſtands for the verſed ſine of the ſupplement.
Alſo $ss$, $tt$, ſtand for the ſquares of the ſine and tangent.
Alſo $s^{\backprime}s^{\backprime}$, $t^{\backprime}t^{\backprime}$, ſtand for the ſquares of the co-ſine and co-tangent.
Radius $= R = AB = AE = AD$; and $H = \frac{1}{2}AB + \frac{1}{2}AC + \frac{1}{2}BC$.





91. $s,\frac{1}{2}\angle A = R \times \sqrt{\frac{\overline{H-AC} \times \overline{H-BC}}{AC \times AB}}$. Here $\angle E = \frac{1}{2}\angle A$.

For $R : s, \frac{1}{2}\angle A :: (DE=)2AB : BD$. (45)
And $RR : ss, \frac{1}{2}\angle A :: 4AB^2 : BD^2$. (II. 161)

Now $ss,\frac{1}{2}\angle A = RR \times \frac{BD^2}{4AB^2}$. (II. 164)

$= \frac{2AB}{4AB^2} \times \frac{\overline{2H-2AC} \times \overline{2H-2BC}}{2AC} \times RR$. (II. 179)

Then $s, \frac{1}{2}\angle A = R \times \sqrt{\frac{\overline{H-AC} \times \overline{H-AB}}{AC \times AB}}$.

92. $s,\angle A = \frac{2R}{AB \times AC} \times \sqrt{H \times \overline{H-CB} \times \overline{H-AC} \times \overline{H-AB}}$.

For $AB : BG :: R : s,\angle A$. (45)
And $AB^2 : BG^2 :: RR : ss, \angle A$. (II. 161)

Now $ss, \angle A = \frac{RR}{AB^2} \times BG^2$. (II. 164)

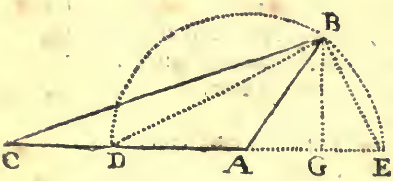
$= \frac{RR}{AB^2} \times \frac{4}{AC^2} \times H \times \overline{H-CB} \times \overline{H-AC} \times \overline{H-AB}$. (II. 180)

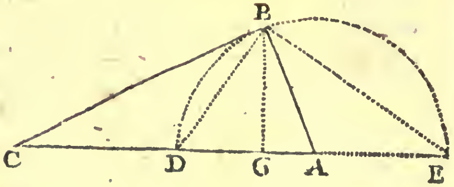
93. $s^{\backprime},\frac{1}{2}\angle A = R \times \sqrt{\frac{H \times \overline{H-CB}}{AC \times AB}}$. Here $\angle E = \frac{1}{2}\angle A$; and $\angle D =$ comp. $\angle E$.

For $R : s^{\backprime}, \frac{1}{2}\angle A :: (DE=)\ 2AB : BE$. (45)
And $RR : s^{\backprime}s^{\backprime}, \frac{1}{2}\angle A :: 4AB^2 : BE^2$. (II. 161)

Now $s^{\backprime}s^{\backprime}, \frac{1}{2}\angle A = RR \times \frac{BE^2}{4AB^2}$. (II. 164)

$= RR \times \frac{2AB \times 4 \times H \times \overline{H-CB}}{4AB^2 \times 2AC}$. (II. 178)





94. $t, \frac{1}{2}\angle A = R \times \frac{\sqrt{\overline{H-AC} \times \overline{H-AB}}}{H \times \overline{H-CB}}$.

For $R : t, \frac{1}{2}\angle A :: BE : BD$. (46)

And $RR : tt, \frac{1}{2}\angle A :: BE^2 : BD^2$. (II. 161)

Now $tt, \frac{1}{2}\angle A = RR \times \frac{BD^2}{BE^2}$. (II. 164)

$= RR \times \frac{\overline{H-AC} \times \overline{H-AB}}{E \times \overline{H-BC}}$. (II. 178, 179)

95. $t', \frac{1}{2}\angle A = R \times \sqrt{\frac{H \times \overline{H-CB}}{\overline{H-AC} \times \overline{H-AB}}}$.

For $BD^2 : BE^2 :: (tt, \frac{1}{2}\angle A : RR ::)\ RR : t't', \frac{1}{2}\angle A$. (36)

Then $t't', \frac{1}{2}\angle A = RR \times \frac{BE^2}{BD^2}$. (II. 178, 179)

96. $v, \angle A = 2R \times \frac{\overline{H-AC} \times \overline{H-AB}}{AC \times AB}$.

For $R : v, \angle A :: AB : GD$. (45)

Then $v, \angle A = R \times \frac{GB}{AB} = R \times \frac{2 \times \overline{H-AC} \times \overline{H-AB}}{AC \times AB}$ (II. 177)

97. $v' \angle A = 2R \times \frac{H \times \overline{H-CB}}{AC \times AB}$.

For $R : v', A :: AB : GE$. (45)

Then $v', \angle A = R \times \frac{GE}{AB} = R \times \frac{2 \times H \times \overline{H-CB}}{AC \times AB}$. (II. 176)

98. $s' \angle A = \frac{1}{2} R \times \frac{CB^2 - AC^2 - AB^2}{AC \times AB}$

For $R : s', \angle A :: AB : AG$. (45)

Then $s', \angle A = R \times \frac{AG}{AB} = R \times \frac{BC^2 - AC^2 - AB^2}{2AC \times AB}$. (II. 174)

## END OF BOOK III.

THE

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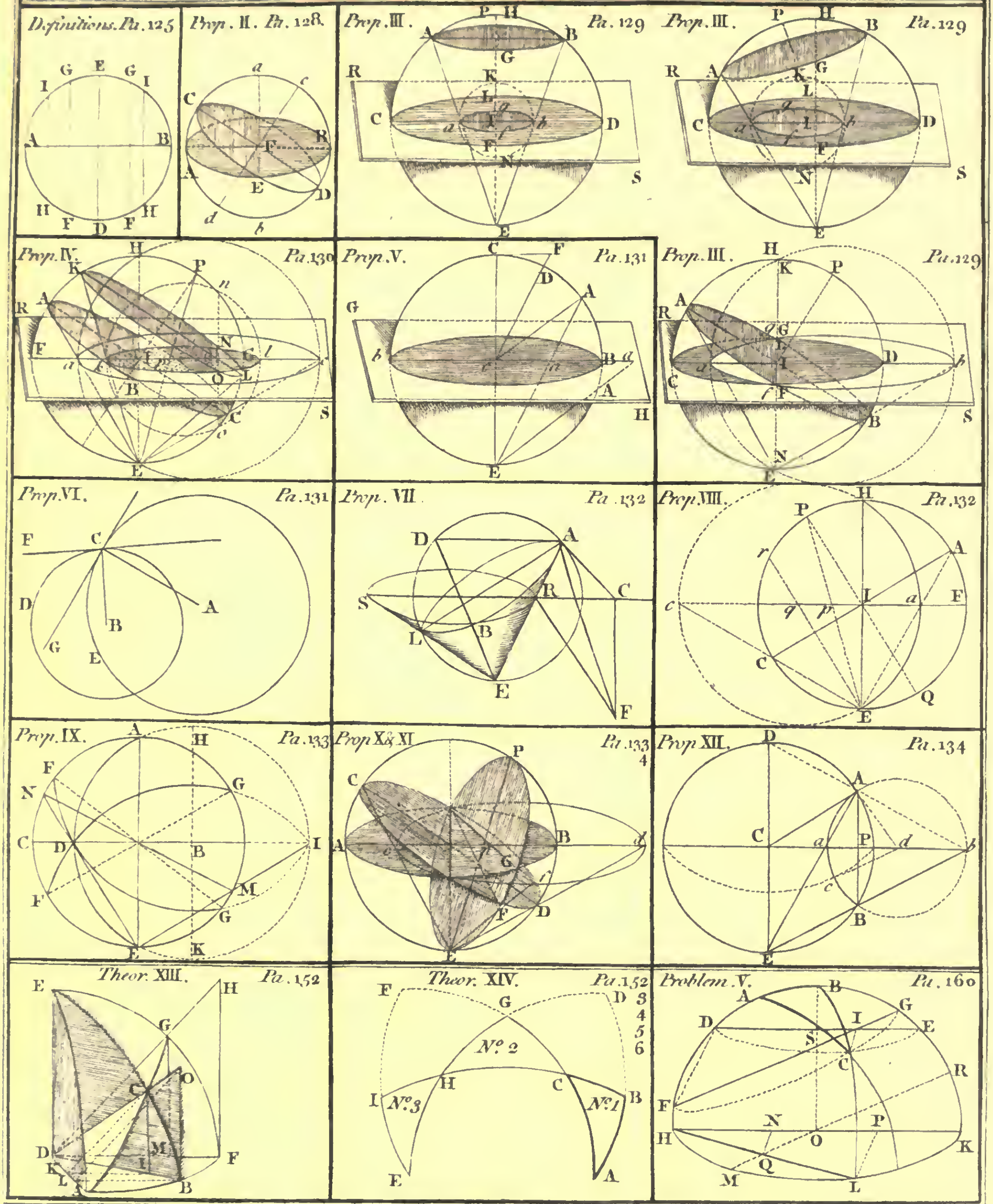

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# THE ELEMENTS OF NAVIGATION.

## BOOK IV.

## OF SPHERICS.

### SECTION I.

### *Definitions* and *Principles.*

1. SPHERICS is that part of the Mathematics which treats of the poſition and magnitude of arcs of circles deſcribed on the ſurface of a ſphere.

2. A SPHERE is a ſolid contained under one uniform round ſurface, ſuch as would be formed by the revolution of a circle about its diameter, that diameter being immoveable during the motion of the circle.

*Thus the circle* AEBD *revolving about the diameter* AB, *will generate a ſphere, the ſurface of which will be formed by the circumference* AEBD. See Plate I.

3. The CENTER and AXIS of a ſphere are the ſame as the center and diameter of a generating circle: And as a circle has an indefinite number of diameters, ſo a ſphere may be conſidered as having alſo an indefinite number of axes, round any one of which the ſphere may be conceived to be generated.

4. CIRCLES OF THE SPHERE are thoſe circles deſcribed on its ſurface by the motion of the extremities of ſuch chords in the generating circle as are at right angles to the diameter, or to the axis of the ſphere.

*Thus by the motion of the circle* AEBD *about the diameter* AB, *the extremities of the chords* ED, GF, IH, *at right angles to* AB, *will deſcribe circles the diameters of which are equal to thoſe chords reſpectively.* Plate I.

5. The POLES of a circle on the ſphere, are thoſe points on its ſurface equally diſtant from the circumference of that circle.

*Thus* A *and* B *are the poles of the circles deſcribed on the ſphere by the ends of the chords* ED, GF, IH. Plate I.

6. A GREAT CIRCLE of the ſphere, is that circle which is equally diſtant from both its poles.

*Thus the circle deſcribed by the extremities* E, D, *of the diameter* ED, *at right angles to* AB, *being equally diſtant from its poles* A *and* B, *is called a great circle.*

7. LESSER CIRCLES of the ſphere, or ſmall circles, are thoſe circles which are unequally diſtant from both their poles.

*Thus the circles of which* FG, HI, *are diameters, having their poles* A *and* B *unequally diſtant from them, are called leſſer circles.*

8. PARALLEL CIRCLES of the ſphere, are thoſe circles, the planes of which are conſidered as parallel to the plane of ſome great circle.

*Thus the circles having the diameters* FG, HI, *are called parallel circles in reſpect of the great circle of which* ED *is the diameter.*

9. A SPHERIC ANGLE is the inclination of two great circles of the ſphere meeting one another.

10. A SPHERIC TRIANGLE is a figure formed on the ſurface of a ſphere by the mutual interſections of three great circles.

11. The STEREOGRAPHIC PROJECTION of the ſphere, is ſuch a repreſentation of its circles, upon the plane of one of them paſſing through the center, and called the PLANE OF PROJECTION, as would appear to an eye placed in one of the poles of that great circle, and thence viewing the circles on the ſphere.

12. The place of the Eye is called the PROJECTING POINT, or lower pole: and the point diametrically oppoſite is called the remoteſt, or oppoſite, or upper pole.

Alſo, the projection of any point on the ſphere, is that point in the plane of projection, through which the viſual ray paſſes to the eye.

13. The PRIMITIVE CIRCLE is that great circle, on the plane of which the repreſentations of all the other circles are ſuppoſed to be drawn.

14. An OBLIQUE CIRCLE is one which has its plane oblique to the eye.

15. A RIGHT CIRCLE is that which is perpendicular to the plane of the primitive circle, and if it be a great circle, its plane paſſes through the eye, and it is ſeen edgewiſe; conſequently it is repreſented by a ſtraight line drawn through the center of the primitive circle.

AXIOMS.

## AXIOMS.

16. The diameter of every great circle paſſes through the center of the ſphere; but the diameters of ſmall circles do not paſs through the ſame center: Alſo the center of the ſphere is the common center of all its great circles.

17. Every ſection of a ſphere, by a plane paſſing though its circumference, is a circle.

18. A ſphere is divided into two equal parts by the plane of every great circle; and into two unequal parts by the plane of every ſmall circle.

19. The pole of every great circle is at 90 degrees diſtance from it on the ſurface of the ſphere: And no two great circles can have a common pole.

20. The poles of a great circle are the extremities of that diameter, or axis, of the ſphere, which is perpendicular to the plane of that circle.

21. Lines flowing to the projecting point, or place of the eye, from every point in the circumference of a circle which it views, form the convex ſurface of a Cone.

22. A plane paſſing through three points on the ſurface of a ſphere, equally diſtant from the pole of a great circle, will be parallel to the plane of that circle.

23. The ſhorteſt diſtance between two points on the ſurface of a ſphere, is the arc of a great circle paſſing through thoſe points.

24. If one great circle meets another, the angles on either ſide are ſupplements to one another; and every ſpheric angle is leſs than 180 degrees.

25. A ſpheric angle is meaſured by an arc of a great circle intercepted between the legs of that angle, 90 degrees diſtant from the angular point.

26. If two circles interſect one another, the oppoſite angles are equal.

27. Two ſpheric triangles are congruous, if two ſides and their contained angle in one, are equal to two ſides and their contained angle in the other, each to each: Or if two angles and the contained ſide in the one, are equal to two angles and their contained ſide in the other, each to each: Or if the three ſides in the one are reſpectively equal to the three ſides in the other.

28. All parallel circles have the ſame pole, and may be conceived to be concentric to the great circle which they are parallel to.

29. All parallel circles on the ſphere, having the ſame pole, are cut into ſimilar arcs by two great circles paſſing through that pole.

## SECTION II.

## *Stereographic Propoſitions.*

### 30. PROPOSITION I.

*Great circles of a ſphere mutually cut one another into two equal parts.*

Dem. Any two great circles have the ſame common center. (16)
And their planes interſect in a right line. (II. 209)
Now the center muſt lie in the line of their interſection.
Therefore this right line is a diameter common to both.
But every circle is biſected by its diameter.
Therefore the circles mutually biſect one another.

31. Corol. I. The circumferences of any two circles interſecting one another twice, make the angles at both ſections equal.

For the planes of thoſe circles have the ſame inclination at both ends of their interſection, or where the circumferences interſect.

32. Corol. II. Two great circles of the ſphere will cut each other twice at the diſtance of 180 degrees, or in oppoſite points of the ſphere.

### 33. PROP. II.

*The diſtance of the poles of two great circles, is equal to the angle formed by the inclination of thoſe circles.* Plate I.

Dem. Let AEB, CED, be two great circles of the ſphere, their planes paſſing through its center F; and let $a$, $b$, be the poles of the circle AEB, and $c$, $d$, the poles of the circle CED.
Then is the arc $\mathrm{A}a =$ arc $\mathrm{C}c = 90^\circ$. (19)
And the arc C$a$ is common to both the arcs A$a$ and C$c$.
Therefore the arc AC, meaſuring the inclination CFA of the circles, is equal to the arc $ac$ meaſuring the diſtance of the poles. (II. 48.)

34. Corol. I. Two great circles are at right angles to one another, when they paſs through each other's poles.

35. Corol. II. The pole of a great circle is 90 degrees diſtant from it, taken in another great circle, or in an arc of it, drawn perpendicular to the former circle.

36. Corol. III. Two or more great circles, at right angles to another great circle, interſect one another 90° diſtant from it, or in the pole of the latter circle. And the like of arcs of great circles.

37. Corol. IV. If ſeveral great circles interſect one another in the pole of another great circle; then are the former circles perpendicular to the latter.

## 38. PROP. III.

*In the ſtereographic projection of the ſphere, the repreſentations of all circles, not paſſing through the projecting point, will be circles.* Plate I. three figures.

Let ACEDB repreſent a ſphere, cut by a plane RS, paſſing through the center I, at right angles to the diameter EH, drawn from E, the place of the eye.

And let the ſection of the ſphere (17) by the plane RS, be the circle CFDL, its poles being H, and E.

Suppoſe AGB is a circle on the ſphere to be projected, its pole, moſt remote from the eye, being P: And the viſual rays from the circle ABG meeting in E, form the cone AGBE (21) of which the triangle AEB is a ſection through the vertex E, and diameter of the baſe AB. (II. 204)

Then will the figure *agbf*, which is the projection of the circle BGA, be a circle.

DEMONSTRATION. Since the $\angle Eab$ is meaſured by $\frac{1}{2}$ arc AC + ($\frac{1}{2}$ arc ED =) $\frac{1}{2}$ arc CE. (II. 137)
And the $\angle EBA$ is meaſured by $\frac{1}{2}$ arc AC + $\frac{1}{2}$ arc CE. (II. 128)
Therefore the angle EBA = angle E*ab*. (II. 50)
And ſo the triangles EAB, E*ba* are ſimilar, the $\angle E$ being common.
Therefore *ab* cuts the ſides EA and EB of the cone, in a ſubcontrary poſition to AB; and conſequently the ſection *afbg* is a circle. (II. 213)

Now ſuppoſe the plane RS to revolve on the line CD, till it coincides with the plane of the circle ACEB;

Then it is evident, that the point L will fall in H, the point F in E, and the circle CFDL will coincide with the circle CEDH, which now becomes the primitive circle, where the point F, or E, is the projecting point: Alſo the projected circle *afbg* will become the circle *a*N*b*K.

39. COROL. I. Hence the middle of the projected diameter is the center of the projected circle, whether it be a great circle or a ſmall one.

40. COROL. II. Hence in all circles parallel to the plane of projection, their centers and poles will fall in the center of the projection.

41. COROL. III. The centers and poles of circles, inclined to the plane of projection, fall in that diameter of the primitive circle which is at right angles to the diameter drawn through the projecting point; but at different diſtances from its center.

42. COROL. IV. All oblique great circles cut the primitive circle in two points diametrically oppoſite.

## 43. PROP. IV.

*The meaſure of the angle which the projected diameter of any circle ſubtends at the eye, is equal to the diſtance of that circle from its pole, which is moſt remote from the projecting point, taken on the ſurface of the ſphere. And that angle is biſected by a right line joining the projecting point and that pole.* Plate I.

Let the plane RS cut the ſphere HFEG, as in the laſt.
And let ABC be any oblique great circle, the diameter of which AC is projected into *ac*; and KOL any ſmall circle parallel to ABC, the diameter of which KL is projected into *kl*.

The diſtances of thoſe circles from the pole P, being the arcs AHP, KHP, and the angles *a*E*c*, *k*E*l*, are angles at the eye ſubtended by their projected diameters *ac*, *kl*.

Then is the angle *a*E*c* meaſured by the arc AHP, the angle *k*E*l* is meaſured by the arc KHP; and thoſe angles are biſected by EP.

DEM. For arc PHA = arc PC; and arc PHK = arc PL. (5)
And the ∠ AEC is meaſured by $\frac{1}{2}$ arc APC = arc PHA. (II. 128)
Alſo the ∠ KEL is meaſured by $\frac{1}{2}$ arc KPL = arc PHK. (II. 128)
Therefore the angles AEC, KEL, are reſpectively meaſured by the arcs PHA, PHK.

And it is evident thoſe angles are biſected by the line EP.

44. COROL. I. Hence as the line EP projects the pole P in *p*; ſo the ſame line refers a projected pole to its place on the ſphere, in the circumference of the primitive circle.

45. COROL. II. Hence, on the plane of the primitive circle, may be deſcribed the repreſentation of any circle whoſe diſtance from its pole, and the projected place of that pole, are given.

For PA and PC are projected into *pa* and *pc*; and the biſection of *ac* gives the center of the circle ſought.

46. COROL. III. Hence every projected oblique great circle cuts the primitive circle in an angle equal to the inclination of the plane of that oblique circle to the plane of projection.

For F*a* is equivalent to FA the inclination.
And F*a* meaſures the angle FH*a*, ſince FH, H*a*, are each 90°. (9)

47. COROL. IV. The diſtance between the projections of a great circle and any of its parallels is equivalent to their diſtance on the ſphere.
Thus the projection *ak* is equivalent to AK.

## 48. PROP. V.

*Any point of a ſphere ſtereographically projected, is diſtant from the center of projection, by the tangent of half the arc intercepted between that point and the pole oppoſite to the eye: The ſemidiameter of the ſphere being made radius.* Plate I.

Let C*b*EB be a great circle of the ſphere, the center of which is *c*, GH the plane of projection cutting the diameter of the ſphere in *b*, B; E, C, the poles of the ſection by that plane; and *a* the projection of A.
Then is *ca* equal to the tangent of half the arc AC.

DEM. Draw CF, a tangent to the arc CD $=\frac{1}{2}$ arc CA, and join *c*F.
Now the triangles CF*c*, *ca*E, are congruous: For C*c* = *c*E, ∠ C = ∠ E*ca* = right ∠, ∠ C*c*F = *c*E*a* (II. 128): Therefore *ca* = CF.
Conſequently *ca* is equal to the tangent of half the arc CA.

## 49. PROP. VI.

*The angle made by the interſection of the circumferences of two circles in the ſame plane, is equal to the angle made by tangents to thoſe circles in the point of ſection; and alſo is equal to the angle made by their radii drawn to that point.* Plate I.

Let CE, CD, be two arcs of circles in the ſame plane cutting in the point C; AC, BC, their radii; GC, FC, tangents at the point C.
Then is the curve-lined angle ECD = ∠ GCF = ∠ ACB.

DEM. Since the radii AC, BC, are at right angles to their tangents GC, FC (II. 126); and are alſo at right angles to the arcs CE, CD. (II. 136)
Therefore the poſition of the tangents and arcs at the point C are the ſame; and conſequently the ∠ ECD = ∠ GCF.
Alſo the ∠ ACB + ∠ BCG = (right ∠ =) ∠ FCG + ∠ BCG.
Therefore the angle ACB is equal to the angle FCG, by taking away the common angle BCG. (II. 48)
Conſequently ∠ ECD = GCF = ∠ ACB.

50. SCHOLIUM. If the arcs CE, CD, were in different planes, the ſame would hold true with regard to their tangents.

For ſuppoſe the circle CD to revolve on the fixed radius BC, ſtill cutting the circle CE in C: Then the tangent CF revolving with it, has ſtill the ſame inclination to BC: And as the inclination of the planes of the circles vary, ſo much will the inclination of the tangents vary.

Therefore the angle made by the tangents, in all poſitions of the circular planes, is the ſame as the angle made by their circumferences.

51. COROL. Hence if a plane touches a ſphere, at the point where two circles of it interſect one another, the tangents to both circles will lie in that plane; and conſequently, in all oblique poſitions, a right line perpendicular to one tangent will cut the other tangent.

## 52. PROP. VII.

*The angle which any two circles make, when ſtereographically projected, is equal to the angle which thoſe circles make on the ſphere.* Plate I.

Suppoſe DAEL a ſphere to be projected on the plane SBRCF, and ALDE a great circle paſſing through the projecting point E. Let LBA be any other circle, cutting the former in L and A, under the angle BAE, which will be repreſented by the circle SBR, (38) as the circle ELDA is by the right line SC (15). The angle BAE is equal to the angle BRC.

From the point A draw AC, AF, to touch the circles AEL, ABS in A, and meet the plane of projection in C and F; alſo draw RF and CF, which will be in that plane; and, in the plane of the great circle AEL, draw AD parallel to SC, and join ED.

DEM. The angular point A is projected into R; (12) conſequently AC is projected into RC, and AF into RF. And ſince SC is the common ſection of the plane of projection with that of the great circle ELDA (II. 210) the lines AC, SC, AD, ED, AE, lie all in the plane of that circle: Alſo becauſe AD is parallel to SC, the $\angle ARC = \angle DAE = \angle ADE = \angle RAC$ (II. 94, 104, 132) conſequently $AC = RC$ (II. 104). Now the plane paſſing through AC and AF touches the ſphere in A, (51) it is therefore perpendicular to the plane of the circle AED; and FC its common ſection with the plane of projection, is at right angles to that plane (II. 210); FC is therefore at right angles both to the lines AC and CR (II. 205): Hence, the triangles ACF, RCF, being right angled at C, having the ſide FC common, and $AC = CR$, are congruous (II. 99), and the $\angle CAF = \angle CRF$. Conſequently the $\angle EAB = \angle FAC$ (51) $= \angle FRC$. Now it is manifeſt that as AF touches the baſe, ABL, of the cone EABL, in the point A, a plane paſſing through AF and AE will touch the ſide of the cone in the line AE; but AF is alſo in that plane (II. 198); therefore AR touches the cone in the line AE; and as AR lies alſo in the plane of the circle SBR, it muſt touch that circle alſo; conſequently (50) $BRC = \angle FRC = \angle FAC = \angle ABE$.

## 53. PROP. VIII.

*The diſtance between the poles of the primitive circle and an oblique great circle, in ſtereographic projections, is equal to the tangent of half the inclination of thoſe circles; and the diſtance of their centers is equal to the tangent of their inclination: The ſemidiameter of the primitive circle being made radius.* Plate I.

Let AC be the diameter of a circle, the poles of which are P and Q, and inclined to the plane of projection in the angle AIF.

And let $a$, $c$, $p$, be the projections of the points A, C, P.

Alſo let H$a$E be the projected oblique circle, the center of which is $q$.

Now when the plane of projection becomes the primitive circle, the pole of which is I,

Then is $Ip$ = tangent of half $\angle AIF$, or of half the arc AF.

And $Iq$ = tangent of AF, or of the $\angle FHa = AIF$.

DEM. For $AH + HP = AH + AF$. Therefore $HP = AF$.

But $Ip$ = tangent of half HP, or of half AF. (48)

Again. As AC is projected in $ac$, then $q$, the middle of $ac$, is the center of the projected circle, and of its repreſentative H$a$E. (45)
Draw E$q$ produced to $r$: Then as $qa = q\mathrm{E}$; the $\angle q\mathrm{EA} = \angle qa\mathrm{E}$. (II. 104)
But the $\angle qa\mathrm{E}$ is meaſured by half the arc EFA. (II. 137)
Therefore the arc $\mathrm{AH}r$ = arc AFE (II. 50): And as the arc $\mathrm{AHP} = \mathrm{FQE}$;
Therefore $\mathrm{P}r = \mathrm{AF} = \mathrm{HP}$; and $\mathrm{HP}r$ = twice the arc AF.
Therefore (II. 127) the $\angle \mathrm{IE}q = \mathrm{AIF}$, the inclination of the circles.
But I$q$ is the tangent of the $\angle \mathrm{IE}q$, EI being the radius.

54. COROL. Hence the radius of an oblique circle is equal to the ſecant of the obliquity of that circle to the primitive.
For E$q$ is the ſecant of the angle IE$q$, to the radius EI.

## 55. PROP. IX.

*If through any given point in the primitive circle an oblique circle be deſcribed; then the centers of all other oblique circles paſſing through that point, will be in a right line drawn through the center of the firſt oblique circle at right angles to a line paſſing through the given point, and the center of the primitive.*

Let GACE be the primitive circle, ADEI a great circle deſcribed through D, its center being B.

HK is a right line drawn through B, perpendicular to a right line CI paſſing through D, and the center of the primitive circle.

Then the centers of all other great circles FDG paſſing through D will fall in the line HK.

DEM. For if E be the projecting point, the circle EDAI will be the projection of a circle, the diameter of which is NM. (38)

Therefore D and I are the projections of N, M, which are oppoſite points on the ſphere; or of points at a ſemicircle's diſtance.

Therefore all circles paſſing through D and I muſt be the projections of great circles on the ſphere.

But DI is a chord in every circle paſſing through the points D, I.

Conſequently the centers of all thoſe circles will be found in HK drawn perpendicularly through B, the middle of DI. (II. 125)

## 56. PROP. X.

*Equal arcs of any two great circles of the ſphere, will be intercepted between two other circles drawn on the ſphere through the remoteſt poles of thoſe great circles.* Plate I.

Let PBEA be a ſphere, on which AGB, CFD, are two great circles, the remoteſt poles of which are E, P; and through theſe poles let the great circle PBEC, and ſmall circle PGE, be drawn, interſecting the great circles AGB, CFD, in the points B, G, and D, F.

Then are the intercepted arcs BG and DF equal to one another.

DEM. For the arcs $\mathrm{ED} + \mathrm{DB}$ = arcs $\mathrm{PB} + \mathrm{DB}$; therefore $\mathrm{ED} = \mathrm{PB}$.
And the arcs $\mathrm{EF} + \mathrm{FG}$ = arcs $\mathrm{PG} + \mathrm{FG}$ (19); therefore $\mathrm{EF} = \mathrm{PG}$.
For the points F and G are equally diſtant from their poles P, E.
Alſo the $\angle \mathrm{DEF} = \mathrm{BPG}$; for interſecting circles make equal angles at the ſections. (31)
Therefore the triangles EFD and PGB are congruous. (27)
Therefore the arc $\mathrm{BG}$ = arc DF.

57. PROP.

## 57. PROP. XI.

*If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle and plane of projection, the intercepted arcs of thoſe circumferences are equal.* Plate I.

On the plane of projection, AGB, let the great circle CFD be projected into *cfd*, and its pole P in *p*; moreover, draw the lines *pd*, *pf*: the arcs GB and *fd* are equal.

Since *pd* lies both in the plane AGB and APBE it is their common ſection. (II. 198)
But the point B is in their common ſection: (56)
Therefore *pd* paſſes through the point B.
And in this manner it may be proved that *pf* paſſes through G.
Now the points D and F are projected into *d* and *f*. (38)
Therefore the arc *fd* is equivalent to the arc FD.
But the arc FD is equal to the arc GB: (56)
Therefore the arc GB is equivalent to the arc *fd*. (II. 46)

## 58. PROP. XII.

*The radius of any ſmall circle, the plane of which is perpendicular to that of the primitive circle, is equal to the tangent of that leſſer circle's diſtance from its pole; and the ſecant of that diſtance, is equal to the diſtance of the centers of the primitive and leſſer circle.* Plate I.

Let P be the pole, and AB the diameter of a leſſer circle, the plane being perpendicular to the plane of the primitive circle, the center of which is C: Then *d* being the center of the projected leſſer circle, *d*A is equal to the tangent of the arc PA, and $dC =$ ſecant of PA.

DEM. Draw the diameter ED parallel to AB, and through P draw C*b*. Now E being the projecting point, the diameter AB is projected in *ab*. (22)
And *d*, the middle of *ab*, is the center of a circle on *ab*. (39)
Then a right line drawn from D through A, will meet *b*: (II. 130)
And draw CA, *d*A.
Now the right-angled triangles DC*b*, DAE, having the angle D common; the $\angle DbC = \angle DEA$. (II. 98)
But $\angle DEA = \frac{1}{2}\angle DCA$; and $\angle DbC = \frac{1}{2}\angle AdC$: (II. 127)
Therefore $\angle DCA = \angle AdC$.
Now $\angle DCA + \angle ACd =$ a right angle.
Then $\angle AdC + \angle ACD =$ a right angle: Therefore $\angle CAd$ is right. (II. 96)
Conſequently *d*A, $=$ radius of the circle A*a*B, is the tangent of the arc PA, to the radius CA. (II. 126)
And *d*C, the diſtance of the centers, is the ſecant of the arc AP. (III. 5)

59. COROL. Hence the tangent and ſecant of any arc of the primitive circle, belongs alſo to an equal arc of any oblique circle; thoſe arcs being reckoned from their interſection.

For the arc P*c* of every oblique circle intercepted between P and the arc of the ſmall circle A*a*B, is equivalent to the arc PA of the primitive circle: Becauſe the arc A*a*B is equally diſtant from its pole P. (5)

## SECTION II.

## *Spherical Geometry.*

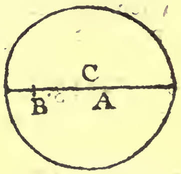
*Spheric Geometry, or ſpheric projection, is the art of deſcribing, or repreſenting, ſuch circles or arcs of circles as are uſually drawn upon a ſphere on the plane of any one of them; and of meaſuring ſuch arcs, and their poſitions to one another, when projected.*

### 60. PROBLEM I.

*To deſcribe a great circle that ſhall paſs through two given points in the primitive circle, or plane of projection.*

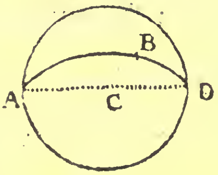
Let the given points be A, B; and C the center of the prim. circle.

CASE 1. *When one point,* A, *is the center of the primitive circle.*



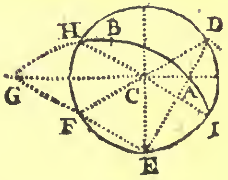
CONST. A diameter drawn through the given points A, B, will be the great circle required. (15)

CASE 2. *When one point* A, *is in the circumference of the primitive circle.*



CONST. Through A draw a diameter AD.
Then an oblique circle deſcribed through the three points A, B, D, (II. 72) will be the great circle required. (42)

61. CASE 3. *When neither point is at the center, or circumference of the primitive circle.*



CONST. Through one point A, and the center C, draw AG, and draw CE at right angles to AG.
A ruler by E and A gives D; by D and C gives F; and by E and F gives G, in AC continued.
Through the three points G, B, A, deſcribe a circumference (II. 72) cutting the primitive circle in H and I.
Then the oblique circle HBAI will be the great circle required.
For AG may be taken as the projection of the great circle FD. (12)
Therefore A and G are the projections of oppoſite points on the ſphere. (32)
Conſequently, all circles paſſing through G and A will be the repreſentatives of great circles on the ſphere.

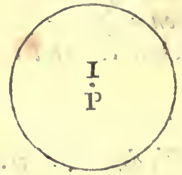
## 62. PROBLEM II.

*About any given point as a pole, to deſcribe a great circle in a given primitive circle.*

Let P be the given point, and I the center of the primitive circle.

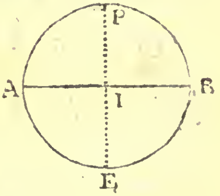
CASE 1. *When the given pole,* P, *is in the center of the primitive circle.*

CONST. The primitive circle will be the great circle required. (13)



CASE 2. *When the given pole,* P, *is in the circumference of the primitive circle.*

CONST. Through the given pole P, draw PE a diameter to the primitive circle.
Then another diam. AB, drawn at right angles to PE, will be the great circle required. (20, 15)

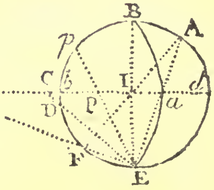


63. CASE 3. *When the given pole,* P, *is neither in the center or circumference of the primitive circle.*

CONST. Through P draw a diameter *bd*, and another BE at right angles to *bd*; then a ruler by E and P gives *p*.

Make the arc *p*A = 90°; a ruler by E and A gives *a* in the diameter *bd*.
Then a circle deſcribed through the three points B, *a*, E, is the great circle required.

Or thus. Make the arc *p*D = arc *p*B; a ruler on E and D gives C in *db* produced.
Then on C, with the radius C*a* deſcribe B*a*E.



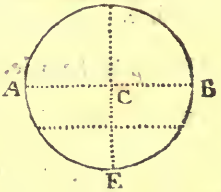
For, As E is the projecting point, and P the projected pole;
Therefore *p* is the pole of the circle AF to be projected. (44)
And B*a*E is the projection of the circle AF. (38)
Now ∠C*a*E is meaſured by half the arc A*d*E. (II. 137)
But arc ABD = arc A*d*E: For A*p* = (B*d* =) *d*E; and *p*D = A*d* by conſtruction.
Therefore ∠AEC = ∠C*a*E; and CE = C*a*. (II. 104)
Conſequently C is the center required.

## 64. PROBLEM III.

*A projected circle being given; to find its poles.*

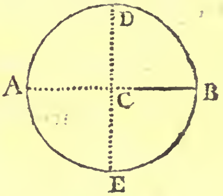
CASE 1. *When the given circle* AEB *is the primitive.*

CONST. Find the center C, (II. 70) and it is the pole ſought.

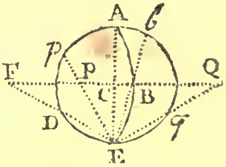


CASE 2. *When the given circle* ACB *is a right circle.*

CONST. Draw a diameter ED at right angles to AB, and the ends or points, D, E, of that diameter are the poles required.



65. CASE 3. *When the given circle* ABE *is oblique.*



CONST. Through the interſections of the primitive and oblique circles draw a diameter AE, and another at right angles to AE, cutting the given oblique circle in B.

A ruler by E and B gives *b*; make *bp*, *bq*, each $=$ an arc of 90°.

A ruler by E and *p* gives, in the diameter through B, the point P, which is the pole required.

And a ruler by E and *q* gives, in CB continued, the point Q for the other, or oppoſite or exterior pole.

Make $pD = pA$; then a ruler by E and D gives, in BC continued, the point F, which is the center of the oblique circle ABE.

THE reaſon of this operation is evident from that of the laſt Problem.

## 66. PROBLEM IV.

*About any given projected pole, to deſcribe a circle at a given diſtance from that pole.*

*Or, at a propoſed diſtance from a given great circle, to deſcribe a parallel circle.*

Let P be the given pole, belonging to the given great circle DFE.

GENERAL SOLUTION. Through the given pole P, and C the center of the primitive circle, draw a diameter, and another DE at right angles to it,

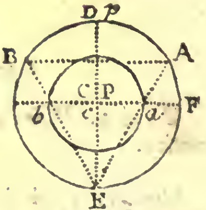
A ruler on E and P gives *p* in the primitive circle.

Make *p*A and *p*B, each equal to the propoſed diſtance from the pole.

A ruler on E and A, and then on E and B, will cut the diameter CP in *a* and *b*.

Biſect *ab* in *c*; and on *c* as a center deſcribe a circle paſſing through *a* and *b*, which will be the circle required.

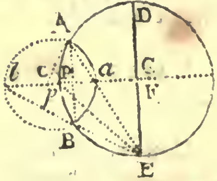
But



But when the parallel circle is to be at a propoſed diſtance from the given great circle DFE,

Find $p$ as before; and make $p$A $=p$B, equal to the complement of the propoſed diſtance; the reſt as before.

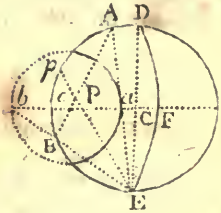
For $p$ is the pole, the projection of which is P. (44)



But $p$ is the pole of a circle, the diameter of which AB is projected in $ab$. (12)

Therefore $c$, the middle of $ab$, is the center of the projected circle. (39)

67. The firſt caſe is readily done, by deſcribing the ſmall circle about the center of the primitive circle with the tangent of half its diſtance from the pole P.



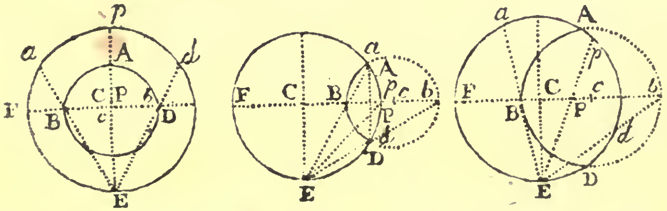
68. The ſecond caſe is ſooneſt performed thus.

From the points A, B, (found as above) with the tangent of their diſtance from P, the pole of the right circle, deſcribe arcs cutting in $c$, which is the center of a ſmall circle parallel to the right circle DFE.

For A$p$ is the tangent of the arc AP. (58)

## 69. PROBLEM V.

*The primitive circle, and the projection of a ſmall circle, being given; to find the pole of that ſmall circle.*



Let C be the center of the primitive circle, and ABD a projected ſmall circle, the center of which is $c$, and radius $c$B.

General Solution. Through $c$ the center of the ſmall circle, and C the center of the primitive, draw a diameter CF, and another, CE, at right angles to it.

Find the projected diameter B$b$ $=2c$B.

Lines drawn from E through B and $b$, cut the primitive circle in $a$, $d$; then biſect the arc $ad$ in $p$.

A ruler by E and $p$ cuts the diameter B$b$ in P, the pole ſought.

The truth of this conſtruction is evident by that of the laſt Prob.

## 70. PROBLEM VI.

*To meaſure any arc of a projected great circle: Or, in a given projected great circle, to take an arc of a given number of degrees.*

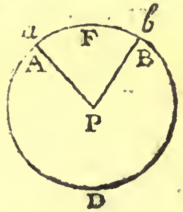
General Solution. Find the pole of the given circle. (64)

From that pole draw lines through the ends of the propoſed arc, cutting the primitive circle.

Then the intercepted arc of the primitive circle applied to the ſcale of chords will give the meaſure ſought.

Thus, if AB be the arc to be meaſured, and P the pole of the given circle DAF.

Then lines drawn from P through A and B, give the arc *ab* in the primitive circle, correſponding to AB in the projected circle.

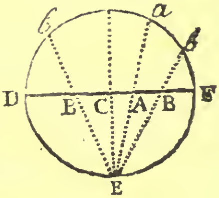


Now if an arc of a given number of degrees was to be taken from a given point A, in the given projected circle DAF.

Draw, from the pole P, through A, the line P*a* to the primitive circle.

Apply the given number of degrees from *a* to *b*.

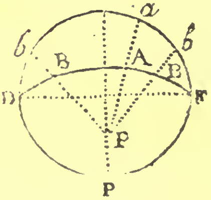
Draw P*b*, and the intercepted arc AB will contain the degrees propoſed.



71. Any number of degrees is readily applied to a right circle by the ſcale of half-tangents. Thus

When the diſtance of the point A from the center C is known, and the given quantity of the arc is to be laid from A towards F;

To the known diſtance CA add the propoſed arc AB, the degrees in the ſum taken from the ſcale of half-tangents, and laid from C to B, will make the arc AB equal to the degrees propoſed.



But when the arc AB is to be laid from A towards D;

Then the difference between the arcs AB and AC, taken from the ſcale of half-tangents and laid towards D from C to B, will make the arc AB equal to the degrees propoſed.

The reaſon of all theſe operations is evident from art. 57.

*Note*, The half, or ſemi-tangents, are only the tangents of half the arcs the ſcale of tangents is made to; their conſtruction depends on art. 48. On the *plane ſcale* they are put under the Tangents, and marked S. T.

## 72. PROBLEM VII.

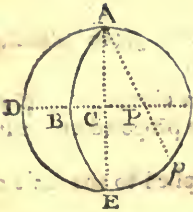
*To meaſure any projected ſpherical angle.*

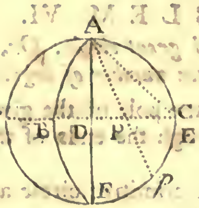
General Solution. Find the poles of the two circles which form the angle (64); and from the angular point draw lines through thoſe poles to cut the primitive circle.

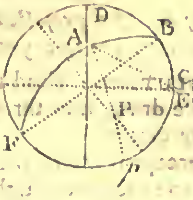
Then the meaſure of that angle, if acute, will be the intercepted arc of the primitive circle; or the ſupplement of that arc will be the meaſure of the angle when obtuſe.

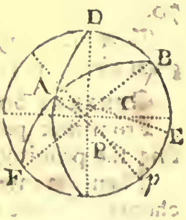
Let the propoſed angle DAB, formed by the great circles AD, AB, the poles of which are C and P; and lines drawn from the angular point A, through the poles C and P, cut the primitive circle in E and *p*.

1ſt.









1ſt. When the angle is formed by the primitive and oblique circles.

Then the arc *p*E meaſures the acute angle DAB.

But the obtuſe angle BAF is meaſured by the ſupplement of *p*E.

2d. When the angle is formed by right and oblique circles meeting in the primitive's circumference.

Then the arc *p*E meaſures the angle DAB.

3d. When the angle is formed by right and oblique circles meeting within the primitive circle.

Then the arc *p*E meaſures the acute angle DAB.

But the obtuſe angle DAF is meaſured by the ſupplement of *p*E.

4th. When the angle is formed by two oblique circles meeting within the primitive circle;

Then the acute angle DAB is meaſured by the arc *p*E.

But the ſupplement of *p*E meaſures the obtuſe angle DAF.

For, as the angular point A is in both circles, and 90° diſtant from their poles C and P (19). Therefore a great circle deſcribed about A, as a pole, will paſs through the poles C and P.

And lines drawn from A through C and P, cut off, in the circumference of the plane of projection, an arc equal to the diſtance of the poles C and P. (57)

But the meaſure of the diſtance of the poles C, P, is equal to the inclination of the planes of the circles AD, AB; (33)

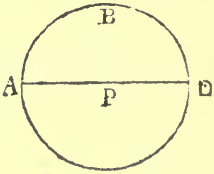
And conſequently meaſures the angle DAB.

## 73. PROBLEM VIII.

*Through a given point in any projected great circle, to deſcribe another great circle at right angles to the given one.*

GENERAL SOLUTION. Find the pole of the given circle. (64)

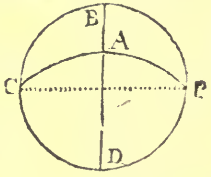
Then a great circle deſcribed through that pole and the given point will be at right angles to the given circle.



Let the given projected great circle be BAD; and A the given point.

1ſt. *When* BAD *is the primitive circle, the pole of which is* P.

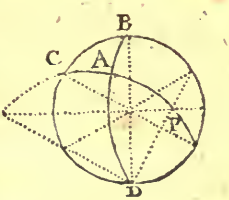
A diameter through A will be perpendicular to BAD. (II. 136)



2d. *When* BAD *is a right circle, the poles of which are* P *and* C.

An oblique circle deſcribed through the points C, A, P, (II. 72) will be at right angles to BAD.

3d. *When* BAD *is an oblique circle, the pole of which is* P.



Through the points P and A, a great circle PAC being deſcribed (61), will be at right angles to BAD.

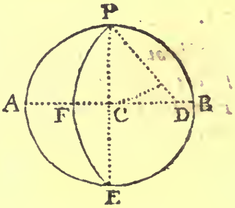
The truth of theſe operations is evident from art. 34.

## 74. PROBLEM IX.

*Through any aſſigned point in a given projected great circle, to deſcribe another great circle cutting the former in an angle of a given number of degrees.*

Let P be a given point in any great circle APB.

1ſt. *When* APB *is the primitive circle.*'



Through the given point P draw a diameter PE, and draw the diameter AB at right angles to PE.

Draw PD cutting AB in D, ſo that the angle CPD be equal to the angle propoſed.

On D with the radius DP deſcribe the great circle PFE.

Then will the angle APF contain the given degrees.

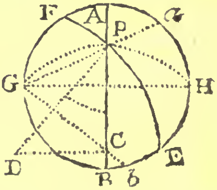
For the ∠FPA=angle made by the radii PC, PD. (49)

And D being equally diſtant from P and E, is the center ſought.

75. Or thus. Make CD equal to the tangent of the given angle to the radius CP.

Or, Make PD equal to the ſecant of that angle.

76. 2d. *When* APB *is a right circle.*



Draw a diameter GH at right angles to APB.

Then a ruler by G and P gives *a* in the primitive circle.

Make H*b*=2A*a*; a ruler by G and *b* gives C in AB.

Draw CD at right angles to AB.

Draw PD cutting CD in D, ſo that the ∠CPD= complement of the degrees given. (II. 84)

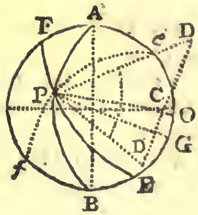
On D with the radius DP deſcribe a circle FPE, which will be a great circle making with APB the angle APF as required.

For, C is the center of a great circle GPH, by the demonſtration to art. 63.

And the centers of all great circles through P, will be in CD. (55)

Now ∠DPE = 90°. (II. 136)

Therefore ∠APF or ∠BPE (26), the compl. of CPD, is the angle ſought.



77. 3d. *When* APB *is an oblique circle.*

From the given point P, draw the lines PG, PC, through the centers of the primitive and given oblique circles, and through C the center of APB draw CD at right angles to PG. (II. 59)

Draw PD, making the ∠CPD = given degrees, and cutting CD in D. (II. 84)

From D with the radius DP, a circle FPE being deſcribed, will be a great circle cutting APB in the angle propoſed.

For C, the center of APD, is in a line perpendicular to PG, drawn through P and the center of the primitive, by conſtruction.

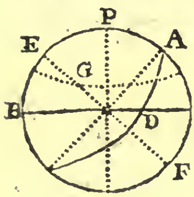
And the centers of all great circles through P will be in CD. (55)

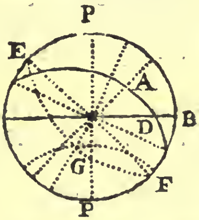
Now the ∠CPD made by the radii PC, PD, contains the given degrees.

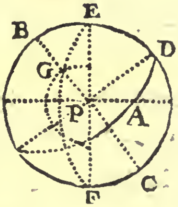
Therefore the angle APF is equal to the angle required. (49)

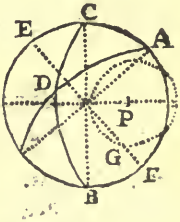
## 78. PROBLEM X.

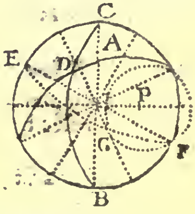
*Through any point in the plane of projection, or primitive circle, to deſcribe a great circle that ſhall cut a given great circle in any angle propoſed: Provided the meaſure of that propoſed angle is not leſs than the diſtance between the given point and circle.*











Let the given point be A, through which a circle is to be deſcribed to cut a given great circle BDC, the pole of which is P, in an angle equal to a propoſed number of degrees.

General Solution. About the given point A, as a pole, deſcribe a great circle EGF. (62)

About P, the pole of the given circle BDC, deſcribe a ſmall circle at a diſtance equal to the given angle, cutting the great circle EGF in G. (66)

About the point G, as a pole (62), deſcribe a great circle cutting the given circle BDC in D. Then will ADC be the angle required.

*Note*, When the given angle is equal to the diſtance between the given point and circle, the problem is limited to one anſwer only: When the meaſure of the angle is greater, the problem has two ſolutions by the circle deſcribed cutting the given one in two points: But when the meaſure of the angle is leſs, the problem is impoſſible. This conſtruction is thus proved.

P and G are the poles of BC and AD.

And the diſtance of P and G is equal to the degrees in the propoſed angle, by conſtruction. But $\angle ADC =$ diſtance of P and G. (33)

Therefore the $\angle ADC$ is the angle required.

79. *When the required circle is to make a given angle with the primitive.*

Then, from the center of the primitive, with the tang. of the given angle, deſcribe an arc; and from the given point A, with the ſecant of the given angle, cut the former arc.

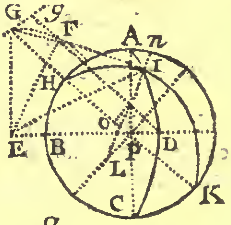
On this interſection, a circle being deſcribed through the given point A, will cut the primitive circle in the angle propoſed.

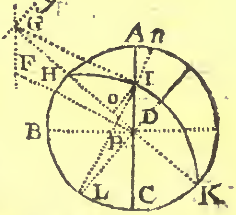
THIS depends on art. 75.

## 80. PROBLEM XI.

*Any great circle, cutting the primitive, being given, to deſcribe another great circle, which ſhall cut the given one in a propoſed angle, and have a given arc intercepted between the primitive and given circles.*

Let ABC be the primitive circle, the center of which is P; and the given great circle be ADC, the center of which is E.





SOLUTION. Draw a diameter EBD at right angles to ADC; and make the angle BDF equal to the complement of the given angle; ſuppoſe = complement of 35°.

Make DF equal to the tangent of the given arc (ſuppoſe 58°); and from P, with the ſecant of that arc, deſcribe an arc *Gg*.

Now when ADC is an oblique circle; from E the center of ADC, with the radius EF, cut the arc *Gg* in G.

But when ADC is a right circle; through F draw FG parallel to ADC, cutting the arc *Gg* in G.

From G, with the tangent DF, deſcribe an arc, *no*, cutting ADC in I; and draw GI.

Through G and the center P draw GK, cutting the primitive circle in H, K; draw PL perpendicular to GK; and IL at right angles to IG, cutting PL in L.

And L will be the center of a circle paſſing through H, I, K, which will be the great circle required.

Then the $\angle AIH = 35°$; and arc $IH = 58°$, as was propoſed.

For GP is the ſecant, and GI is the tang. of the arc HI. (59)

And as the triangles EGI, EFD, are congruous; the $\angle EIG = \angle EDF$. (II. 101)

But the $\angle EIG$ made by the tangent of the arc HI and the radius of the arc AI, is the complement of the angle made by thoſe arcs. (49)

Conſequently the $\angle AIH$ is the complement of the $\angle EDF$.

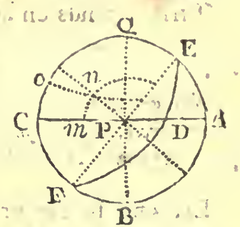
The center of the right circle AC being ſuppoſed at an infinite diſtance, therefore any circle FG deſcribed from that center, will be parallel to AC.

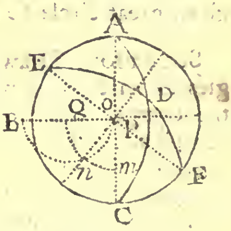
When the given arc is more than 90°, the tangent and ſecant of its ſupplement is to be applied on the line DF the contrary way, or towards the right; the former conſtruction being reckoned to the left.

## 81. PROBLEM XII.

*Any great circle in the plane of projection being given; to deſcribe another great circle, which ſhall make given angles with the primitive and given circles.*

Let the given great circle be ADC, and its pole Q.





SOLUTION. About P, the pole of the primitive circle, deſcribe an arc *mn*, at the diſtance of as many degrees as are in the angle which the required circle is to make with the primitive: Suppoſe 62°. (67)

About Q, the pole of the other given circle, and at a diſtance equal to the meaſure of the angle which the required circle is to make with the given circle ADC (ſuppoſe 48°), deſcribe an arc *on*, cutting *mn* in *n*. (66)

About *n*, as a pole, deſcribe the great circle EDF, cutting the given circles in E and D. (62)

Then is the angle AED = 62°; and ADE = 48°.

FOR the diſtance of the poles of any two great circles, is equal to the angle which thoſe circles make with one another. (33)

REMARK. The 11th Problem, which is particularly uſeful in conſtructing a ſpherical triangle, *in which are given two angles and a ſide oppoſite to one of them*, includes only two caſes of a more general Problem, viz.

*Any two great circles being given in poſition; to deſcribe a third, which ſhall cut one of thoſe given in an angle propoſed, and have a given arc intercepted between the given circles.*

Alſo the 12th Problem, uſed when *the three angles are given*, contains only two caſes of another Problem; viz.

*Any two great circles being given in poſition, to deſcribe a third which ſhall cut the given circles in given angles.*

The ſolution of theſe two general Problems not being wanted in any part of this work; it was not thought neceſſary here to annex them; more having been already delivered in the preceding pages than it is uſual to meet with on this ſubject. However, their ſolution is recommended as exerciſes to ſpeculative learners.

# SECTION IV.

## *Spheric Trigonometry.*

### DEFINITIONS.

82. SPHERIC TRIGONOMETRY is the art of computing the meaſures of the ſides and angles of ſuch triangles as are formed on the ſurface of a ſphere, by the mutual interſections of three great circles deſcribed thereon.

83. A SPHERIC TRIANGLE conſiſts of three ſides and three angles.

The meaſures of unknown ſides or angles of ſpheric triangles are eſtimated by the relations between the ſines, or the tangents, or the ſecants, of the ſides or angles known, and of thoſe that are unknown.

84. A RIGHT ANGLED SPHERIC TRIANGLE has one right angle: The ſides about the right angle are called Legs; and the ſide oppoſite to the right angle is called the Hypothenuſe.

85. A QUADRANTAL SPHERIC TRIANGLE has one ſide equal to ninety degrees.

86. An OBLIQUE SPHERIC TRIANGLE has all its angles oblique.

87. The CIRCULAR PARTS of a triangle, are the arcs which meaſure its ſides and angles.

88. Two ſpheric triangles are ſaid to be ſupplements to one another, when the ſides and angles of the one are reſpective ſupplements of the angles and ſides of the other: And one, in regard to the other, is called the ſupplemental triangle.

89. Two arcs or angles, when compared together, are ſaid to be alike, or of the ſame kind, when both are acute, or leſs than 90°, or when both are obtuſe, or greater than 90°: But when one is greater and the other leſs than 90°, they are ſaid to be unlike.

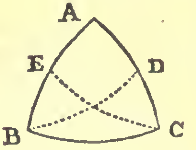
The leſſer circles of the ſphere do not enter into Trigonometrical computations, becauſe of the diverſity of their radii.

## SECTION V.

## *Spherical Theorems.*

### 90. THEOREM I.

*In every ſpheric triangle,* ABC, *equal angles,* B, C, *are oppoſite to equal ſides,* AC, AB: *And equal ſides,* AB, AC, *are oppoſite to equal angles,* C, B.



DEM. Since AB=AC, make AE=AD; and draw BD, CE.
Then is BD=CE; and ∠AEC=∠ADB. (27)
For the triangles AEC, ADB are congruous,
Since AB=AC; AD=AE; ∠A common.
Alſo, the triangles BEC, CDB are congruous; (27)
Therefore ∠EBC=∠DCB.
For EC=BD; EB=(AB−AE=) DC (=AC−AD). (II. 48)
And ∠BEC=∠CDB, they being the ſuppl. of equal angles AEC, ADB.
Again, if ∠ABC=∠ACB: Then is AB=AC.
For take BE=CD; and deſcribe the arcs CE, BD.
Then is EC=DB, ∠BEC=∠CDB; ∠BCE=∠CBD. (27)
For △[*] BCE=△CBD; ſince BC is common, BE=CD, ∠EBC=∠DCB.
Alſo the triangles ABD, ACE are congruous;
Since EC=DB, ∠ACE=(∠BCA−BCE=) ∠ABD (=∠CBA−CBD). (II. 48)
And ∠AEC=(ſup. ∠BEC=) ∠ADB (=ſup. ∠CDB). (II. 48)
Therefore AE=AD; and AB=(AE+EB=) AC (=AD+DC). (II. 47)

91. COROL. A line drawn from the vertex of an iſoſceles ſpheric triangle, to the middle of the baſe, is perpendicular to the baſe.
This is eaſily proved from art. 90, 27.

### 92. THEOREM II.

*Either ſide of a ſpheric triangle is leſs than the ſum of the other two ſides.*

DEM. For on the ſurface of the ſphere, the ſhorteſt diſtance between two points, is an arc of a great circle paſſing through thoſe points. (23)
But each ſide of a ſpheric triangle is an arc of a great circle. (10)
Therefore either ſide being the ſhorteſt diſtance between its extremities, is leſs than the ſum of the other two ſides.

### 93. THEOREM III.

*Each ſide of a ſpheric triangle is leſs than a ſemicircle, or* 180 *degrees.*

DEM. Two great circles interſect each other twice at the diſtance of 180 degrees. (32)
The ſides about any ſpheric angle are arcs of two great circles. (10)
But a ſpheric triangle has three ſides.
Therefore every two ſides before their ſecond meeting muſt be interſected by the third ſide.
Conſequently each ſide is leſs than a ſemicircle.

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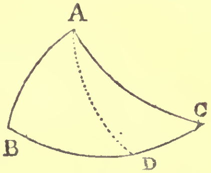
[*] The mark △ ſtands for the word triangle.

## 94. THEOREM IV.

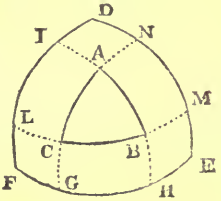
*In every ſpheric triangle,* ABC, *the greateſt ſide,* BC, *is oppoſite the greateſt angle,* A.



DEM. Make ∠BAD = ∠ABC.
Then AD = BD (90); and BC = AD + DC.
But AD + DC is greater than AC. (92)
Therefore BC is greater than AC.

## 95. THEOREM V.

*If from the three angles of a ſpheric triangle,* ABC, *as poles, be deſcribed three arcs of great circles, forming another ſpheric triangle,* EDF; *then will the ſides of the latter, and the oppoſite angles of the former, be the ſupplements of one another: Alſo the angles in the latter, and their oppoſite ſides in the former, are the ſupplements of one another.*



That is, FE and ∠CAB, FD and ∠ABC, DE and ∠ACB, are ſupplements to one another.
Alſo ∠E and AC, ∠D and CB, ∠F and AB, are the ſupplements to one another.
DEM. The interſection E of the arcs about the poles A and C, being 90° diſtant from them, is the pole of the arc AC. (19)

And for the ſame reaſon, D is the pole of CB, and F of AB.

Let the ſides of the triangle ABC be produced to meet the ſides of the triangle DEF in G and H, I and L, M and N.

Then FI = DL = 90°: Therefore (DL + FI = DL + FL + LI =) DF + LI = 180°. (II. 47)

Therefore DF and LI are ſupplements to one another.

But LI meaſures the angle ABC. (9)

Therefore ∠ABC and DF are the ſupplements to one another.
And in the ſame manner it may be demonſtrated, that the ∠BAC and FE, ∠ACB and DE, are the ſupplements of one another.

Again, ſince BI = AH = 90 degrees; (19)

Therefore (IB + AH = IB + BH + AB =) IH + AB = 180 degrees.

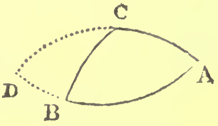
But IH meaſures the angle F; (9)

Therefore AB and ∠F are the ſupplements of one another.

And the ſame may be ſhewn of AC and ∠E, CB and ∠D.

## 96. THEOREM VI.

*The ſum of the three ſides of every ſpheric triangle,* ABC, *is leſs than a circumference, or 360 degrees.*



DEM. Continue the ſides AC, AB, till they meet in D.
Then the arcs ACD, ABD, are each 180°. (32)
But DC + DB is greater than BC. (92)
Therefore AC + AB + DC + DB is greater than AC + AB + BC.

Or the ſemicircles ACD + ABD is greater than AC + AB + BC.

That is, 360° is greater than the three ſides of the triangle ABC.

### 97. THEOREM VII.

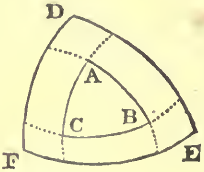
*The ſum of the three angles of a ſpheric triangle,* ABC, *is greater than two right angles, and leſs than ſix; or will always fall between* 180 *and* 540 *degrees.*

DEM. Since ∠A and FE, ∠B and FD, ∠C and DE, are ſupplements to one another. (95)

Therefore the three angles A, B, C, together with the three ſides FE, FD, DE make thrice 180°, or 540°.

Now the ſum of the three ſides FE+FD+DE, is leſs than twice 180°. (96)

Therefore the ſum of the three angles A+B+C is greater than 180°.



Again, as a ſpheric angle is ever leſs than 180; (24)

Therefore the ſum of any three ſpheric angles is ever leſs than thrice 180°, or 540 degrees.

### 98. THEOREM VIII.

*If one ſide,* AB, *of a ſpheric triangle,* ABC, *be produced, then the outward angle,* CBD, *is either equal to, leſs, or greater than the inward oppoſite angle* A, *adjacent to that ſide; according as the ſum of the other two ſides,* CA+CB, *is equal to, greater, or leſs than* 180 *degrees.*

DEM. Produce AC, AB, to meet in D.

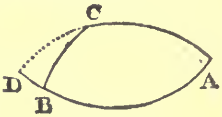
Then arc ACD=arc ABD=180°. (32)

And ∠D=∠A. (31)

Now if AC+CB is equal to 180°; then CB=CD.

And ∠CBD=(∠D=) ∠A. (90)

If AC+CB is greater than 180°; then CB is greater than CD.



And ∠CBD is leſs than (∠D=) ∠A. (94)

If AC+CB is leſs than 180°; then CB is leſs than CD. (94)

And ∠CBD is greater than (∠D=) ∠A.

### 99. THEOREM IX.

*In right angled ſpheric triangles, the oblique angles and their oppoſite ſides are of the ſame kind: That is, if a leg is leſs or greater than* 90°, *its oppoſite angle is alſo leſs or greater than* 90°.

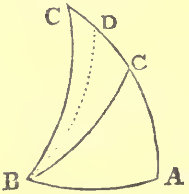
In the right angled ſpheric triangle ABC, right angled at A.

If AC is greater than 90°; then ∠ABC is greater than 90°.

If AC is leſs than 90°; then ∠ABC is leſs than 90°.

DEM. Let the leg AC be leſs, AD equal, AC greater, than 90°, and deſcribe the arc DB.

Now D being the pole of AB (37). Therefore ∠DBA is right.



Conſequently if AC is leſs than AD, the ∠CBA is leſs than ∠DBA.

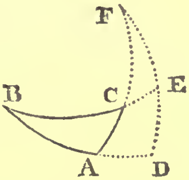
But if AC is greater than AD, the ∠CBA is greater than ∠DBA.

And the ſame may be proved of the leg AB and its oppoſite angle.

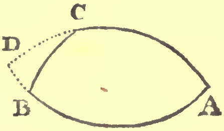
## 100. THEOREM X.

*In right angled ſpheric triangles*, BAC, *the hypothenuſe*, BC, *is leſs than* 90°, *when the legs*, AB, AC, *are of a like kind: But the hypothenuſe is greater than* 90°, *when the legs are of different kinds.*

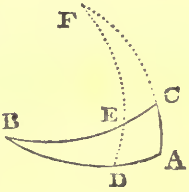
1ſt. When the legs AB, AC, are both leſs than 90°.
DEM. In BA, AC produced, take BD, AF equal to quadrants; through F and D deſcribe an arc FD meeting BC produced in E.
Now F being the pole of BD (19). Therefore B is the pole of ED.
Conſequently BC is leſs than (BE=) 90°.



2d. When the legs AB, AC, are both greater than 90°.
Produce AC, AB, till they meet in D.
Now the hypothenuſe CB is common to both the right angled triangles BAC and BDC.
And the legs DC, DB, being both leſs than 90°.
Therefore the hypothenuſe BC is leſs than 90°, by the firſt caſe of this Theorem.



3d. When the legs AB, AC, are one greater, the other leſs than 90°.
In AB, and AC produced, take BD, AF each of 90°, and deſcribe the arc FED.
Then B is the pole of FD; and ſince F is the pole of BA, and FD is at right angles to BD. Therefore BE=90°. (37)
Conſequently BC is greater than 90 degrees.



101. COROL. I. The hypothenuſe is leſs or greater than 90°, according as the oblique angles are of a like, or different kinds.
For if legs are like, or unlike, the angles are like or unlike. (99)
And if legs are like, or unlike, the hypoth. is acute or obtuſe. (100)
Therefore if the angles are like, the hypothenuſe is acute, or leſs than 90°; but if unlike, the hypothenuſe is obtuſe, or greater than 90°.

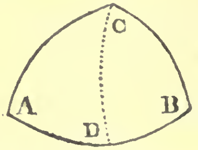
102. COROL. II. The legs and their adjacent angles are like, or unlike, as the hypothenuſe is leſs, or greater than 90 degrees.
For like legs, or like angles, make the hypothenuſe acute (by 1ſt and 2d of 100).
And unlike legs, or unlike angles, make the hypothenuſe obtuſe (by 3d of 100 and by 101).

103. COROL. III. A leg and its oppoſite angle are both acute, or both obtuſe, according as the hypothenuſe and other leg are like, or unlike.
This is evident from the three caſes of this Theorem.

104. COROL. IV. Either angle is acute, or obtuſe, as the hypothenuſe and the other angle are like, or unlike.
This follows from caſe 1ſt and 2d of this Theorem.

## 105. THEOREM XI.

*In every ſpheric triangle,* ABC, *if the angles adjacent to either ſide,* AB, *be alike, then a perpendicular,* CD, *drawn to that ſide from the other angle, will fall within the triangle: But the perpendicular* CD *falls without the triangle, when the angles adjacent to the ſide* AB *it falls on are unlike.*

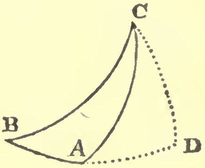


DEMONST. Since in all right angled triangles the perpendicular and its oppoſite angle are of the ſame kind. (99)

Therefore the ∠s CAD, CBD, are each like CD.

Now in Fig. 1. the angles CAD, CBD, or CAB, CBA, are angles adjacent to the baſe AB within the triangle, and are therefore alike.

Therefore the perpendicular falling between A and B, falls within the triangle.



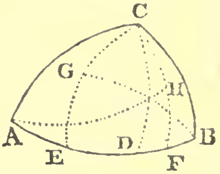
In Fig. 2. the angles CAD and CAB are the ſupplements of each other, and are therefore unlike, as CA falls obliquely on AB.

Therefore ∠CAB is unlike to ∠CBA.

Conſequently the perpendicular CD cannot fall between A and B: Therefore it muſt fall without.

## 106. THEOREM XII.

*If the two leſſer ſides,* CA, CB, *of a ſpheric triangle,* ABC, *are of the ſame kind; then an arc,* CD, *drawn from their included angle,* ACB, *perpendicular to the oppoſite ſide,* AB, *will fall within the triangle.*



DEMONST. In AB take AF=AC; draw CF and AH at right angles to CF.

Then CH=HF (91) are each leſs than 90°. (93)

Alſo take BE=BC; draw CE and BG at right angles to CE.

Then CG=GE (91) are each leſs than 90°. (93)

Now in the right angled triangles FHA, EGB; if the hypothenuſes AF (=AC), and BE (=BC), are acute, or like FH and EG;

Then the angles AFH and BEG are acute, and like AC and BC; (103)

Therefore the perpendicular CD falls on EF, within the triangle. (105)

Alſo if the hypothenuſes AF and BE are obtuſe, or unlike to FH and GE;

Then the angles AFH and BEG are obtuſe, and alſo like CA and CB. (103)

Conſequently the perpendicular will fall on EF. (105)

Therefore in either caſe the perpendicular falls within the triangle.

## 107. THEOREM XIII.

*In all right angled ſpheric triangles,*

*As ſine hypoth. : Rad : : ſine of a leg : ſine of its oppoſite angle.*

108. *And ſine a leg : Rad. : : tan. other leg : tan. of its oppoſite angle.* Pl. I.

DEMONST. Let FDAFG repreſent the eighth part of a ſphere, where the quadrantal planes EDFG, EDBC, are both perpendicular to the quadrantal plane ADFB; and the quadrantal plane ADGC is perpendicular to the quadrantal plane EDFG: and the ſpheric triangle ABC is right angled at B, where CA is the hypothenuſe, and BA, BC, are the legs.

To the arcs GF, CE, draw the tangents HF, OB, and the ſines GM, CI, on the radii DF, DB; alſo draw BL, the ſine of the arc AB, and CK, the ſine of AC; then join IK and OL.

Now HF, OB, GM, CI, are all perpendicular to the plane ADFB.

And HD, CK, OL, lie all in the ſame plane ADGC.

Alſo FD, IK, BL, lie all in the ſame plane ADFB.

Therefore the right angled triangles HFD, CIK, OBL, having the equal angles HDF, CKI, OLB, (II. 199) are ſimilar. (II. 167)

Therefore CK : DG :: CI : GM.

That is, As ſin. hyp. : Rad. :: ſin. of a leg : ſin. opp. angle.

For GM is the ſine of the arc GF, which meaſures the angle CAB. (9)

Alſo, As LB : DF :: BO : FH.

That is, As ſin. of a leg : Rad. :: tan. of other leg : tan. opp. angle.

## 109. THEOREM XIV.

*In right angled ſpheric triangles,* ABC, *if about the oblique angles,* A, C, *as poles, at 90° diſtance, there be deſcribed arcs,* DE, FE, *cutting one another in* E; *and the ſides* AB, AC, BC, *of the triangle be produced to cut thoſe arcs in* D; G, F; H, I; *there will be conſtituted two other triangles,* CGH, HIE, *the parts of which are either equal to, or are the complements of, the parts of the given triangle,* ABC. Pl. I.

DEMONST. Now ſince A is the pole of ED (19). Therefore AD, AG are at right angles to ED; and ſo is ED to AD. (37)

And ſince BI and DE are at right angles to AD, their interſection H is the pole of AD (36). Therefore HB, HD are quadrants. (35)

Then in the triangle CGH, right angled at G.

CG = complement of AC.

HG = comp. ∠A; For HG is the comp. of GD, which mea. ∠A. (9)

HC the hypoth. is the comp. of CB.

The ∠HCG = ∠ACB. (26)

The ∠CHG = comp. AB: For BD, the comp. of AB, meaſures ∠CHG.

Alſo in the triangle EIH, right angled at I: Becauſe CF, CI are at right angles to EF; and EF, EG being alſo at right angles to AF; therefore E is the pole of AF; (36) conſequently EF and EG are quadrants. (35)

Then the hypoth. EH = ∠A; For GH = comp. of EH and GD; and GD meaſures the angle A.

HI = CB; for HC = comp. of HI and CB.

EI = comp. ∠C; For EI = comp. of IF, which meaſures ∠C.

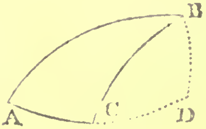
The ∠H = comp. AB; For BD, the comp of AB, meaſures ∠H.

The ∠E = AC; For GF, the meaſure of ∠E, is equal to ∠AC.

## 110. THEOREM XV.

*In every ſpheric triangle, it will be,*
*As the ſine of either angle, is to the ſine of its oppoſite ſide;*
*So is the ſine of another angle, to the ſine of its oppoſite ſide.*

Let ABC be a ſpheric triangle, where BD is perpendicular to AC produced; forming the two right angled triangles ADB, CDB.



DEM. Now as ſin. AB : rad. :: ſin. BD : ſin. ∠A. (167)

And ſin. BC : rad. :: ſin. BD : ſin. ∠C.

Therefore ſin. AB × ſin. ∠A = rad. × ſin. BD. (II. 162)

And ſin. BC × ſin. ∠C = rad. × ſin. BD. (II. 162)

Therefore ſin. AB × ſin. ∠A = ſin. BC × ſin. ∠C. (II. 46)

Therefore ſin. ∠A : ſin. BC :: ſin. ∠C : ſin. AB. (II. 165)

## SECTION VI.

## *Of the Solution of right angled ſpheric Triangles.*

In every caſe of right angled ſpheric triangles, three things beſide the radius enter the proportion, of which two are given, and the third is ſought.

Now the ſolution of every caſe will be obtained by the application of the two following rules to Theorem XIII. and XIV. (107, 108, 109.)

111. RULE I. If of the three things concerned, or their complements, two are oppoſite to one another, and the third is oppoſite to the right angle, in one of the triangles marked 1, 2, 3, in the fig. to Theo. XIV. Pl. I. Then the thing ſought will be found by the firſt proportion (107) either directly, or by inverſion.

112. RULE II. If of the three things concerned, or their complements, two are ſides, and the third is an oblique angle, in either of the three triangles marked 1, 2, 3, in fig. to Theo. XIV. Pl. I. Then the thing ſought will be found by the ſecond proportion (108) either directly, or by inverſion.

### 113. PROBLEM I.

In the right angled ſpheric triangle ABC, Plate I. Theorem XIV.

Given the hypothenuſe AC } required the reſt.
and one of the legs AB }

1ſt. *To find the angle* ACB *oppoſite the given leg* AB.

Here the things concerned are AC, ∠B, AB, ∠C; which are found in the triangle, N° 1, to be oppoſite; and ſo fall under Rule I. (111)

Then ſin. AC : rad. : : ſin. AB : ſin. ∠ACB. (107)

Or ſin. hyp. : rad. : : ſin. g. leg : ſin. op. ∠. Like the g. leg. (99)

2d. *To find the angle* CAB *adjacent to the given leg* AB.

Here the things concerned are AC, ∠B, AB, ∠A.

Now trying in the triangle, N° 1, I find the things concerned will fall under neither of the Rules.

But trying in the triangle, N° 2, the things concerned, or their complements, fall under Rule II. (112)

Then ſin. HG : rad. : : tan. GC : tan. ∠CHG. (108)

Or co-ſ. ∠CAB : rad. : : co-t. AC : co-t. AB.

Or co-ſ. ∠CAB : co-t. AC : : (rad. : co-t. AB) : : tan. AB : rad. (III. 36)

Therefore rad. : co-t. hyp. : : tan. g. leg : co-ſ. adj. angle. (II. 145)

Like, or unlike the given leg; as the hyp. is acute, or obtuſe. (102)

3d. *To find the other leg* BC.

Here the things concerned are AC, ∠B, AB, BC; which in the triangle, N° 1, do not fall under either Rule: But in N° 2 they will be found to fall under the firſt Rule. (111)

Then ſin. HE : rad. : : ſin. CG : ſin. ∠CHG. (107)

Or co-ſ. CB : rad. : : co-ſ. AC : co-ſ. AB.

Therefore co-ſ. g. leg, AB, : rad. : : co-ſ. hyp. AC, : co-ſ. req. leg CB.

And is acute, if hyp. and given leg are like; but obtuſe, if unlike. (103)

## 114. PROBLEM II.

In the right angled ſpheric triangle ABC. Pl. I. Theorem XIV.
Given the hypothenuſe AC
And one of the oblique angles A } Required the reſt.

1ſt. *To find the leg* CB *oppoſite to the given angle* A.
In the triangle, N° 1. the things concerned fall under Rule I. (111)
Then rad. : ſin. AC : : ſin. ∠CAB : ſin. CB. (107)
Or rad. : ſin. hyp. : : ſin. given angle : ſin. opp. ſide.
And is like the given angle. (99)

2d. *To find the leg* AB *adjacent to the given angle* A.
In the triangle, N° 2. the things concerned fall under Rule II. (112)
Then ſin. HG : rad. : : tan. CG : tan. ∠CHG. (108)
Or coſ. ∠BAC : rad. : : (co-t. AC : co-t. AB : :) tan. AB : tan. AC. (III. 37)
Therefore rad. : tan. AC : : co-ſ. ∠BAC : tan. AB.
Or rad. : tan. hyp. : : co-ſ. given angle : tan. adjacent leg.
And is acute, if hyp. and given angle are alike; but obtuſe if unlike. (104)

3d. *To find the other angle* ACB.
In the triangle, N° 2, the things concerned fall under Rule II. (112)
Then ſin. CG : rad. : : tan. GH : tan. ∠HCG. (108)
Or co-ſ. AC : rad. : : co-t. ∠ BAC : tan. ∠BCA : :) co-t. ∠ BCA. : tan. ∠BAC. (III. 37)
Therefore rad. : tan. ∠BAC : : co-ſ. AC : co-t. ∠BCA. (II. 145)
Or rad. : co-ſ. hyp. : : tan. given angle : co-t. req. angle.
And is acute, if hyp. and given angle are alike; but obtuſe, if unlike. (104)

## 115. PROBLEM III.

In the right angled ſpheric triangle ABC. Plate I. Theorem XIV.
Given one of the legs AB
And its oppoſite angle ACB } Required the reſt.

1ſt. *To find the hypothenuſe* AC.
In the triangle, N° 1. the things concerned fall under Rule I. (111)
Then ſin. ∠ACB : ſin. AB : : rad. : ſin. AC. (107)
Or ſin. given angle : ſin. given leg : : rad. : ſin. hyp.
And is either acute or obtuſe.

2d. *To find the other leg* CB.
In the triangle, N° 1. the things concerned fall under Rule II. (112)
Then ſin. CB : (rad. : : ) tan. AB (: tan. ∠ ACB) : : co-t. ∠ ACB : rad. (III. 36)
Or rad. : co-t. given angle : : tan. given leg : ſin. req. leg.
And is either acute or obtuſe.

3d. *To find the other angle* CAB.
In the triangle, N° 3. the things concerned fall under Rule I. (111)
Then ſin. EH : rad. : : ſin. EI : ſin ∠ IHE. (107)
Or ſin. ∠BAC : rad. : : co-ſ. ∠ ACB : co-ſ. AB.
Or co-ſ given leg : co-ſ. given angle : : rad. : ſin. required angle.
And is either acute or obtuſe.

## 116. PROBLEM IV.

In the right angled ſpheric triangle ABC, Plate I. Theorem XIV.
Give one of the legs AB } Required the reſt.
And its adjacent angle BAC }

1ſt. *To find the other angle* BCA.
In the triangle N° 3. the things concerned fall under Rule I. (111)
Then rad. : ſin. EH : : ſin. ∠EHI : ſin. EI. (107)
Or rad. : ſin. ∠BAC : : co-ſ. AB : co-ſ. ∠ACB.
Therefore rad. : co-ſ. given leg : : ſin. given angle : co-ſ. req. angle.
And is like the given leg. (99)

2d. *To find the other leg* BC.
In the triangle, N° 1. the things concerned fall under Rule II. (112)
Then ſin. AB : rad. : : tan. BC : tan. ∠CAB. (108)
Or rad. : ſin. AB : : tan. ∠CAB : tan. BC.
Therefore rad. : ſin. given leg : : tan. given angle : tan. req. leg.
And is like the given angle. (99)

3d. *To find the hypothenuſe* AC.
In the triangle, N° 2. the things concerned fall under Rule II. (112)
Then ſin. GH : rad. : : tan. CG : tan. ∠CHG. (108)
Or co-ſ. ∠CAB : rad. : : co-t. AC : co-t. AB.
Therefore rad. : co-ſ. given angle : : co-t. given leg : co-t. hpothenuſe.
And is acute, if the given leg and angle are alike; but obtuſe, if unlike. (102)

## 117. PROBLEM V.

In the right angled ſpheric triangle ABC, Plate I. Theorem XIV.
Given both the legs AB, BC.
Required the reſt.

1ſt. *To find either of the oblique angles, as* BAC.
In the triangle, N° 1. the things concerned fall under Rule II. (112)
Then, as ſin. AB : rad. : : tan. BC : tan. ∠BAC. (108)
Or rad. : ſin. AB : : (tan. ∠BAC : tan. BC : : ) co-t. BC : co-t. ∠BAC. (III. 37)
Therefore rad. : ſin. one leg : : co-t. oth. leg : co-t. opp. angle.
And is like its oppoſite leg. (99)

2d. *To find the hypothenuſe* AC.
In the triangle, N° 2. the things concerned fall under Rule I. (111)
Then, As rad. : ſin. HC : : ſin. ∠CHG : ſin. CG. (107)
Or rad. : co ſ. BC : : co-ſ AB : co-ſ. AC.
Therefore rad. : co-ſ. one leg : : co-ſ. oth. leg : co-ſ. hypothenuſe.
And is acute, if the legs are alike; but obtuſe, if unlike. (100)

## 118. PROBLEM VI.

In the right angled ſpherical triangle ABC, Plate I. Theorem XIV.
Given both the angles BAC, BCA.
Required the reſt.

1ſt. *To find either of the legs, as* BC.
In the triangle, N° 2 or 3. the things concerned fall under Rule I. (111)
Then, rad. : ſin. HC : : ſin. ∠HCG : ſin. HG. (107)
Or rad. : co-ſ. BC : : ſin. ∠ACB : co-ſ. ∠BAC.
Therefore ſine of one angle : rad. : : co-ſ. oth. angle : co-ſ. oppoſite ſide.
And is like its oppoſite angle. (99)

2d. *To find the hypothenuſe* AC.
In the triangle, N° 2. the things concerned fall under Rule II. (112)
Then, As ſin. CG : rad. : : tan. GH : tan. ∠HCG. (108)
Or co-ſ. AC : (rad. : :) co-t. ∠BAC (: tan. ∠BCA) : : co-t. ∠BCA : rad. (III. 36)
Therefore rad. : co-t. one angle : : co-t. oth. angle : co-ſ. hypothenuſe.
And is acute, if the angles are like. (101)
But obtuſe, if unlike.

In theſe ſix Problems are contained ſixteen proportions, which are applicable to the like number of caſes uſually given to right angled ſpheric triangles; and theſe proportions being collected and diſpoſed in a Table, will readily ſhew, by inſpection, how any of the caſes are to be ſolved.

The celebrated Lord NEPIER, the inventor of logarithms, contrived a general rule, eaſy to be remembered, by which the ſolution of every caſe in right angled ſpheric triangles is readily obtained, where the table of proportions is wanting; which rule is as follows.

### GENERAL RULE.

119. *Radius multiplied by the ſine of the middle part, is either equal to the product of the tangents of extremes conjunct.*
*Or to the product of the co-ſines of extremes disjunct.*
*Obſerving ever to uſe the complements of the hypoth. and angles.*

Lord *Nepier* called the five parts of every right angled ſpheric triangle, omitting the right angle, circular parts; which he thus diſtinguiſhed; the *two legs*, the *complements of the two angles*, and the *complement of the hypothenuſe*; and any two of theſe circular parts being given, the others are to be found by this rule, as is ſhewn in what follows.

Now, In all the proportions about right angled ſpheric triangles, there are, beſides the radius, three things concerned; one of which may be called the middle term in reſpect of the other two; and theſe two, in reſpect of the middle term, may be called extremes.

When the two extremes are joined to the middle, they are called extremes conjunct: But when each of them is disjoined from the middle, by an intermediate term (not concerned), they are then called extremes diſjunct; taking notice that the right angle does not disjoin the legs.

If the three parts under conſideration do all join, the middle one of thoſe three is readily ſeen, and the other two are extremes conjunct.

But if only two of the three parts are joined, theſe two are extremes disjunct, and the other term is the middle part.

Theſe things duly obſerved, the practice of the Rule will appear in the following examples.

Example I. *When the hypothenuſe and the angles are concerned.*

The hypoth. is the middle term, and the two angles are extremes conjunct; then by the rule.

Rad. × ſin. hyp. = tan. one angle × tan. other angle.

But the comp. of the hypoth. and angles are always to be uſed.

Therefore rad. × co-ſ. hyp. = co-t. one angle × co-t. other angle.

Hence rad. : co-t. one angle : : co-t. other angle : co-ſ. hypoth. (II. 163)

From whence are deduced the 6th and 15th caſes.

Exam. II. *When the hypothenuſe and legs are under conſideration.*

The hypothenuſe is the middle term, and the two legs are extremes disjunct, having the angles between them and the hypothenuſe.

Then by the rule. Rad. × ſin. hyp. = co-ſ. one leg × co-ſ. other leg.

But the complement of the hypothenuſe is to be uſed.

Therefore rad. × co-ſ. hypoth. = co-ſ. one leg × co-ſ. other leg.

Hence rad. : co-ſ. one leg : : co-ſ. other leg : co-ſ. hypoth. (II. 163)

From whence are deduced the 3d and 13th caſes.

Exam. III. *The legs and an angle under conſideration.*

Here the angle and its oppoſite leg are extremes conjunct; and the other leg is the middle part.

And theſe being reſolved into a proportion by the rule, will produce the 8th, 11th, and 14th caſes.

Exam. IV. *The angles and a leg under conſideration.*

Here one angle is the middle, and the other angle and leg are extremes disjunct, the hypothenuſe and other leg intervening.

Now theſe being reſolved into a proportion, give the 9th, 12th, and 16th caſes.

Exam. V. *The hypothenuſe, a leg, and the angle between them, being under conſideration.*

Here the angle is the middle term, and the hypothenuſe and leg are extremes conjunct.

And theſe being reſolved into a proportion, will give the 2d, 4th, and 10th caſes.

Exam. VI. *The hypothenuſe, a leg, and its oppoſite angle, being under conſideration.*

Here the leg is the middle term, and the hypothenuſe and angle are extremes disjunct, the other leg and other angle falling between them and the middle.

And theſe being converted into a proportion, from thence the 1ſt, 5th, and 7th caſes are deduced.

## SECTION VII.

## *Of the Solution of oblique angled ſpheric Triangles.*

120. All the caſes in oblique angled ſpheric triangles, except where the three ſides, or the three angles are given, are moſt conveniently reſolved by drawing a perpendicular from one of the angles to its oppoſite ſide, continued if neceſſary; which perpendicular will either divide the given triangle into two right angled triangles, or make two that are right angled, by joining a right angled one to the given triangle.

In drawing this perpendicular, obſerve,

1ſt. It muſt be drawn from the end of a given ſide, and oppoſite to a given angle.

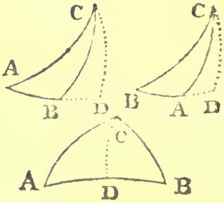
2d. It muſt be ſo drawn, that two of the given things in the oblique triangle may remain known in one of the right-angled triangles.

3d. This perpendicular is to be uſed as a known quantity; and being drawn as here directed, will either fall within or without the triangle, as the angles, next the ſide on which it falls, are of the ſame or of different kinds. (105)

### 121. PROBLEM I.

In the oblique angled ſpheric triangle ABC.
Given two ſides CA, CB } Requir. the reſt
And the angle opp. to one, ∠ CAB }

1ſt. *To find the angle oppoſite to the other given ſide* (∠ CBA).
As ſin. BC : ſin. ∠ CAB : : ſin. AC : ſin. ∠ CBA. (110)



Or, *As ſin. one ſide : ſin. oppoſite angle : : ſin. other ſide : ſin. oppoſite angle.
Which may be either acute or obtuſe.*

2d. *To find the angle between the given ſides* (∠ ACB).
Now rad. : tan. ∠ CAB : : co-ſ. AC : co-t. (ACD, call it) m. (3d 114)
*Or rad. : tan. given* ∠ : : *co-ſ. adj. ſide : co-t.* (*of a fourth* =) *m.*
*And is acute, if* AC *and* ∠ CAB *are like; but obtuſe, if unlike.*
But rad. : tan. CD : : co-t. AC : co-ſ. (ACD =) m. (2d 113)
rad. : tan. CD : : co-t. CB : co-ſ. (BCD, call it) n
Therefore co-t. AC : co-t. CB : : co-ſ. m : co-ſ. n. (II. 155)
*Or co-t. ſide adj. given* ∠ : *co-t. other ſide : : co-ſ. m : co-ſ. n.*
*And is like the ſide oppoſite the given angle, if that angle is acute.
But unlike that ſide, if the given angle is obtuſe.*

*Then the angle ſought,* viz. $\angle \text{ACB} = \begin{cases} \textit{ſum of m and n, if } \perp^{*} \textit{ falls within} \\ \textit{diff. of m and n, if } \perp \textit{ falls without.} \end{cases}$

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[*] The mark ⊥ ſignifies the perpendicular.

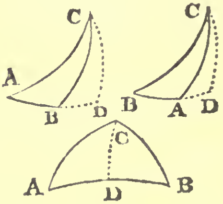
3d. *To find the other ſide* AB.

Now rad. co-ſ. ∠CAB : : tan. AC : tan. (AD, call it) M. (2d 114)
*Or rad. : co-ſ. given angle : : tan. adj. ſide : tan. (of a fourth =)* M.
*Acute, if the angle and its adj. ſide are like; but obtuſe, if unlike.*
But co-ſ. CD : rad. : : co-ſ. AC : co-ſ. (AD =) M.
co-ſ. CD : rad. : : co-ſ. CB : co-ſ. (DB call it) N. (3d 113)
Therefore co-ſ. AC : co-ſ. CB : : co-ſ. M. : co-ſ. N. (II. 155)
*Or co-ſ. ſide adj. given angle : co-ſ. other ſide : : co-ſ.* M. *: co-ſ.* N.
*Like the ſide oppoſite the given angle, if that angle be acute;*
*But unlike that ſide, if the angle be obtuſe.*

*Then the ſide ſought* $AB = \begin{cases} \textit{ſum of } M \textit{ and } N, \textit{ if the } \perp \textit{ falls within.} \\ \textit{diff. of } M \textit{ and } N, \textit{ if the } \perp \textit{ falls without.} \end{cases}$

But if CA = CB, or if CA = 180° − CB, or if CA is between BC and 180 − BC;
Then ∠B is like BC only.

And if BC is $\begin{cases} \text{like} \\ \text{unlike} \end{cases} \angle A$; $\begin{cases} \text{Then } \angle ACB = m \pm n \text{ only; and } AB = M \\ \pm N \text{ only.} \end{cases}$



## 122. PROBLEM II.

In the oblique angled ſpheric triangle ABC.
Given two angles CAB, CBA } Required
And a ſide oppoſite one of them AC } the reſt.

1ſt. *To find the ſide oppoſite the other given angle, viz.* CB.

Then, As ſin. ∠CBA : ſin. AC : : ſin. ∠CAB : ſin. CB. (110)
*Or ſin. one angle : ſin. oppoſite ſide : : ſin. other angle : ſin. oppoſite ſide.*
*Which may be either acute or obtuſe.*

2d. *To find the ſide included by the given angles, viz.* AB.

Now rad. : co-ſ. ∠CAB : : tan. AC : tan. (AD, call it) M. (II. 114)
*Or rad. : co-ſ. ∠adj. given ſide : : tan. the given ſide : tan. (of a fourth =)* M.
*Like the angle adj. the ſide given, if that ſide is acute; but unlike, if obtuſe.*
But rad. : tan. CD : : co-t. ∠CAB : ſin. (AD =) M
rad. : tan. CD : : co-t. ∠CBD : ſin. (DB, call it) N. (2d 115)
Therefore co-t. ∠CAB : co-t. ∠CBD : : ſin. M : ſin. N. (II. 155)
*Or co-t. ∠adj. given ſide : co-t. other angle : : ſin.* M *: ſin.* N.
*Which may be either acute or obtuſe.*

*Then the ſide ſought* $AB = \begin{cases} \textit{ſum of } M \textit{ and } N, \textit{ if the given angles are alike.} \\ \textit{diff. of } M \textit{ and } N, \textit{ if the given angles are unlike.} \end{cases}$

3d. *To find the other angle, viz.* ∠ACB.

Now rad. : tan. ∠CAD : : co-ſ. AC : co-t. (∠ACD, call it) m. (3d 114)
*Or rad. : tan. ∠adj. ſide given : : co-ſ. of given ſide : co-t. (of a fourth =)*m.
*Like ∠adj. ſide given, if that ſide is acute; but unlike, if obtuſe.*
But co-ſ. CD : rad. : : co-ſ. ∠CAB : ſin. (∠ACD =)m.
co-ſ. CD : rad. : : co-ſ. ∠ABC : ſin. (∠BCD, call it) n. (3d 115)
Therefore co-ſ. ∠CAB : co-ſ. ∠ABC : : ſin. m. to ſin. n. (II. 155)
*Or co-ſ. ∠adj. ſide given : co-ſ. other angle : : ſin. m : ſin. n.*
*Which may be either acute or obtuſe.*

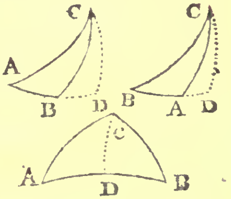
*Then ∠ ſought* ABC = { *ſum of m and n, if the given angles are alike.*
{ *diff. of m and n, if the given angles are unlike.*

But if AC=BC, or to 180°—BC, or is between BC and 180°—BC.
Then BC cannot be unlike its oppoſite angle.
Neither can DB, or the ∠ BCD be obtuſe.

## 123. PROBLEM III.

In the oblique angled ſpheric triangle ABC.

Given two ſides AC, AB }
And their contained angle BAC } Required the reſt.



1ſt. *To find either of the other angles, as* ∠ ABC.

As rad. : co-ſ. ∠ CAB : : tan. AC : tan. (AD, call it) M. (2d 114)
*Or rad. : co-ſ. given ∠ : : tan. ſide oppoſite ∠ ſought : tan. (of a fourth=)* M.
*Like the ſide oppoſite ∠ ſought, if the given ∠ is acute;*
*But unlike that ſide, if the given ∠ is obtuſe.*
*Take the diff. between* AB, *ſide adj. ∠ ſought, and* (AD=)M; *call it* N.
Now rad. : co-t. CD : : ſin. (AD=) M : co-t. ∠ CAB. (1ſt 117)
rad. : co-t. CD : : ſin. (DB=) N : co-t. ∠ ABC.
Therefore ſin. N : ſin M. : : co-t. ∠ ABC : co-t. ∠ CAB. (II. 155)
: : tan. ∠ CAB : tan. ∠ CBA. (III. 37)
*Or ſin.* N : *ſin.* M : : *tan. given ∠ : tan. ∠ ſought.*
*Like the given angle,* BAC, *if* M *is leſs than* AB, *the ſide adjacent the angle ſought; but unlike, if* M *is greater.*

2d. *To find the other ſide* CB.

As rad : co-ſ. ∠ CAB : : tan. AC : tan. (AD, call it) M. (2d 114)
*Or rad. : co-ſ. given ∠ : : tan. of either given ſide : tan. (of a fourth=)* M.
*Like the ſide uſed in this proportion, if the given ∠ is acute;*
*But unlike that ſide, if the angle is obtuſe.*
*Take the difference between the other ſide,* AB, *and* (AD=) M; *call it* N.
Now rad. : co-ſ. CD : : co-ſ. (AD=) M : co-ſ. AC. (2d 117)
rad. : co-ſ. CD : : co-ſ. (DB=) N : co-ſ. CB.
Therefore co-ſ. M : co-ſ. N : : co-ſ. AC : co-ſ. CB. (II. 155)
*Or co-ſ.* M : *co-ſ.* N : : *co-ſ. ſide uſed in firſt proportion : co-ſ. ſide required.*
*Like* N, *if the given ∠ is acute; but unlike* N, *if that ∠ is obtuſe.*

## 124. PROBLEM IV.

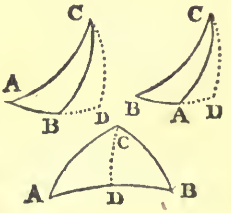
In the oblique angled ſpheric triangle ABC.

Given two angles ∠ CAB, ∠ ACB }
And their included ſide AC } Required the reſt.

1ſt. *To find either of the other ſides, as* CB.

As rad. : co-ſ. AC : : tan. ∠ CAB : co-t. (∠ ACD, call it) m. (3d 114)
*Or rad. : co-ſ. given ſide : : tan. ∠ oppoſite ſide ſought : co-t. (of a fourth=) m.*
*Like the angle oppoſite ſide ſought, if the given ſide is acute;*
*But unlike that angle, if the given ſide be obtuſe*
*Take the diff. between ∠* ACB, *adj. ſide ſought, and* (∠ ACD=) *m, call it n.*
Then rad. : co-t. CD : : co-ſ. (∠ ACD=) m : co-t. AC. (3d 116)
rad. : co-t. CD : : co-ſ. (∠ BCD=) n : co-t. CB.

Therefore co-ſ. n. : co-ſ. m : : co-t. CB : co-t. AC. (II. 155)
: : tan. AC : tan. CB. (III. 37)
*Or co-ſ. n : co-ſ. m. : : tan. given ſide : tan. ſide required.*
*Like n, if the angle oppoſite the ſide ſought be acute;*
*But unlike n, if the angle is obtuſe.*
2d. *To find the other angle* ABC.



As rad. : co-ſ. AC : : tan. ∠CAB : co-t. (∠ACD, call it) m. (3d 114)
*Or rad.:co-ſ. given ſide : : tan. either given ∠ : co-t. (of a fourth =) m.*
*Like ∠ uſed in this proportion, if the given ſide,* AC, *is acute;*
*But unlike that ∠, if the given ſide is obtuſe.*
*Take the difference between the other ∠,* ACB, *and ∠(*ACD =*) m, call it n.*
Now rad. : co-ſ. CD : : ſin. (∠ACD =) *m* : co-ſ. ∠CAB. (1ſt 116)
rad. : co-ſ. CD : : ſin. (∠BCD =) *n* : co-ſ. ∠ABC.
Therefore ſin. m : ſin. n : : co-ſ. ∠CAB : co-ſ. ∠ABC. (II. 155)
*Or ſin.* m : *ſin.* n : : *co-ſ. ∠ uſed in firſt prop. : co-ſ. ∠ ſought.*
*Like the ∠ uſed in both proportions, if* m *is leſs than the other ∠;*
*But unlike, if* m *is greater than the other angle.*

## 125. PROBLEM V.

In the oblique angled ſpheric triangle ABC. Plate I. Problem V.
Given the three ſides AB, BC, AC; Required the angles.
*To find the angle* ABC.

Let HBKLM repreſent the quarter of a ſphere, the center of which is O. Where the ſemicircular ſections HBK, HLK, are at right angles to one another; and OB is perpendicular to HK.

Then, continuing the ſide BC to L, the arc HML meaſures ∠ABC. (9)
And HQ = $\frac{1}{2}$ chord HL, will be the ſine of $\frac{1}{2}$ (arc HMC = $\frac{1}{2}$) ∠ABC.
Draw the radius OQM; and draw LP, QN at right angles to HK.
Then LP = ſine, HP = verſed ſine, of ∠ ABC; And HN = NP. (II. 165)
But as HQO is a right-angled triangle; OQ being perp. to HL. (II. 125)
Therefore OH : HQ : : HQ : HN. (II. 170)
And OH × HN = $\overline{HQ}^2$ (II. 162) = ſquare of the ſine of $\frac{1}{2}$ ∠ABC.
Make BD = BE = BC; and AF = AG = AC.

Then the ſemicircular plane DCE, which is parallel to HLK (23), will be cut by the ſemicircular plane FCG, drawn at right angles to the plane HBK, in the line CI (II. 209) at right angles to DE. (II. 210)

And the arc DC, and its verſed ſine DI, are ſimilar to the arc HL and its verſed ſine HP. (29. III. 15)

Then rad. OH : rad. DS : : PH : ID = $\left(\frac{PH}{OH}\times DS=\right)\frac{2HN}{OH}\times DS$.

Draw OR parallel to FG; then arc AR = (90° =) arc BK, and RK = AB.
Therefore ∠DIF = (∠KOR = arc RK =) arc AB.
Now DS = (SE = ſine arc BE =) ſine arc BC.
And AD = (BD − BA = BC − BA) = diff. ſides about ∠ ſought.
Alſo ∠DFI = $\frac{1}{2}$ arc(DG = AG + AD =) $\overline{AC+AD}$, the ſine of which is $\frac{1}{2}$ ID. Schol. to art. III. 45.
And arc FD = (AF − AD =) AC − AD, the ſine of which is $\frac{1}{2}$ DF.

Now $\text{ſin.}\angle\mathrm{DIF} : \text{ſin.}\angle\mathrm{DFI} :: (\mathrm{FD} : \mathrm{ID} ::)\ \tfrac{1}{2}\mathrm{FD} : \tfrac{1}{2}\mathrm{ID}$. (Schol. III. 45)

Or $\text{ſin.}\angle\mathrm{DIF} : \text{ſin.}\angle\mathrm{DFI} :: \tfrac{1}{2}\mathrm{FD} : \dfrac{\mathrm{HN}}{\mathrm{OH}}\times\mathrm{DS}$.

Therefore $\text{ſin.}\angle\mathrm{DIF}\times\mathrm{DS}\times\dfrac{\mathrm{HN}}{\mathrm{OH}} = \text{ſin.}\angle\mathrm{DFI}\times\tfrac{1}{2}\mathrm{FD}$. (II. 163)

Therefore $\text{ſin.}\angle\mathrm{DIF}\times\mathrm{DS}\times\dfrac{\mathrm{HN}}{\mathrm{OH}}\times\mathrm{OH} = \text{ſin.}\angle\mathrm{DFI}\times\tfrac{1}{2}\mathrm{FD}\times\mathrm{OH}$ (II. 156)

Or $\text{ſin.}\angle\mathrm{DIF}\times\mathrm{DS}\times\mathrm{HN} = \text{ſin.}\angle\mathrm{DFI}\times\tfrac{1}{2}\mathrm{FD}\times\mathrm{OH}$. (II. 149)
Theref. $\text{ſin.}\angle\mathrm{DIF}\times\mathrm{DS} : \text{ſin.}\angle\mathrm{DFI}\times\tfrac{1}{2}\mathrm{FD} :: \mathrm{OH} : \mathrm{HN}$. (II. 163)
$:: \mathrm{OH}\times\mathrm{OH} : (\mathrm{HN}\times\mathrm{OH} =)\,\overline{\mathrm{HQ}}^2$. (II. 155)
Therefore $\text{ſin.}\angle\mathrm{DIF}\times\mathrm{DS} : \text{ſin.}\angle\mathrm{DFI}\times\tfrac{1}{2}\mathrm{FD} :: \overline{\mathrm{OH}}^2 : \overline{\mathrm{HQ}}^2$.
Or $\text{ſin.}\,\mathrm{AB}\times\text{ſin.}\,\mathrm{BC} : \text{ſin.}\,\tfrac{1}{2}\overline{\mathrm{AC}+\mathrm{AD}}\times\text{ſin.}\,\tfrac{1}{2}\overline{\mathrm{AC}-\mathrm{AD}} :: \mathrm{Rad}^2 : \overline{\text{ſin.}\,\tfrac{1}{2}\angle\mathrm{ABC}}^2$

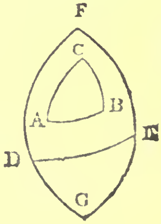
$$\text{Theref.ſqu.ſin.}\tfrac{1}{2}\angle\mathrm{ABC} = \frac{\text{ſin.}\,\tfrac{1}{2}\,\overline{\mathrm{AC}+\mathrm{AD}}\times\text{ſin.}\,\tfrac{1}{2}\,\overline{\mathrm{AC}-\mathrm{AD}}.}{\text{ſin.}\,\mathrm{AB}\times\text{ſin.}\,\mathrm{BC}}\times\text{ſqu. Rad.}\quad\text{(II.164)}$$

Now ſuppoſing Rad. $=1$, and L. to ſtand for logarithm.
Then $2\mathrm{L},\ \text{ſin.}\ \tfrac{1}{2}\angle\mathrm{ABC} = \mathrm{L.}\ \text{ſin.}\ \tfrac{1}{2}\overline{\mathrm{AC}+\mathrm{AD}} + \mathrm{L},\ \text{ſin.}\ \overline{\mathrm{AC}-\mathrm{AD}} - \mathrm{L.}\ \text{ſin.}\ \mathrm{AB} - \mathrm{L.}\ \text{ſin.}\ \mathrm{BC}$. (I. 90, 85, 86)
And putting *l* for the arithmetic complement of a logarithm.

$$\text{Then L, ſin.}\,\tfrac{1}{2}\angle\mathrm{ABC} = \frac{l.\,\text{ſin.}\ \mathrm{AB} + l.\,\text{ſin.}\ \mathrm{BC} + \mathrm{L.}\ \text{ſin.}\ \tfrac{1}{2}\overline{\mathrm{AC}+\mathrm{AD}} + \mathrm{L.}\ \text{ſin.}\ \tfrac{1}{2}\overline{\mathrm{AC}-\mathrm{AD}}}{2}$$

That is, having determined which angle to find,
*To the arithmetic complement of log. ſin. of one containing ſide,*
*Add the arithmetic complement of log. ſin. of the other containing ſide,*
*And the log. ſin. of the* ½ *ſum of 3d ſide and difference of the containing ſides,*
*Alſo the log. ſin. of the* ½ *difference of 3d ſide and diff. of the containing ſides,*
*Then the degrees anſwering to half the ſum of theſe four logarithms, found among the ſines, being doubled, will give the angle ſought.*

## 126. PROBLEM VI.



In the oblique angled ſpheric triangle ABC.
Given the three angles A, B, C; Requ. the ſides.
*To find the ſide* AB.
About the given angles as poles, deſcribe arcs of great circles meeting one another, and forming the triangle FDE.
Then are the ſides of FDE, the ſupplements of the angles A, B, C. (95)
Continue FD, FE, the ſupplements of the angles B, A, adjacent to the ſide AB required, till they meet in G.

Then in the triangle DGE, the ſides GD, GE, are the meaſures of the angles B and A, adjacent to the ſide ſought.

The ſide DE is the ſupplement of ∠C oppoſite the ſide AB.

Now ∠G (= ∠F, by 31) is the ſupplement of AB.

Therefore the ∠G being found in the triangle DGB by PROB. V. (125) will give the ſupplement of the ſide AB required.

*That is, Let the given angles be taken as the ſides of another triangle, obſerving to uſe the ſupplement of that angle oppoſite to the ſide required.*

*In this new triangle find (by* PROB. V.*) the angle oppoſite to that ſide where the ſupplement is uſed.*

*Then will the ſupplement of the angle thus found be the ſide required.*

*A* TABLE *containing all the caſes of right angled, or Quadrantal, ſpheric Triangles, with the Solutions and Determinations.*

| | *Pr.* | *Given.* | *Required.* | S O L U T I O N. | | | | *Determination.* |
|---|---|---|---|---|---|---|---|---|
| 127. | I. | Hyp. and a leg | ∠ op. gn. leg | ſin. Hyp. | : Rad. | : : ſin. gn. leg | : ſin. op. ∠ | Like given leg. |
| 128. | | | ∠ ad. gn. leg | Rad. | : co-t. Hyp. | : : tan. gn. leg | : co-ſ. adj. ∠ | ac. or ob. as hyp. is like or unl. gn. leg. |
| 129. | | | other leg | co-ſ. gn. leg | : Rad. | : : co-ſ. Hyp. | : co-ſ. req. leg | ac. or ob. as hyp. is like or unl. gn. leg. |
| 130. | II. | Hyp. and an angle | leg. op. gn. ∠ | Rad. | : ſin. Hyp. | : : ſin. gn. ∠ | : ſin. op. leg | Like given angle. |
| 131. | | | leg. ad. gn. ∠ | Rad. | : tan. Hyp. | : : co-ſ. gn. ∠ | : tan. adj. leg | ac. or ob. as hyp. is like or unl. gn. ∠. |
| 132. | | | other angle | Rad. | : co-ſ. Hyp. | : : tan. gn. ∠ | : co-t. req. ∠ | ac. or ob. as hyp. is like or unl. gn. ∠. |
| 133. | III. | A leg and its op. ∠ | Hypoth. | ſin. gn. ∠ | : ſin. gn. leg | : : Rad. | : ſin. Hyp. | either ac. or ob. |
| 134. | | | other leg | Rad. | : co-t. gn. ∠ | : : tan. gn. leg | : ſin. req. leg | either ac. or ob. |
| 135. | | | other angle | co-ſ. gn. leg | : co-ſ. gn. ∠ | : : Rad. | : ſin. req ∠ | either ac. or ob. |
| 136. | IV. | A leg. and its adj. ∠ | other angle | Rad. | : co-ſ. gn. leg | : : ſin. gn. ∠ | : co-ſ. req. ∠ | Like given leg. |
| 137. | | | other leg | Rad. | : ſin. gn. leg | : : tan. gn. ∠ | : tan. req. leg | Like given angle. |
| 138. | | | Hypoth. | Rad. | : co-ſ. gn. ∠ | : : co-t. gn. leg | : co-t. Hyp. | ac. or ob. as gn. leg like or unlike gn. ∠. |

| | | | | | |
|---|---|---|---|---|---|
| 139. | V. | Both legs | either angle | Rad. : ſin. either leg : : co-t. oth. leg : co-t. op. ∠ | Like oppoſite leg. |
| 140. | | | Hypoth. | Rad. : co-ſ. either leg : : co-ſ. oth. leg : co-ſ. Hyp. | ac. or ob. as legs are like or unlike. |
| 141. | VI. | Both angles | either leg | ſin. eith. ∠ : Rad. : : co-ſ. other ∠ : co-ſ. op. leg | Like oppoſite angle. |
| 142. | | | Hypoth. | Rad. : co-t. either ∠ : : co-t. oth. ∠ : co-ſ. Hyp. | ac. or ob. as ∠s are like or unlike. |

143. In a quadrantal triangle, if the quadrantal ſide be called radius; the ſupplement of the angle oppoſite to that ſide be called hypothenuſe; the other ſides be called angles, and their oppoſite angles be called legs: Then the ſolution of all the caſes will be as in this table; obſerving, that where the kind of a ſide or angle is determined by the hypothenuſe; or the hypothenuſe is to be determined; to uſe unlike inſtead of like, and like inſtead of unlike.

In this table, beſide the contractions for ſine, tangent, co-ſine, co-tangent; *op.* ſtands for oppoſite; *adj.* for adjacent; *oth.* for other; *eith.* for either; *ac.* for acute; *ob.* for obtuſe; *gn.* for given; *lik. unl.* for like, unlike; *req.* for required; *ang.* or ∠, for angle.

*A* TABLE *containing all the caſes of Oblique angled Spheric Triangles, with the Solutions and Determinations.*

| | Pr. | *Given.* | *Required.* | SOLUTION. | *Determination.* |
|---|---|---|---|---|---|
| 144. | | | ∠ op. oth. ſid. | ſin. one ſide : ſin. op. ∠ : : ſin. other ſide : ſin. op. ∠ | either acute or obtuſe. |
| 145. | I. | Two ſides and an angle op. to one | ∠ bet. gn. ſid. | Rad. : tan. given ∠ : : co-ſ. adj. ſide : co-t. m<br>co-t. ſid. adj. gn. ∠ : co-t. other ſide : : co-ſ. m : co-ſ. n<br>Then req. angle is either equal to ſum, or diff. of m and n | acute or obtuſe as given angle and its adjacent ſide are like or unlike.<br>like or unlike ſide op. given angle as that angle is acute or obtuſe.<br>as given ſides are like or unlike. |
| 146. | | | other ſide | Rad. : co-ſ. given ∠ : : tan. adj. ſide : tan. M.<br>co-ſ. S. adj. gn. ∠ : co-ſ. other S. : : co-ſ. M : co-ſ. N<br>Then required ſide = ſum or difference of M and N | acute or obtuſe as given angle and its adjacent ſide are like or unlike.<br>like or unlike ſide op. given angl. as that angle is acute or obtuſe.<br>as the given ſides are like or unlike. |
| 147. | | | S. op. other ∠ | ſin. one angle : ſin. oppoſite S. : : ſin. other ∠ : ſin. op. S. | either acute or obtuſe. |
| 148. | II. | Two angles and a ſide op. to one | S. bet. gn. ∠s | Rad. : co-ſ. ∠ adj. gn. S. : : tan. given S. : tan. M<br>co-t. ∠ adj. gn. S : co-t. other angle : : ſin. M : ſin. N<br>Then required ſide = ſum or difference of M and N | like or unlike ang. adj. given ſide, as that ſide is acute or obtuſe.<br>either acute or obtuſe.<br>as the given angles are like or unlike. |
| 149. | | | other angle | Rad. : tan. ∠ adj. gn. S : : co-ſ. given ſide : co-t. m<br>co-t. ∠ adj. gn. S. : co-ſ. other angle : : ſin. m : ſin. n<br>Then required angle = ſum or difference of M and N | like or unlike ang. adj. given ſide, as that ſide is acute or obtuſe.<br>either acute or obtuſe.<br>as the given angles are like or unlike. |
| 150. | III. | Two ſides | either of other angles | Rad. : co-ſ. given angle : : tan. S. op. req. ∠ : tan. M<br>Take the difference between ſide adj. req. angle and M, call it N<br>ſine N : ſine M : : tan. given ∠ : tan. req. ∠ | like or unlike ſide op. req. angle, as given angle is acute or obtuſe.<br>like or unlike given angle as M is leſs or greater than S. adj. req. ∠. |

| | | | | | |
|---|---|---|---|---|---|
| 151. | | and their included ∠ | other ſide | Rad. : co-ſ. given angle : : tan. eith. gn. S. : tan. M<br>Take the difference between the other ſide and M, call it N<br>co-ſ. M : co-ſ. N : : co-ſ. S. 1ſt uſed : co-ſ. req. S. | like or unlike ſide uſed, as given angle is acute or obtuſe.<br>like or unlike N, as given angle is acute or obtuſe. |
| 152. | IV. | Two angles and their included ſide | either of other ſides | Rad. : co-ſ. given ſide : : tan. ∠ op. req. S : co-t. m<br>Take the difference between ∠ adj. required ſide and m, call it n<br>co ſ. n : co-ſ. m : : tan. given ſide : tan. req. S | like or unlike angle op. req. ſide, as given ſide is acute or obtuſe.<br>like or unlike n, as the angle oppoſite req. ſide is acute or obtuſe. |
| 153. | | | other angle | Rad. : co-ſ. given ſide : : tan. eith. gn. ∠ : co-t. m<br>Take the difference between the other angle and m, call it n<br>ſine m. : ſine n : : coſ. ∠ 1ſt uſed : co-ſ. req. ∠ | like or unlike angle uſed, as given ſide is acute or obtuſe.<br>like or unlike angle here uſed, as m is leſs or greater than other ang. |
| 154. | V. | Three ſides | either angle | Call the ſides including the angle ſought E and F; the oppoſite ſide call G.<br>Put D equal to difference between E and F; find the half ſum and half difference of G and D.<br>Then. To the Ar. Co. of Log. ſine of E, add the Ar. Co. of Log. ſine of F,<br>And the Log. ſine of $\frac{1}{2}$ ſum of G and D; Alſo the Log. ſine of $\frac{1}{2}$ difference of G and D.<br>Take half the ſum of theſe four Logarithms, which ſeek among the Log. ſines;<br>And the degrees and minutes anſwering being doubled, will give the angle ſought. | |
| 155. | VI. | Three angles | either ſide | Let the given angles be taken as the ſides of another triangle, obſerving to uſe the ſupplement of that angle oppoſite the ſide required.<br>In this new triangle, find the angle oppoſite to that ſide where the ſupplement is uſed, by the precepts in Problem V.<br>Then will the ſupplement of the angle thus found be the ſide required. | |

## SECTION VIII.

## *The Conſtruction and numerical Solution of the caſes of right angled ſpheric Triangles.*

156. Exam. I. In the right-angled ſpheric triangle ABC.
Given the hypoth. AC = 64° 40′ } Required the reſt.
And one leg BC = 42 12 }

### Constructions.

1ſt. *To put the given leg on the primitive circle.*

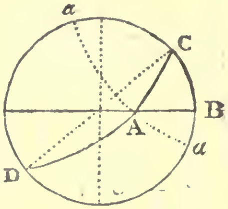
Deſcribe the primitive circle, and draw the right circle AB.

Apply the given leg (42° 12′) to the primitive circle from B to C.

About C, as a pole, at a diſtance equal to the hypothenuſe (64° 40′) deſcribe (68) a ſmall circle *aa*, cutting the right circle AB in A; and draw the right circle CD.

Through C, A, D, deſcribe an oblique circle.

And ABC is the triangle ſought.



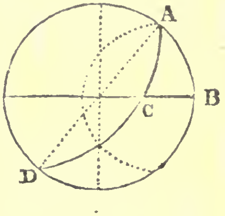
(II. 72)

2d. *To put the required leg on the primitive circle.*

Deſcribe the primitive circle, and draw the right circle CB; on which lay the given leg (42° 12′) from B to C. (70)

About C, as a pole (66), at a diſtance equal to the hypothenuſe (64° 40′) deſcribe a ſmall circle cutting the primitive in A; and draw AD.

Through A, C, D, deſcribe an oblique circle. (II. 72)



Then ABC is the triangle required: Whoſe ſides and angles are meaſured by art. 70, 72.

### Computation.

| *To find ∠ oppoſ. the given leg.* | | | (127) |
|---|---|---|---|
| As ſin. hyp. | = 64° | 40′ | 0,04391 |
| To Rad. | = 90 | 00 | 10,00000 |
| So ſin. gn. leg | = 42 | 12 | 9,82719 |
| To ſin. op. ∠ | = 48 | 00 | 9,87110 |

This angle is acute, becauſe it is to be like the given leg, which is acute.

| *To find ∠ adj. the given leg.* | | | (128) |
|---|---|---|---|
| As Rad. | = 90° | 00′ | 10,00000 |
| To co-t. hyp. | = 64 | 40 | 9,67524 |
| So tan. gn. leg. | = 42 | 12 | 9,95748 |
| To co-ſ. adj ∠ | = 64 | 35 | 9,63272 |

This angle is acute, becauſe the hyp. and given leg are of like kinds.

| *To find the other leg.* | | | (129) |
|---|---|---|---|
| As co-ſ. gn. leg | = 42° | 10′ | 0,13030 |
| To Rad. | = 90 | 00 | 10,00000 |
| So co-ſ. hyp. | = 64 | 40 | 9,63133 |
| To co-ſ. req. leg | = 54 | 43 | 9,76163 |

This leg is acute, becauſe the hyp. and given leg are of like kinds.

Note, In theſe operations, and in all the following ones, although the word co-ſine, or co-tangent, is uſed in the proportions, yet the degrees and minutes ſet down, are not the complements, but the real ſides or angles.

157. **Example II.** In the right-angled ſpheric triangle ABC.

Given the hyopth. AC = 64° 40′ } Required the reſt.
And one angle ACB = 64 35 }

## CONSTRUCTIONS.

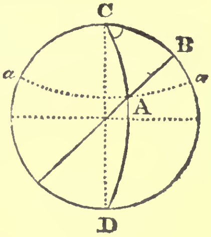
1ſt. *To put the leg adjacent to the given angle on the primitive circle.*

Through any point C, in the primitive circle, deſcribe (75) the oblique circle CAD, making with the primitive circle the angle BCA, equal to the given angle 64° 35′.

In the oblique circle CAD, take CA equal to the given hypothenuſe 64° 40′. (70)

Through A deſcribe the right circle AB.

And CAB is the triangle required.



2d. *To put the leg oppoſite the given angle on the primitive circle.*

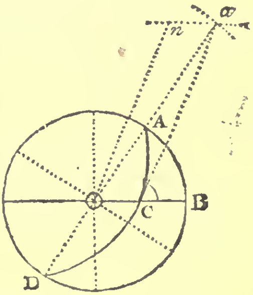
Having deſcribed the primitive circle, and drawn the right circle OB;

Deſcribe (80) an oblique circle ACD, cutting the right circle OB in C, with the given angle 64° 35′, and having the part AC intercepted between the right circle OB and the primitive circle, equal to the given hypothenuſe 64° 40′;

Then ABC is the triangle required.

The ſides required are meaſured by art. 70.

And the required angle by art. 72.



## COMPUTATION.

*To find the leg opp. the giv.* ∠ (130)

| | | | |
|---|---|---|---|
| As Rad. | =90° | 00′ | 10,00000 |
| To ſin. hyp. | =64 | 40 | 9,95609 |
| So ſin. given ∠ | =64 | 35 | 9,95579 |
| To ſin. op. leg | =54 | 43 | 9,91188 |

Like the given angle.

*To find the leg adj. the giv.* ∠ (131)

| | | | |
|---|---|---|---|
| As Rad. | =90° | 00′ | 10,00000 |
| To tan. hyp. | =64 | 40 | 10,32476 |
| So co-ſ. given ∠ | =64 | 35 | 9,63266 |
| To tan. adj. leg | =42 | 12 | 9,95742 |

Acute, as the hypothenuſe and given angle are of like kind.

*To find the other angle.* (132)

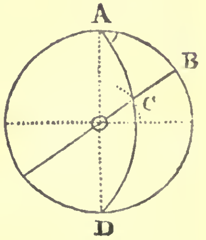
| | | | |
|---|---|---|---|
| As Rad. —— | =90° | 00′ | 10,00000 |
| To co-ſ. hyp. —— | =64 | 40 | 9,63133 |
| So tan. given angle | =64 | 35 | 10,32313 |
| To co-t. required angle | =48 | 00 | 9,95446 |

And is acute, as the hypothenuſe and given angle are of like kind.

158. EXAMPLE III. In the right angled ſpheric triangle ABC.
Given one leg CB = 42° 12′ }
And its opp. angle CAB = 48 00 } Required the reſt.

### CONSTRUCTIONS.

1ſt. *To put the required leg on the primitive circle.*

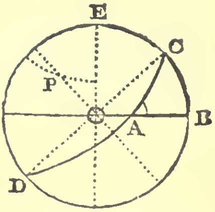


Deſcribe an oblique circle ACD (75), making with the primitive circle the angle CAB, equal to the given angle 48° 00′.

About the center O of the primitive circle deſcribe (67) a ſmall circle at the diſtance of the complement of the given leg 42° 12′, cutting ACD in C.

Draw the right circle OCB, and ACB is the triangle ſought.

2d. *To put the given leg on the primitive circle.*



Draw the right circle OAB, and another OE at right angles.

Make BC equal to the given leg 42° 12′; draw the diameter CD, and another OP at right angles.

About F, the pole of AB, deſcribe a ſmall circle (68), at the diſtance of the given angle 48° 00′, cutting OP in P.

About P, as a pole (62), deſcribe the oblique circle CAD, cutting AB in A. Then CBA is the triangle required.

The ſides are meaſured by art. 70, and the angles by art. 72.

### COMPUTATION.

| *To find the hypothenuſe.* | | (133) | *To find the other leg.* | | (134) |
|---|---|---|---|---|---|
| As ſin. giv.∠ | =48° 00′ | 0,12893 | As Rad. | =90° 00′ | 10,00000 |
| To ſin. giv. leg | =42 12 | 9,82719 | To co-t. giv.∠ | =48 00 | 9,95444 |
| So Rad. | =90 00 | 10,00000 | So tan. giv. leg | =42 12 | 9,95748 |
| To ſin. hyp. | =64 40½ | 9,95612 | To ſin. req. leg | =54 44 | 9,91192 |
| And is either acute or obtuſe. | | | And is either acute or obtuſe. | | |

*To find the other angle.* (135)

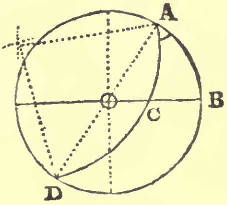
| | | |
|---|---|---|
| As co-ſ. given leg —— | = 42° 12′ | 0,13030 |
| To co-ſ. given ∠ —— | = 48 00 | 9,82551 |
| So Rad. —— | = 90 00 | 10,00000 |
| To ſin. required ∠ —— | = 64 35 | 9,95581 |

And is either acute or obtuſe.

159. **Example IV.** In the right angled ſpheric triangle ABC.
Given a leg AB $=54^\circ\ 43'$ }
And its adj. angle CAB $=48\ \ 00$ } Required the reſt.

### CONSTRUCTIONS.

1ſt. *To put the given leg on the primitive circle.*



Having deſcribed the primitive, and right circle OB;

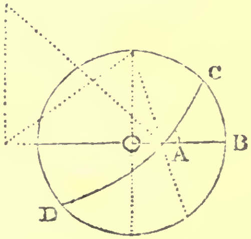
Make BA equal to the given leg 54° 43′

Draw the diameter AD.

Through A deſcribe the oblique circle ACD (75) making with the primitive the given angle BAC 48° 00′, cutting OB in C.

Then is ACB the triangle required.

2d. *To put the required leg on the primitive.*



In the right circle OB, take (71) AB, equal to the given leg 54° 43′.

Through the point A, deſcribe (76) the oblique circle CAD, making with AB the angle BAC, equal to the given angle 48° 00′, cutting the primitive circle in C.

Then is ABC the triangle ſought.

The ſides required are meaſured by art. 70.

And the required angle by art. 72.

### COMPUTATION.

*To find the other angle.* (136)

| | | | |
|---|---|---|---|
| As Rad. | =90° | 00′ | 10,00000 |
| To co-ſ. giv. leg | =54 | 43 | 9,76164 |
| So ſin. given ∠ | =48 | 00 | 9,87107 |
| To co-ſ. req. ∠ | =64 | 35 | 9,63271 |

And is like the given angle.

*To find the other leg.* (137)

| | | | |
|---|---|---|---|
| As Rad. | =90° | 00′ | 10,00000 |
| To ſin. giv. leg | =54 | 43 | 9,91185 |
| So tan. giv. ∠ | =48 | 00 | 10,04556 |
| To tan. req. leg | =42 | 12 | 9,95741 |

And is like the given leg.

*To find the hypothenuſe.* (138)

| | | | |
|---|---|---|---|
| As Rad. | = 90° | 00′ | 10,00000 |
| To co-ſ. given ∠ | = 48 | 00 | 9,82551 |
| So co-t. given leg | = 54 | 43 | 9,84979 |
| To co-t. hypoth. | = 64 | 40 | 9,67530 |

And is acute, as the given leg and angle are of a like kind.

160. EXAMPLE V. In the right angled ſpheric triangle ABC.

Given one leg BA = 54° 43′ } Required the reſt.
And the other leg BC = 42 12 }

## CONSTRUCTION.

*To put either leg on the primitive circle.*

Deſcribe the primitive circle, and draw the right circle OB.

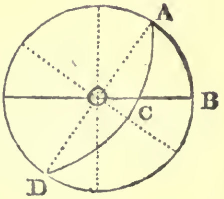
Then, let the given legs 54° 43′, and 42° 12′, be applied, one from B to A, and the other from B to C (74); and draw the diameter AD.

Through the points A, C, D, deſcribe an oblique circle, (II. 72)

Then is ABC the triangle required.

The angles A and C may be meaſured by art. 72.

And the hypothenuſe AC by art. 70.



## COMPUTATION.

*To find the angle* A. (139)

| | | | |
|---|---|---|---|
| As Radius | —— | = 90° 00′ | 10,00000 |
| To ſin. of leg | AB | = 54 43 | 9,91185 |
| So co-t. other leg | BC | = 42 12 | 10,04251 |
| To co-t. op. angle | A | = 48 00 | 9,95436 |

And is acute, as the oppoſite leg CB is acute.

*To find the angle* C. (139)

| | | | |
|---|---|---|---|
| As Radius | —— | = 90° 00′ | 10,00000 |
| To ſin. of leg | CB | = 42 12 | 9,82719 |
| So co-t. other leg | AB | = 54 43 | 9,84979 |
| To co-t. op. angle | C | = 64 35 | 9,67698 |

And is acute, becauſe the oppoſite leg AB is acute.

*To find the hypothenuſe* AC. (140)

| | | | |
|---|---|---|---|
| As Radius | —— | = 90° 00′ | 10,00000 |
| To co-ſ. either leg | AB | = 54 43 | 9,76164 |
| So co-ſ. other leg | CB | = 42 12 | 9,86970 |
| To co-ſ. hypoth. | AC | = 64 40 | 9,63134 |

And is acute, as the legs are of the ſame kind.

161. EXAMPLE VI. In the right angled ſpheric triangle ABC.

Given one angle A = 48° 00′ }
And the other angle C = 64 35 } Required the reſt.

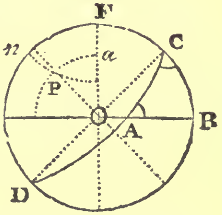
### CONSTRUCTION.

*To put either leg, as* CB, *on the primitive circle.*

Having deſcribed the primitive circle, and drawn the right circle OB;

Then (81) deſcribe the oblique great circle CAD, cutting the primitive circle in the given angle C, and the right circle OB in the given angle A.

The ſides are to be meaſured by art. 70.



### COMPUTATION.

*To find the leg* CB. (141)

| | | | |
|---|---|---|---|
| As ſin.∠ adj. req. leg | C | = 64° 35 | 0,04421 |
| To Radius —— | | = 90 00 | 10,00000 |
| So co-ſ. other angle | A | = 48 00 | 9,82551 |
| To co-ſ. of its op. leg | CB | = 42 12 | 9,86972 |

And is acute, becauſe the oppoſite angle is acute.

*To find the leg* AB. (141)

| | | | |
|---|---|---|---|
| As ſin.∠ adj. req. leg | A | = 48° 00′ | 0,12893 |
| To Radius —— | | = 90 00 | 10,00000 |
| So co-ſ. other angle | C | = 64 35 | 9.63266 |
| To co-ſ. of its op. leg | AB | = 54 43 | 9,76159 |

And is acute, becauſe the oppoſite ∠ is acute.

*To find the hypothenuſe* AC. (142)

| | | | |
|---|---|---|---|
| As Radius —— | | = 90° 00′ | 10,00000 |
| To co-t. either angle as | A | = 48 00 | 9,95444 |
| So co-t. other angle as | C | = 64 35 | 9,67687 |
| To co-ſ. hypoth. | AC | = 64 40 | 9,63131 |

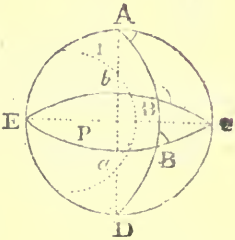
And is acute, becauſe the angles are both acute, or like.

162. EXAMPLE VII. In the quadrantal triangle ABC.

Given the quadrantal ſide AC = 90° 00′ }
an adjacent angle A = 42 12 } Required the reſt.
And the oppoſite angle B = 64 40 }

### CONSTRUCTION.

*To put the quadrantal ſide on the primitive circle.*



Having deſcribed the primitive circle, and drawn the diameters AD, BC, at right angles;

Deſcribe the oblique circle ABD, making with AC an angle of 42° 12′. (75)

Through C deſcribe a great circle CBE, cutting the circle ABD in an ∠ of 64° 40′. (74)

Then is ABC the triangle ſought.

The angle C is to be meaſured by art. 72.

And the ſides AB, CB, are meaſured by art. 70.

### COMPUTATION.

Imagine the given triangle ABC to be changed into a right angled triangle, where the ſupplement of the angle B is to repreſent the hypothenuſe, and the angle A to be one of the legs.

Then will the ſolution fall under art. 127, 128, 129, in the table; and the numerical computations will be the ſame as in Example I. Obſerving that the angles there found are, in this example, the meaſures of the ſides AB, CB; and the ſide AB in that example ſtands for the angle C in this.

Now in determining the value of the parts of this triangle, as they ariſe in the computation, the words like and unlike are to be changed one for the other, where the hypothenuſe is concerned in the determination: Thus the leg AB is taken acute, becauſe the ſupplement of the angle oppoſite to the quadrantal ſide, which is here uſed as the hypothenuſe, is unlike the other given angle; and its oppoſite angle C is to be acute for the ſame reaſon: But the kind of the ſide BC being known by the kind of its oppoſite angle A, it muſt be taken acute, as the oppoſite angle is acute.

In the conſtruction there ariſes two triangles, either of which will anſwer the conditions in the example: For the ſmall circle deſcribed about P, the pole of the oblique circle ABD, cuts the diameter AD in the points, *a*, *b*; and either of theſe points may be taken for the pole of the oblique circle wanting to complete the triangle.

Now if *a* be taken for the pole, then in the triangle ABC, the meaſure of the things ſought, will be equal to thoſe ariſing from the computation: But the angle B is the ſupplement of what was given.

And if *b* is taken for the pole; then the triangle ABC will ariſe from the conſtruction; wherein the angles A and B are reſpectively equal to what is propounded: But then the ſide AB, and the angle C, will both be obtuſe.

SECTION

## SECTION IX.

## *The Conſtruction and numerical Solution of the caſes of oblique angled ſpheric Triangles.*

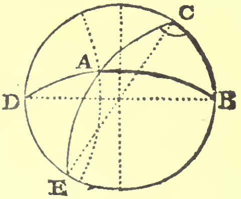
163. EXAMPLE I. In the oblique angled ſpheric triangle ABC.

| | | | |
|---|---|---|---|
| Given the ſide | AB = 114° 30′ | Required the reſt. |
| the ſide | BC = 56 40 | |
| And an angle oppoſite to one ſide, | BCA = 125 20 | |

CONSTRUCTION.

*To put the given ſide, adjacent to the known angle, on the primitive circle.*

Deſcribe the primitive circle, and draw the diameter BD.



Make BC equal to the ſide adjacent to the given angle = 56° 40′. (70)

Deſcribe the great circle CAE, making the angle BCA equal to the given one, = 125° 20′. (75)

Through B deſcribe a great circle BAD, cutting AE in A, at the diſtance of AB, the other given ſide from B, = 114° 30. (68)

Then ABC is the triangle ſought.

And the parts required are meaſured by art. 70, 72.

COMPUTATION.

*To find the angle* A, *oppoſite to the other given ſide.* (144)

| | | | |
|---|---|---|---|
| As ſin. one ſide | AB = 114° 30′ | 0,04098 | |
| To ſin. op. ∠ | C = 125 20 | 9,91158 | |
| So ſin. oth. ſide | CB = 56 40 | 9,92194 | |
| To ſin. op. ∠ | A = 48 30 | 9,87450 | |

Which may be either acute or obtuſe from the things given: But the conſtruction ſhews it to be acute.

---

*To find the angle* B *between the given ſides.* (145)

| | | | | | |
|---|---|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | As co-t. S. ad. g. ∠ BC | = 56° 40′ | 0,18197 |
| To tan. giv. ∠ C | = 125 20 | 10,14941 | To co-t. oth. ſide AB | = 114 30 | 9,65870 |
| So co-ſ. adj ſid. BC | = 56 40 | 9,73997 | So co-ſ. m | = 127 47 | 9,78723 |
| To co-t. m | = 127 47 | 9,88938 | To co-ſ. n | = 64 53 | 9,62790 |
| And is obtuſe, as the given angle and its given adjacent ſide are unlike. | | | Which is acute, being unlike ſide oppoſite given ∠, that ∠ being obtuſe. | | |

Then as the given ſides are unlike, the diff. of m and n, or 62° 54′ = ∠ B.

---

*To find the other ſide* AC. (146)

| | | | | | |
|---|---|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | As co-ſ. S. ad. g. ∠ BC | = 56° 40′ | 0,26002 |
| To co-ſ. giv. ∠ C | = 125 20 | 9,76218 | To co-ſ. oth. ſide AB | = 114 30 | 9,61773 |
| So tan. adj. ſid. BC | = 56 40 | 10,18197 | So co-ſ. M | = 138 40 | 9,87557 |
| To tan. M | = 138 40 | 9,94415 | To co-ſ. N | = 55 29 | 9,75332 |
| And is obtuſe, as ∠ C and CB are unl. | | | And is acute, being unl. AB as above. | | |

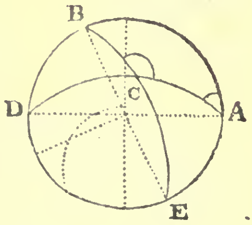
Then as BC and BA, are unlike the diff. of M and N, or 83° 11 = AC.

164. EXAMPLE II. In the oblique angled ſpheric triangle ABC.

| | | |
|---|---|---|
| Given the angle | BAC = 48° 30′ | Required the reſt. |
| the angle | BCA = 125 20 | |
| And the ſide oppoſite to one angle, | AB = 114 30 | |

## CONSTRUCTION.

*To put the given ſide* AB *on the primitive circle.*



Deſcribe the primitive circle; draw the diameter DA; and through A deſcribe the great circle ACD, making the given angle BAC = 48° 30′. (75)

Make the arc AB equal to the given ſide = 114° 30′ (70); and draw the diameter BE.

Through B, deſcribe the great circle BCE, cutting ACD in an angle equal to the given angle BCA = 125° 20′. (78)

Then is ACB the triangle ſought.

And the parts required are to be meaſured by art. 70, 72.

## COMPUTATION.

*To find the ſide oppoſite the other given angle.* (147)

| | | |
|---|---|---|
| As ſin. one ∠ C | = 125° 20′ | 0,08842 |
| To ſin. op. ſide AB | = 114 30 | 9,95902 |
| So ſin. other ∠ A | = 48 30 | 9,87446 |
| To ſin. op. ſide BC | = 56 40 | 9,92190 |

Which may either be acute or obtuſe from what is given. But the conſtruction ſhews it to be acute.

---

*To find the ſide* AC *between the given angles.* (148)

| | | | | | |
|---|---|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | As co-t. ∠ ad. g. S. A | = 48° 30′ | 0,05319 |
| To co-ſ ∠ ad. g. S. A | = 48 30 | 9,82126 | To co-t. other ∠ C | = 125 20 | 9,85059 |
| So tan. gn. S. AB | = 114 30 | 10,34130 | So ſine M | = 124 31 | 9,91591 |
| To tan. M | = 124 31 | 10,16256 | To ſine N | = 41 19 | 9,81969 |

And is obtuſe, being unlike ∠ A, as AB is greater than 90°.

Which may be either acute or obtuſe; either 41° 20′ or 138° 40′.

Then as the given angles are unlike, the difference of M and N, or 83° 12′, is the ſide AC. Or the ſum of 138° 41′, and 124° 31′, leſſened by 180°, leaves 83° 12′.

---

*To find the other angle* ABC. (149)

| | | | | | |
|---|---|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | As co-ſ. ∠ ad. g. S. A | = 48° 30′ | 0,17874 |
| To tan. ∠ ad. g. S. A | = 48 30 | 10,05319 | To co-ſ. other ∠ C | = 125 20 | 9,76218 |
| So co-ſ. gn. S. AB | = 114 30 | 9,61773 | So ſine m | = 115 07 | 9,95686 |
| To co-t. m | = 115 07 | 9,67092 | To ſine n | = 52 13 | 9,89778 |

And is obtuſe, being unlike ∠ A, as its adj. ſide AB is greater than 90°.

Which may be either acute or obtuſe, viz. 52° 13′, or 127° 47′.

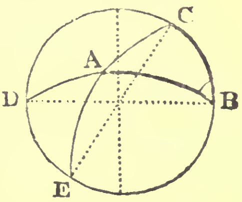
Then as the given angles are unlike, the difference of m and n, or 62° 54′, is the angle B required. Or the ſum of 115° 07′, and 127° 47′, leſſened by 180°, leaves 62° 54′.

165. **Example III.** In the oblique angled ſpheric triangle ABC.

| | | | | |
|---|---|---|---|---|
| Given the ſide | AB = | 114° | 30′ | Required the reſt. |
| the ſide | BC = | 56 | 40 | |
| And the contained angle | ABC = | 62 | 54 | |

## CONSTRUCTION.

*To put either of the given ſides, as* BC, *on the primitive circle.*



Deſcribe the primitive circle; draw the diameter BD; and through B deſcribe a great circle BAD, making the given angle ABC = 62° 54′. (75)

On the circles BCD, BAD, take the arcs BC, BA, reſpectively equal to the given ſides, *viz.* BC = 56° 40′, and BA = 114° 30′. (70)

Draw the diameter CE, and through C, A, E, deſcribe the great circle CAE; then ABC is the triangle ſought.

The required parts of ABC are meaſured by art. 70, 72.

## COMPUTATION.

*To find the angle* C. (150)

| | | | |
|---|---|---|---|
| As Rad. | = 90° | 00′ | 10,00000 |
| To co-ſ. given ∠ B = | 62 | 54 | 9,65853 |
| So t. S. op. re. ∠ AB = | 114 | 30 | 10,34130 |
| To tan. M | = 135 | 01 | 9,99983 |

Obtuſe, being like ſide op. req. ∠, the given angle being acute.

Take the difference between M and BC, and it is 78° 21′; call it N.

| | | | |
|---|---|---|---|
| As ſine N | = 78° | 21′ | 0,00904 |
| To ſine M | = 135 | 01 | 9,84936 |
| So tan. given ∠ B = | 62 | 54 | 10,29096 |
| To tan. req. ∠ C = | 125 | 20 | 10,14936 |

And is obtuſe, being unlike the given angle, becauſe M is greater than BC, the ſide adjacent to the required angle.

*To find the angle* A. (150)

| | | | |
|---|---|---|---|
| As Rad. | = 90° | 00′ | 10,00000 |
| To co-ſ. given ∠ B = | 62 | 54 | 9,65853 |
| So t. S. op. re. ∠ BC = | 56 | 40 | 10,18197 |
| To tan. of M | = 34 | 42 | 9,84050 |

Acute, being like ſide op. req. ∠, the given angle being acute.

Take the difference between M and BA, and it is 79° 48′; call it N.

| | | | |
|---|---|---|---|
| As ſine N | = 79° | 48′ | 0,00692 |
| To ſine M | = 34 | 42 | 9,75533 |
| So tan. given ∠ B = | 62 | 54 | 10,29096 |
| To tan. req. ∠ A = | 48 | 30 | 10,05321 |

And is acute, being like the given angle, as M is leſs than AB, the ſide adjacent to the required angle.

*To find the other ſide* AC. (151)

| | | | |
|---|---|---|---|
| As Rad. | = 90° | 00′ | 10,00000 |
| To co-ſ. given ∠ B = | 62 | 54 | 9,65853 |
| So tan. eith. S. AB = | 114 | 30 | 10,34130 |
| To tan. M | = 135 | 01 | 9,99983 |

Obtuſe, being like AB, the ſide uſed, becauſe the given angle is acute.

| | | | |
|---|---|---|---|
| As co-ſ. M | = 135° | 01′ | 0,15039 |
| To co-ſ. N | = 78 | 21 | 9,30521 |
| So co-ſ. S. uſed AB = | 114 | 30 | 9,61773 |
| To co-ſ. S. req. AC = | 83 | 12 | 9,07333 |

And is acute, being like N, becauſe the given angle is acute.

The diff. of M and BC, or 78° 21′ = N.

166. **Example IV.** In the oblique angled ſpheric triangle ABC.

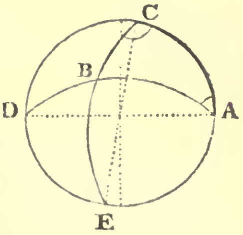
| | | | | |
|---|---|---|---|---|
| Given the angle | BCA = | 125° | 20′ | Required the reſt. |
| the angle | BAC = | 48 | 30 | |
| And the included ſide | AC = | 83 | 12 | |

### Construction.

*To put the given ſide on the primitive circle.*

Deſcribe the primitive circle; draw the diameter AD; and through A deſcribe the great circle ABD, making the given ∠BAC=48° 30′. (75)

Make AC equal to the given ſide=83° 12′. (70)



Draw the diameter CE, and through C deſcribe the great circle CBE, making the given angle BCA=125° 20′ (75), cutting ABD in B.

Then is ABC the triangle ſought.

And the parts required are meaſured by art. 70, 72.

### Computation.

*To find the ſide* AB. (152)

| | | | |
|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | Obtuſe, being like ∠op. ſide req. the given ſide being acute.<br>Take the diff. between m and ∠ A, and it is 50° 59′; call it n. |
| To co-ſ.gn.ſide AC | = 83 12 | 9,07337 | |
| So ta.∠op. r.S. C | = 125 20 | 10,14941 | |
| To co-t. M | = 99 29 | 9,22278 | |
| As co-ſ. n | = 50° 59′ | 0,20097 | And is obtuſe, being unlike n, becauſe the angle oppoſite to the ſide required is obtuſe. |
| To co-ſ. m | = 99 29 | 9,21615 | |
| So tan.gn.ſide AC | = 83 12 | 10,92357 | |
| To tan.req.ſid. AB | = 114° 30′ | 10,34139 | |

---

*To find the ſide* BC. (152)

| | | | |
|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | Acute, being like ∠ op. ſide required, the given ſide being acute.<br>Take the diff. between m and ∠ C, and it is 42° 57′½; call it n. |
| To co-ſ.gn.ſid. AC | = 83 12 | 9,07337 | |
| So tan.∠op.r.S. A | = 48 30 | 10,05319 | |
| To co-t. of m | = 82 22½ | 9,12656 | |
| As co-ſ. n | = 42° 57′½ | 0,13558 | And is acute, being like n, becauſe the angle A oppoſite to BC, the ſide required, is acute. |
| To co-ſ. m | = 82 22½ | 9,12283 | |
| So tan. gn. ſid. AC | = 83 12 | 10,92357 | |
| To tan.req.ſid. BC | = 56 40 | 10,18198 | |

---

*To find the other angle* B. (153)

| | | | | | |
|---|---|---|---|---|---|
| As Rad. | = 90° 00′ | 10,00000 | As ſine m | = 99° 29′ | 0,00598 |
| To co-ſ.gn.ſid. AC | = 83 12 | 9,07337 | To ſine n | = 50 59 | 9,89040 |
| So tan. either ∠C | = 125 20 | 10,14941 | So co-ſ. ∠ uſed, C | = 125 20 | 9,76218 |
| To co-t. m | = 99 29 | 9,22278 | To co-ſ. req. ∠ B | = 62 54 | 9,65856 |

Obtuſe, being like ∠ C here uſed, becauſe the given ſide is acute.

Take difference of m and ∠A, *viz.* 42° 57′; and call it n.

And is acute, being unlike the angle C here uſed, as m is greater than the other angle A.

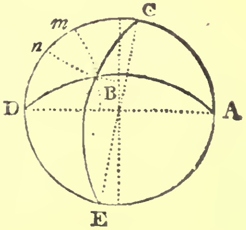
167. **Example V.** In the oblique angled ſpheric triangle ABC.

Given the ſide AB = 114° 30′ }
the ſide AC = 83 13 } Required the reſt.
the ſide BC = 56 40 }

## Construction.

*To put either ſide, as* AC, *on the primitive circle.*

Deſcribe the primitive circle, and from any point in the circumference, as A, ſet off one of the given ſides, as AC, = 83° 13′ (70); and draw the diameters AD, CE.



About C, as a pole, and at a diſtance equal to the given ſide BC, = 56° 40′, deſcribe a ſmall circle *n*B. (68)

About A, as a pole, and at a diſtance equal to the given ſide AB (when AB is leſs than 90°) deſcribe another ſmall circle *m*B (68), cutting the former in B: But when the ſide, as AB, = 114° 30′, is greater than 90°; then about D, the oppoſite pole to A, deſcribe a ſmall circle with the ſupplement of AB, as *m*B, cutting the former ſmall circle *n*B in B.

Thro' the points A, B, D, and C, B, E, deſcribe the great circles ABD, CBE. Then is ABC the triangle ſought, and the angles are meaſured by art. 72.

## Computation.

*To find the angle* C. (154)

| | | | |
|---|---|---|---|
| Here AC = E = | 83° | 13′ | |
| CB = F = | 56 | 40 | |
| E — F = D = | 26 | 33 | |
| AB = G = | 114 | 30 | |
| G + D = | 141 | 03 | 70° 31′½ = ½ ſum |
| G — D = | 87 | 57 | 43 58½ = ½ diff. |

| | | |
|---|---|---|
| Ar. Co ſine E | = 83° 13′ | 0,00305 |
| Ar. Co. ſine F | = 56 40 | 0,07806 |
| Sine ½ ſum | = 70 31½ | 9,97441 |
| Sine ½ diff. | = 43 58½ | 9,84158 |
| Sum of the four Log. | - | 19,89710 |
| ½ ſum is ſin. of 62° 39½′ | - | 9,94855 |
| Which doubled gives 125° 19′ = ∠C. | | |

*To find the angle* A. (154)

| | | | |
|---|---|---|---|
| Here AB = E = | 114° | 30′ | |
| AC = F = | 83 | 13 | |
| E — F = D = | 31 | 17 | |
| BC = G = | 56 | 40 | |
| G + D = | 87 | 57 | 43° 58′½ = ½ ſum |
| G — D = | 25 | 23 | 12 41½ = ½ diff. |

| | | |
|---|---|---|
| Ar. Co. ſine E | = 114° 30′ | 0,04098 |
| Ar. Co. ſine F | = 83 13 | 0,00305 |
| Sine ½ ſum | = 43 58½ | 9,84158 |
| Sine ½ diff. | = 12 41½ | 9,34184 |
| Sum of four Log. | - - | 19,22745 |
| ½ ſum is ſin. of 24° 15′½ | - | 9,61372 |
| Which doubled gives 48° 31′ = ∠A. | | |

*To find the angle* B. (154)

| | | | |
|---|---|---|---|
| Here AB = E = | 114° | 30′ | |
| BC = F = | 56 | 40 | |
| E — F = D = | 57 | 50 | |
| AC = G = | 83 | 13 | |
| G + D = | 141 | 03 | 70° 31′½ = ½ ſum |
| G — D = | 25° | 23 | 12 41½ = ½ diff. |

| | | |
|---|---|---|
| Ar. Co. ſine E | = 114° 30′ | 0,04098 |
| Ar. Co. ſine F | = 56 40 | 0,07806 |
| Sine ½ ſum | = 70 31½ | 9,97441 |
| Sine ½ diff. | = 12 41½ | 9,34184 |
| Sum of four Log. | - - | 19,43529 |
| ½ ſum is ſin. of 31° 28′ | - | 9,71764 |
| Which doubled gives 62° 56′ = ∠B. | | |

168. Example VI. In the oblique angled ſpheric triangle ABC.

| | | |
|---|---|---|
| Given the angle A = 48° 31′ | | |
| the angle B = 62 52 | } | Required the reſt. |
| the angle C = 125 20 | | |

### Construction.

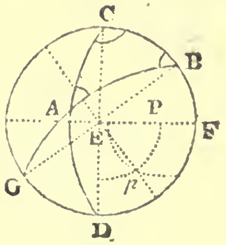
*To put either two angles, as* C *and* B, *at the primitive.*

Deſcribe the primitive circle, draw the diameters CD and EF at right angles to one another; and thro' C deſcribe a great circle CAD, making the angle BCA equal to the given angle C = 125° 20′. (75)

Deſcribe a great circle BAG, cutting the given great circles CFD, CAD, in the given angles B = 62° 52′, and A = 48° 31′. (81)

Then is ABC the triangle ſought.

Where the ſides are meaſured by art. 70.



### Computation.

*To find the ſide* AB. (155)

Here ∠B = E = 62° 52′
∠A = F = 48 31
E − F = D = 14 21
Sup. ∠C = G = 54 40
G + D = 69 0 | 34° 30½′ = ½ ſum
C − D = 40 19 | 20 09½ = ½ diff.

| | | |
|---|---|---|
| Ar. Co. ſine E | = 62° 52′ | 0,05064 |
| Ar. Co. ſine F | = 48 31 | 0,12543 |
| Sine ½ ſum | = 34 30½ | 9,75322 |
| Sine ½ diff. | = 20 09½ | 9,53733 |
| Sum of the four Log. | | 19,46662 |
| ½ ſum is ſin. of 32° 45½′ | | 9,73331 |

The ſup. of its double is 114° 29′ = AB.

*To find the ſide* AC. (155)

Here ∠C = E = 125° 20′
∠A = F = 48 31
E − F = D = 76 49
Sup. ∠B = G = 117 08
G + D = 193 57 | 96° 58½′ = ½ ſum
G − D = 40 19 | 20 09½ = ½ diff.

| | | |
|---|---|---|
| Ar. Co. ſine E | = 125° 20′ | 0,08842 |
| Ar. Co. ſine F | = 48 31 | 0,12543 |
| Sine ½ ſum | = 96 58½ | 9,99677 |
| Sine ½ diff. | = 20 09½ | 9,53733 |
| Sum of four Log. | | 19,74795 |
| ½ ſum is ſin. of 48° 25½′ | | 9,87397 |

The ſup. of its double is 83° 09′ = CA.

*To find the ſide* BC. (155)

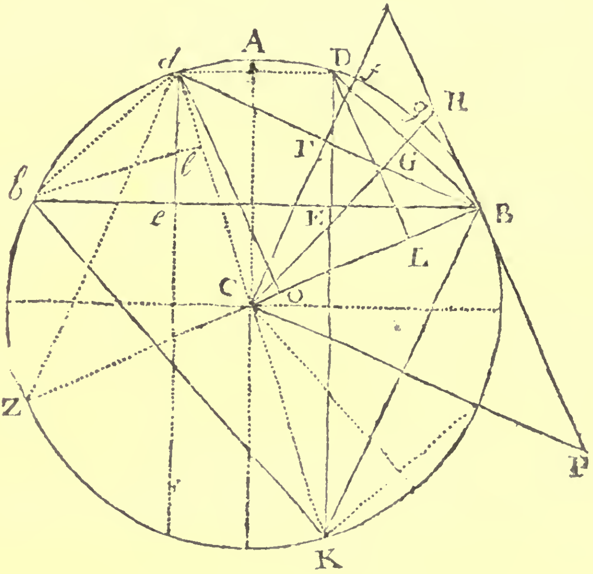
Here ∠C = E = 125° 20′
∠B = F = 62 52
E − F = D = 6[illegible] 28
Sup. ∠A = G = 131 29
G + D = 193 57 | 96° 58½′ = ½ ſum
G − D = 69 01 | 34 30½ = ½ diff.

| | | |
|---|---|---|
| Ar. Co. ſine E | = 125° 20′ | 0,08842 |
| Ar. Co. ſine F | = 62 52 | 0,05064 |
| Sine ½ ſum | = 96 58½ | 9,99677 |
| Sine ½ diff. | = 34 30½ | 9,75322 |
| Sum of four Log. | | 19,88905 |
| ½ ſum is the ſin. of 61° 39′ | | 9,94452 |

The ſup. of its double is 56° 42′ = BC.

## SECTION X.

169. The principles already delivered have been ſhewn ſufficient for deriving methods for the ſolution of all the caſes in ſpherical Trigonometry: yet as there are many other uſeful and curious particulars which appertain to the ſubject, it was thought proper to add ſome of them for the entertainment of ſpeculative readers. The chief of theſe relations cannot, perhaps, be better inveſtigated, than by imitating the method of the late William Jones, Eſq. who publiſhed in the year 1747, in the Philoſophical Tranſactions, N° 483, ſome properties of Goniometrical lines; which properties are moſtly derived from a general figure which Mr. Jones improved from one communicated to him by the great Dr. Halley. See *Synopſis Palmariorum Matheſeos*, p. 245.



Let AB, AD; or A*b*, A*d*, be any two arcs, each leſs than 90 degrees, B*e* and BE, or *b*E and *be*, be the ſum and difference of their right ſines. KE and DE, the ſum and difference of their co-ſines.

The arcs B*d*, BD; or *b*D, *bd*; expreſs the ſum and difference of the arcs AB, AD. *d*O, DL, are ſines of the arcs B*d*, BD, the ſum and difference of arcs AB, AD: BO, BL, the verſed ſines of that ſum and difference. ZO, ZL = B*d*, the verſed ſines of the ſupplements of their ſum and diff.

Let the arcs B*f*, B*g*, be the half ſum, and half diff. of the arcs AB, AD.

$$\left.\begin{array}{l}\text{BF, BG, the ſines}\\ \text{CF, CG, the co-ſines}\\ \text{BI, BH, the tangents}\\ \text{CI, CH, the ſecants}\\ \text{B}d\text{, BD, twice the ſines}\\ \text{KB, K}b\text{, twice the co-ſines}\end{array}\right\}\text{of the half ſum and half diff. of AB and AD.}$$

PB, PC, the co-tangent and the co-ſecant of the half ſum of the arcs AB, AD.

Now the following ſet of triangles being ſimilar,

*viz.* CBG, B*de*, KBE, K*bl*, DBL, CHB, BHG, K*db*.

$$\text{Then}\,\frac{CB}{CG}=\frac{Bd}{Be}=\frac{KB}{KE}=\frac{Kb}{Kl}=\frac{BD}{DL}=\frac{CH}{CB}=\frac{BH}{BG}=\frac{Kd}{Kb}$$

$$\frac{CB}{BG}=\frac{Bd}{de}=\frac{KB}{BE}=\frac{Kb}{bl}=\frac{ED}{BL}=\frac{CH}{BH}=\frac{BH}{HG}=\frac{Kd}{db}$$

$$\frac{CG}{EG}=\frac{Be}{de}=\frac{KE}{BE}=\frac{Kl}{bl}=\frac{DL}{BL}=\frac{CB}{BH}=\frac{BG}{HG}=\frac{Kb}{db}.$$

The following ſet of triangles being alſo ſimilar,

*viz.* CBF, BDE, K*b*E, Z*d*O, *d*BO, CIB, BIF, PCB, PIC, K*d*B.

There will reſult,

$$\frac{CB}{CF}=\frac{BD}{BE}=\frac{Kb}{KE}=\frac{Zd}{ZO}=\frac{Bd}{dO}=\frac{CI}{CB}=\frac{BI}{BF}=\frac{PC}{PB}=\frac{PI}{PC}=\frac{Kd}{BK}$$

$$\frac{CB}{BF}=\frac{BD}{DE}=\frac{Kb}{Eb}=\frac{Zd}{dO}=\frac{Bd}{BO}=\frac{CI}{BI}=\frac{BI}{IF}=\frac{CP}{CB}=\frac{PI}{CI}=\frac{dK}{dB}$$

$$\frac{CF}{EF}=\frac{BE}{DE}=\frac{KE}{Eb}=\frac{ZO}{dO}=\frac{dO}{BO}=\frac{CB}{BI}=\frac{BF}{IF}=\frac{PB}{CB}=\frac{PC}{CI}=\frac{BK}{Bd}.$$

170. Now the ſeveral values of the radius CB being collected, are placed in annexed table; where the letters $s$, $t$, $\text{ſ}$, $v$, ſtand for the ſine, tangent, ſecant, ſed ſine; and the letters $s^{\backprime}$, $t^{\backprime}$, ${}^{\backprime}\text{ſ}$, the co-ſine, co-tangent, co-ſecant, of the arcs $a$; or of the arcs $\tfrac{1}{2}\overline{A+a}$, $\tfrac{1}{2}\overline{A-a}$; and $v^{\backprime}$, the verſed ſine of the ſupplement.

## Goniometrical Properties.

| | | | |
|---|---|---|---|
| (171) $\frac{s^{\backprime},A+s^{\backprime},a}{s,A+s,a}\times t,\tfrac{1}{2}\overline{A+a}.$ | (172) $\frac{s^{\backprime},A+s^{\backprime},a}{s,A-s,a}\times t,\tfrac{1}{2}\overline{A-a}.$ | (173) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A-a}}{s,A+s,a}\times s,\tfrac{1}{2}\overline{A+a}.$ | (174) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A-a}}{s^{\backprime},A+s^{\backprime},a}\times s^{\backprime},\tfrac{1}{2}\overline{A+a}.$ |
| (175) $\frac{s,A+s,a}{s^{\backprime},A+s^{\backprime},a}\times t^{\backprime},\tfrac{1}{2}\overline{A+a}.$ | (176) $\frac{s,A-s,a}{s^{\backprime},A+s^{\backprime},a}\times t^{\backprime},\tfrac{1}{2}\overline{A-a}.$ | (177) $\frac{s,A+s,a}{2s^{\backprime}\,\tfrac{1}{2}\overline{A-a}}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ | (178) $\frac{s^{\backprime},A+s^{\backprime},a}{2s,\tfrac{1}{2}\overline{A-a}}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ |
| (179) $\frac{s,A-s,a}{s^{\backprime},a-s^{\backprime},A}\times t,\tfrac{1}{2}\overline{A+a}.$ | (180) $\frac{s,A+s,a}{s^{\backprime},a-s^{\backprime},A}\times t,\tfrac{1}{2}\overline{A-a}.$ | (181) $\frac{2s,\tfrac{1}{2}\overline{A-a}}{s^{\backprime},a-s^{\backprime},A}\times s,\tfrac{1}{2}\overline{A+a}.$ | (182) $\frac{2s,\tfrac{1}{2}\overline{A-a}}{s,A-s,a}\times s^{\backprime},\tfrac{1}{2}\overline{A+a}.$ |
| (183) $\frac{s^{\backprime},a-s^{\backprime},A}{s,A-s,a}\times t^{\backprime},\tfrac{1}{2}\overline{A+a}.$ | (184) $\frac{s^{\backprime},a-s^{\backprime},A}{s,a+s,A}\times t^{\backprime},\tfrac{1}{2}\overline{A-a}.$ | (185) $\frac{s^{\backprime},a-s^{\backprime},A}{2s,\tfrac{1}{2}\overline{A-a}}\times {}^{\backprime}\text{ſ},\tfrac{1}{2}\overline{A-a}.$ | (186) $\frac{s,A-s,a}{2s,\tfrac{1}{2}\overline{A-a}}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ |
| (187) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A+a}}{s,\tfrac{1}{2}\overline{A+a}}\times t,\tfrac{1}{2}\overline{A+a}.$ | (188) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A-a}}{s,\tfrac{1}{2}\overline{A-a}}\times t,\tfrac{1}{2}\overline{A-a}.$ | (189) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A+a}}{s,\overline{A+a}}\times s,\tfrac{1}{2}\overline{A+a}.$ | (190) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A-a}}{s,\overline{A-a}}\times s,\tfrac{1}{2}\overline{A-a}.$ |
| (191) $\frac{v^{\backprime},\overline{A+a}}{s,\overline{A+a}}\times t,\tfrac{1}{2}\overline{A+a}.$ | (192) $\frac{v^{\backprime},\overline{A-a}}{s,\overline{A-a}}\times t,\tfrac{1}{2}\overline{A-a}.$ | (193) $\frac{2s,\tfrac{1}{2}\overline{A+a}}{v,\overline{A+a}}\times s,\tfrac{1}{2}\overline{A+a}.$ | (194) $\frac{2s,\tfrac{1}{2}\overline{A-a}}{v,\overline{A-a}}\times s,\tfrac{1}{2}\overline{A-a}.$ |
| (195) $\frac{s,\overline{A+a}}{v,\overline{A+a}}\times t,\tfrac{1}{2}\overline{A+a}.$ | (196) $\frac{s,\overline{A-a}}{v,\overline{A-a}}\times t,\tfrac{1}{2}\overline{A-a}.$ | (197) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A+a}}{v^{\backprime},\tfrac{1}{2}\overline{A+a}}\times s^{\backprime},\tfrac{1}{2}\overline{A+a}.$ | (198) $\frac{2s^{\backprime},\tfrac{1}{2}\overline{A-a}}{v^{\backprime},\overline{A-a}}\times s^{\backprime},\tfrac{1}{2}\overline{A-a}.$ |
| (199) $\frac{t,\tfrac{1}{2}\overline{A+a}}{\text{ſ},\tfrac{1}{2}\overline{A+a}}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ | (200) $\frac{s,\tfrac{1}{2}\overline{A-a}}{t,\tfrac{1}{2}\overline{A-a}}\times \text{ſ},\tfrac{1}{2}\overline{A-a}.$ | (201) $\frac{s,\tfrac{1}{2}\overline{A+a}}{t,\tfrac{1}{2}\overline{A+a}}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ | (202) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A+a}}{t^{\backprime},\tfrac{1}{2}\overline{A+a}}\times {}^{\backprime}\text{ſ},\tfrac{1}{2}\overline{A+a}.$ |
| (203) $\frac{t^{\backprime},\tfrac{1}{2}\overline{A+a}}{r}\times t,\tfrac{1}{2}\overline{A+a}.$ | (204) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A-a}}{r}\times \text{ſ},\tfrac{1}{2}\overline{A-a}.$ | (205) $\frac{s,\tfrac{1}{2}\overline{A+a}}{r}\times {}^{\backprime}\text{ſ},\tfrac{1}{2}\overline{A+a}.$ | (206) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A+a}}{r}\times \text{ſ},\tfrac{1}{2}\overline{A+a}.$ |
| (207) $\frac{s,\tfrac{1}{2}\overline{A+a}}{\text{ſ}-s,\tfrac{1}{2}\overline{A+a}}\times t,\tfrac{1}{2}\overline{A+a}.$ | (208) $\frac{s,\tfrac{1}{2}\overline{A-a}}{\text{ſ}-s,\tfrac{1}{2}\overline{A-a}}\times t,\tfrac{1}{2}\overline{A-a}.$ | (209) $\frac{s,\tfrac{1}{2}\overline{A+a}}{\text{ſ},\tfrac{1}{2}\overline{A+a}}\times \overline{t+t^{\backprime}},\tfrac{1}{2}\overline{A+a}.$ | (210) $\frac{s^{\backprime},\tfrac{1}{2}\overline{A+a}}{\text{ſ},\tfrac{1}{2}\overline{A+a}}\times \overline{t+t^{\backprime}},\tfrac{1}{2}\overline{A+a}.$ |

From the preceding table a very great number of properties are readily deduced; ſome of which are here annexed, as examples of its uſe; where analogies are, in general, expreſſed by equal ratios.

211. The $\frac{\text{ſum of the ſines of two arcs}}{\text{diff. of the ſines of thoſe arcs}} = \frac{\text{tan. of half the ſum of thoſe arcs}}{\text{tan. of half the diff. of the arcs}}$.
(171, 172)

212. The $\frac{\text{ſum of the co-ſin. of two arcs}}{\text{diff. of the co-ſin. of thoſe arcs}} = \frac{\text{co-tan. of half the ſum of the arcs}}{\text{tan. of half the diff. of the arcs}}$.
(175, 180)

213. The $\frac{\text{ſine of the ſum of two arcs}}{\text{ſine of the diff. of thoſe arcs}} = \frac{\text{ſum of the tan. of thoſe arcs}}{\text{diff. of the tan. of thoſe arcs}}$.

For $\frac{s,A + s,a}{s,A - s,a} = \frac{t,\frac{1}{2}\overline{A+a}}{t,\frac{1}{2}\overline{A-a}}$ (211.) And $\frac{s,A + s,A + s,A - s,a}{s,A + s,a - s,A + s,A} = \frac{t,\frac{1}{2}\overline{A+a} + t,\frac{1}{2}\overline{A-a}}{t,\frac{1}{2}\overline{A+a} - t,\frac{1}{2}\overline{A-a}}$ by Compoſition.

Then $\frac{t,\frac{1}{2}\overline{A+a} + t,\frac{1}{2}\overline{A-a}}{t,\frac{1}{2}\overline{A+a} - t,\frac{1}{2}\overline{A-a}} = \left(\frac{s,A + s,A}{s,a + s,a}\right.$ (III. 47, 48) $\left. = \frac{2s,A}{2s,a} = \right) \frac{s,A}{s,a}$.
Here the arcs A, $a$, are the ſum and diff. of the arcs $\frac{1}{2}\overline{A+a}$, $\frac{1}{2}\overline{A-a}$.

214. The $\frac{\text{coſ. of the ſum of two arcs}}{\text{coſ. of the diff. of the arcs}} = \frac{\text{diff. of tan. of one and cot. of other}}{\text{ſum of tan. of one and cot. of other}}$; taking the tan. of the ſame arc.

For $\frac{\grave{t},\frac{1}{2}\overline{A+a}}{t,\frac{1}{2}\overline{A-a}} = \frac{\grave{s},A + \grave{s},a}{\grave{s},a - \grave{s},A}$ (212. And $\frac{\grave{t},\frac{1}{2}\overline{A+a} - t,\frac{1}{2}\overline{A-a}}{\grave{t},\frac{1}{2}\overline{A+a} + t,\frac{1}{2}\overline{A-a}} = \frac{\grave{s},A + \grave{s},a - \grave{s},a + \grave{s},A}{\grave{s},A + \grave{s},a + \grave{s},a - \grave{s},A}$

Then $\frac{\grave{t},\frac{1}{2}\overline{A+a} - t,\frac{1}{2}\overline{A-a}}{\grave{t},\frac{1}{2}\overline{A+a} + t,\frac{1}{2}\overline{A-a}} = \left(\frac{2\grave{s}A}{2\grave{s}a} = \right)\frac{\grave{s},A}{\grave{s},a}$.
Here the arcs A, $a$, are the ſum and diff. of the arcs $\frac{1}{2}\overline{A+a}$, $\frac{1}{2}\overline{A-a}$.

215. The ſine of the ſum of two arcs, into radius; is equal to the ſum of the products, of the ſine of the greater by the co-ſine of the leſs, and the ſine of the leſs by the co-ſine of the greater. And,

The ſine of the difference of two arcs, into radius; is equal to the difference of the products, of the ſine of the greater by the co-ſine of the leſs, and the ſine of the leſs by the co-ſine of the greater.

For $\left\{\begin{array}{l} R \times \overline{\tfrac{1}{2}s,A + \tfrac{1}{2}s,a} = s,\tfrac{1}{2}\overline{A+a} \times \grave{s},\tfrac{1}{2}\overline{A-a}\,(173). \\ R \times \overline{\tfrac{1}{2}s,A - \tfrac{1}{2}s,a} = s,\tfrac{1}{2}\overline{A-a} \times s,\tfrac{1}{2}\overline{A+a}\,(182). \end{array}\right\}$ Here $\frac{1}{2}\overline{A+a}$ and $\frac{1}{2}\overline{A-a}$ are the arcs.

Hence $\left\{\begin{array}{l} R \times s,A\text{(the ſum)} = s,\tfrac{1}{2}\overline{A+a} \times \grave{s},\tfrac{1}{2}\overline{A-a} + s,\tfrac{1}{2}\overline{A-a} \times \grave{s},\tfrac{1}{2}\overline{A+a} \\ R \times s,a\text{(the diff.)} = s,\tfrac{1}{2}\overline{A+a} \times s,\tfrac{1}{2}\overline{A-a} - s,\tfrac{1}{2}\overline{A-a} \times \grave{s},\tfrac{1}{2}\overline{A+a}. \end{array}\right.$

216. The co-ſine of the ſum of two arcs, into radius; is equal to the difference, between the product of the co-ſines, and product of the ſines, of thoſe arcs.

The co-ſine of the difference of two arcs, into radius; is equal to the ſum, of the product of the co-ſines, and product of the ſines, of thoſe arcs.

For $\left\{\begin{aligned} &R\times\overline{\tfrac{1}{2}s,A+\tfrac{1}{2}s^{\backprime},a}=s^{\backprime},\tfrac{1}{2}\overline{A+a}\times s^{\backprime},\tfrac{1}{2}\overline{A-a}\,(174).\\ &R\times\tfrac{1}{2}\overline{s^{\backprime},a}-\tfrac{1}{2}s^{\backprime},A=s,\tfrac{1}{2}\overline{A+a}\times s,\tfrac{1}{2}\overline{A-a}\,(181).\end{aligned}\right\}$ $\tfrac{1}{2}\overline{A+a}$ and $\tfrac{1}{2}\overline{A-a}$ being the arcs.

Then $\left\{\begin{aligned} &R\times s^{\backprime},A(\text{the ſum})=s^{\backprime},\tfrac{1}{2}\overline{A+a}\times s^{\backprime},\tfrac{1}{2}\overline{A-a}-s,\tfrac{1}{2}\overline{A+a}\times s,\tfrac{1}{2}\overline{A-a}.\\ &R\times s^{\backprime},a(\text{the diff.})=s^{\backprime},\tfrac{1}{2}\overline{A+a}\times s^{\backprime},\tfrac{1}{2}\overline{A-a}+s,\tfrac{1}{2}\overline{A+a}\times s,\tfrac{1}{2}\overline{A-a}.\end{aligned}\right.$

217. $\dfrac{\text{Radius, leſs the co-ſine of an arc}}{\text{Radius, more the co-ſine of an arc}}=\dfrac{\text{Square the tan. of half that arc}}{\text{Square of the Radius}}$

$$\text{For}\left\{\begin{aligned} &R-s^{\backprime},\overline{A+a}=\left(v,\overline{A+a}=\right)\frac{t,\tfrac{1}{2}\overline{A+a}}{R}\times s,\overline{A+a}. &&(195)\\ &R+s^{\backprime},\overline{A+a}=\left(v^{\backprime},\overline{A+a}=\right)\frac{R}{t,\tfrac{1}{2}\overline{A+a}}\times s,\overline{A+a}. &&(191)\end{aligned}\right.$$

Then $\dfrac{R-s^{\backprime},\overline{A+a}}{R+s^{\backprime},\overline{A+a}}=\dfrac{tt.\tfrac{1}{2}\overline{A+a}.}{RR}$

218. The $\dfrac{\text{ſum of the ſine \& co-ſine of an arc}}{\text{diff. of the ſine \& co-ſine of that arc}}=\dfrac{\text{Radius}}{\text{tan. of diff. of that arc \& }45^\circ}.$

The $\dfrac{\text{ſum of Rad. and tan. of an arc}}{\text{diff. of Rad. and tan. of that arc}}=\dfrac{\text{Radius}}{\text{tan. of diff. of that arc \& }45^\circ}.$

For if $A+a=90^\circ$; then $\tfrac{1}{2}A=45^\circ-\tfrac{1}{2}a$; and $\tfrac{1}{2}a=45^\circ-\tfrac{1}{2}A$.

Alſo $s^{\backprime},a=s,A$ : $s,a=s^{\backprime},A$ : and $s,A=t,A\times s^{\backprime},A\div R$. (III. 33)

Then $\dfrac{s^{\backprime},A+s^{\backprime},a}{s^{\backprime},a-s^{\backprime},A}=\dfrac{t^{\backprime},\tfrac{1}{2}\overline{A+a}.}{t,\tfrac{1}{2}\overline{A-a}.}$ (212)

Or $\dfrac{s^{\backprime},A+s,A}{s,A-s,A}=\left(\dfrac{t^{\backprime},45^\circ-\tfrac{1}{2}a+\tfrac{1}{2}a}{t,A-45^\circ+\tfrac{1}{2}A}\right)=\dfrac{R}{t,\overline{A\backsim 45^\circ}}$*.

Again, $\dfrac{s,A+s^{\backprime},A}{s,A-s^{\backprime},A}=\left((\text{III. }33)\dfrac{t,A\times s^{\backprime},A\div R+s^{\backprime},A}{t,A\times s^{\backprime},A\div R-s^{\backprime},A}=\right)\dfrac{t,A\times s^{\backprime},A+s^{\backprime},A\times R}{t,A\times s^{\backprime},A-s,A\times R}.$

Then $\left(\dfrac{s,A+s^{\backprime},A}{s,A-s^{\backprime},A}=\dfrac{t,\overline{A+R}\times s^{\backprime},A}{t,\overline{A-R}\times s^{\backprime},A}=\right)\dfrac{R+t,A}{R\backsim t,A}=\dfrac{R}{t,\overline{A\backsim 45^\circ}}.$

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* This mark $\backsim$ ſhews the difference of the values it ſtands between.

219. The difference of the co-ſines of two arcs, is equal to the difference of the verſed ſines of thoſe arcs.

220. The product of the ſines of two arcs, is equal to the product of half the radius into the difference of the co-ſines, of the ſum and difference of thoſe arcs.

That is, $s,\tfrac{1}{2}\overline{A+a}\times s\,\tfrac{1}{2}\overline{A-a}=\tfrac{1}{2}R\times\overline{s^{\backprime},a-s^{\backprime},A}=s^{\backprime},\overline{\tfrac{1}{2}\overline{A+a}-\tfrac{1}{2}\overline{A-a}}-s^{\backprime},\overline{\tfrac{1}{2}\overline{A+a}+\tfrac{1}{2}\overline{A-a}}.$ (181)

Or $s,Z\times s,X=\tfrac{1}{2}R\times\overline{s^{\backprime},\overline{Z+X}-s^{\backprime},\overline{Z-X}}.$ Putting $Z=\tfrac{1}{2}\overline{A+a}$; $X=\tfrac{1}{2}\overline{A-a}.$

221. The product of the ſines of two arcs, is equal to the product of half the Radius into the difference between the verſed ſines, of the ſum and difference of thoſe arcs.

That is, $s,Z\times s,X=(\tfrac{1}{2}R\times\overline{s^{\backprime},\overline{Z+X}-s^{\backprime},\overline{Z-X}}(220)=)\overline{v,\overline{Z+X}-v,\overline{Z-X}}\times\tfrac{1}{2}R.$ (219)

222. Half the Radius $=\dfrac{\text{ſquare of the ſine of an arc}}{\text{verſed ſine of twice that arc}}.$ (193)

$=\dfrac{\text{ſquare of the co-ſine of an arc}}{\text{ſup-verſed ſine of twice that arc}}.$ (197)

$=\dfrac{\text{product of the ſines of two arcs}}{\text{diff. of ver. ſines of the ſum and diff. of thoſe arcs}}.$ (221)

223. The ſq. of Rad. $=\dfrac{\text{prod. of the ſquares, of the ſine and cot. of an arc}}{\text{ſquare of the co-ſine of that arc}}.$

$=\dfrac{\text{prod. of the ſquares of the co-ſine \& tan. of an arc}}{\text{ſquare of the ſine of that arc}}.$

For $R=\dfrac{s,A\times t^{\backprime},A}{s^{\backprime},A}=\dfrac{s^{\backprime},A\times t,A}{s,A}$(187.) Then $RR\dfrac{ss,A\times t^{\backprime}t^{\backprime},A}{s^{\backprime}s^{\backprime},A}=\dfrac{s^{\backprime}s^{\backprime},A\times tt,A}{ss,A}.$

224. The product of Radius, and the co-ſine of an arc, is equal to the difference of the ſquares, of the ſine and co-ſine of half that arc.

For $\dfrac{s^{\backprime}s^{\backprime},\tfrac{1}{2}}{ss,\tfrac{1}{2}}=\left(\dfrac{v^{\backprime},\overline{A+a}}{v,\overline{A+a}}(193,\ 197)=\right)\dfrac{R\times s,\overline{A+a}}{R-s^{\backprime},\overline{A+a}}.$ Put $Z=\overline{A+a}.$

Then $\dfrac{R+s^{\backprime},Z-R+s^{\backprime},Z}{R+s^{\backprime},Z+R-s^{\backprime},Z}=\dfrac{s^{\backprime}s^{\backprime},\tfrac{1}{2}Z-ss,\tfrac{1}{2}Z}{s^{\backprime}s^{\backprime},\tfrac{1}{2}Z+ss,\tfrac{1}{2}Z}.$ By compoſition and diviſion.

$\dfrac{2s^{\backprime},Z}{2R}=\dfrac{s^{\backprime}s^{\backprime},\tfrac{1}{2}Z-ss,\tfrac{1}{2}Z}{RR}$(II. 111.) And $R\times s^{\backprime},Z=s^{\backprime}s^{\backprime},\tfrac{1}{2}Z-ss,\tfrac{1}{2}Z.$

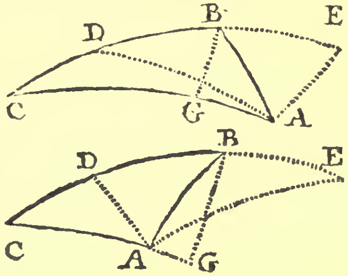
$=\overline{s,\tfrac{1}{2}Z+s,\tfrac{1}{2}Z}\times\overline{s^{\backprime},\tfrac{1}{2}Z-s,\tfrac{1}{2}Z}.$ (II. 119)

In any ſpheric triangle ABC, if in the ſide CB produced, be taken BE, BD, each equal to BA, and BG be drawn at right angles to CA.

Then $\left.\begin{array}{l} CE = BC + BA \text{ is the ſum} \\ CD = BC - BA \text{ is the diff.} \end{array}\right\}$ of the legs including the angle at B.

CG and AG are the ſegments of the baſe, or ſide oppoſite to the angle B. $\angle A$ and $\angle C$ are called baſe angles. $\angle CBG = a$, $\angle ABG = c$ are the vertical angles.

Now a very great number of relations may be formed between the ſides and angles; ſome of which are here enumerated.



225. The ſines of the legs, are as the ſines of the oppoſite baſe angles.
That is, $s,BC : s,BA :: s,A : s,C$. (110)

$$\text{Hence } \frac{\text{ſum of the ſines of the legs}}{\text{diff. of the ſines of the legs}} = \frac{\text{ſum of the ſines of the baſe angles}}{\text{diff. of the ſines of the baſe angles}}$$

by compoſition.

226. The co-ſines of the baſe angles, are as the ſines of the vertical angles.
That is, $\grave{s},C : \grave{s},A :: s,a : s,c$. (3d of 122, and 2d of 124)

$$\text{Hence } \frac{\text{ſum of co-ſines of baſe angles}}{\text{diff. of co-ſines of baſe angles}} = \frac{\text{ſum of the ſines of vertical angles}}{\text{diff. of ſines of vertical angles}}$$

by compoſition and diviſion of ratios.

227. The co-ſines of the legs, are as the co-ſines of the adjacent ſegments of the baſe.
That is, $\grave{s},BC : \grave{s},BA :: \grave{s},CG : \grave{s},AG$. (3d of 121, and 2d of 123)

$$\text{Hence } \frac{\text{ſum of co-ſines of the legs}}{\text{diff. of co-ſines of the legs}} = \frac{\text{ſum of co-ſines of baſe ſegments}}{\text{diff. of co-ſines of baſe ſegments}}$$

by compoſition and diviſion of ratios.

228. The co-tangents of the legs, are as the co-ſines of the adjacent vertical angles.
That is, $\grave{t},BC : \grave{t},BA :: \grave{s},a : \grave{s},c$. (2d of 121, and 2d of 124)

$$\text{Hence } \frac{\text{ſum of co-t. of the legs}}{\text{diff. of co-t. of the legs}} = \frac{\text{ſum of co-ſines of vert. angles}}{\text{diff. of co-ſines of vert. angles}}$$

by compoſition and diviſion of ratios.

229. The tangents of the legs, are as the co-ſ. of the adjacent vertical angles reciprocally.

$$\text{Hence } \frac{\text{ſum of tan. of the legs}}{\text{diff. of tan. of the legs}} = \frac{\text{ſum of co-ſ. of vert. angles}}{\text{diff. of co-ſ. of vert. angles}}$$

by comp. &c.

230. The ſines of the baſe ſegments, are as the tangents of the adjacent baſe angles reciprocally.

That is, $s,CG : s,AG :: (t^{\backprime},C : t^{\backprime},A ::)\ t,A ; t,C.$ (2d of 122, and 1ſt of 123)

Hence $\frac{\text{ſum of ſines of baſe ſegments}}{\text{diff. of ſines of baſe ſegments}} = \frac{\text{ſum of tan of baſe angles}}{\text{diff. of tan. of baſe angles}}$ by compoſition and diviſion.

231. The tangents of the baſe ſegments, are as the tangents of the oppoſite vertical angles.

That is, $t,CG : t,AG :: t,a : t,c.$ (108)

Hence $\frac{\text{ſum of tan. of baſe ſegments}}{\text{diff. of tan. of baſe ſegments}} = \frac{\text{ſum of tan. of vert. angles}}{\text{diff. of tan. of vert, angles}}$ by compoſition and diviſion of ratios.

232. The $\frac{\text{tan. of half the ſum of the legs}}{\text{tan. of half the diff. of the legs}} = \frac{\text{tan. of half the ſum of the baſe ang.}}{\text{tan. of half the diff. of the baſe ang.}}$

For $\frac{s,BC+s,BA}{s,BC,BA} = \frac{s,A+s,C}{s,A-s,C}$ (225) $= \frac{t,\frac{1}{2}\overline{BC+BA}}{t,\frac{1}{2}\overline{BC-BA}} = \frac{t,\frac{1}{2}\overline{A+C}}{t,\frac{1}{2}\overline{A-C}}.$ (211)

233. The $\frac{\text{tan. of }\frac{1}{2}\text{ ſum of baſe ſegments}}{\text{tan. of }\frac{1}{2}\text{ ſum of the legs}} = \frac{\text{tan. of }\frac{1}{2}\text{ diff. of the legs}}{\text{tan. of }\frac{1}{2}\text{ diff. of the baſe ſegments}}.$

For $\frac{s^{\backprime},BA+s^{\backprime},BC}{s^{\backprime},BA-s^{\backprime},BC} = \frac{s^{\backprime},GA+s^{\backprime},GC}{s^{\backprime},GA-s^{\backprime},GC}$ (227) $= \frac{t^{\backprime},\frac{1}{2}\overline{BC+BA}}{t,\frac{1}{2}\overline{BC-BA}} = \frac{t^{\backprime},\frac{1}{2}\overline{CG+GA}}{t,\frac{1}{2}\overline{CG-GA}}.$ (212)

Then $\frac{t,\frac{1}{2}\overline{BC-BA}}{G-GA} = \left(\frac{t^{\backprime},\frac{1}{2}\overline{BC+BA}}{t^{\backprime},\frac{1}{2}\overline{CG+GA}}\right.$ (II. 145) $\left.=\right)\frac{t,\frac{1}{2}\overline{CG+GA}}{t,\frac{1}{2}\overline{CB+BA}}.$ (III. 37)

234. The $\frac{\text{ſine of ſum of legs}}{\text{ſine of diff. of legs}} = \frac{\text{co-tan. of }\frac{1}{2}\text{ ſum of vert. angles}}{\text{tan. of }\frac{1}{2}\text{ diff. of vert. angles}}$

$= \frac{\text{co-tan. }\frac{1}{2}\text{ diff. of vert. angles}}{\text{tan. of }\frac{1}{2}\text{ ſum of vert. angles}}.$

For $\frac{s,\overline{BC+BA}}{s,\overline{BC-BA}} = \left(\frac{t,BC+t,BA}{t,BC-t,BA}\right.$ (213) $= \frac{s^{\backprime},c+s^{\backprime},a}{s^{\backprime},c-s^{\backprime},a}$ (226) $\left.=\right)\frac{t^{\backprime},\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{a-c}} = \frac{t^{\backprime},\frac{1}{2}\overline{a-c}}{t,\frac{1}{2}\overline{a+c}}.$ (212)

235. The $\frac{\text{cot. of }\frac{1}{2}\text{ ſum of vert. angles}}{\text{tan. of }\frac{1}{2}\text{ ſum of the baſe angles}} = \frac{\text{tan. of }\frac{1}{2}\text{ diff. of baſe angles}}{\text{tan. of }\frac{1}{2}\text{ diff. of vert. angles}}.$

For $\frac{t^{\backprime},\frac{1}{2}\overline{A+C}}{t^{\backprime},\frac{1}{2}\overline{A-C}} = \left(\frac{s^{\backprime},C+s^{\backprime},A}{s^{\backprime},C-s^{\backprime},A}\right.$ (212) $= \frac{s,a+s,c}{s,a-s,c}$ (226) $\left.=\right)\frac{t,\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{a-c}}$ (211)

Hence $\frac{t,\frac{1}{2}\overline{A-C}}{t,\frac{1}{2}\overline{a-c}} = \left(\frac{t^{\backprime},\frac{1}{2}\overline{A+C}}{t,\frac{1}{2}\overline{a+c}}\right.$ (II. 145) $\left.=\right)\frac{t^{\backprime},\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{A+C}}.$ (III. 37)

236. The $\frac{\text{ſine of ſum of the legs}}{\text{ſine of diff. of the legs}} = \frac{\text{ſquare of co-t. of } \frac{1}{2} \text{ ſum of vert. angles}}{\text{tan. } \frac{1}{2} \text{ ſum, into tan. } \frac{1}{2} \text{ diff. of baſe } \angle^{s}}$.

For $\frac{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C}}{\grave{t},\frac{1}{2}\overline{a+c}} = t,\frac{1}{2}\overline{a-c}$. (235)

Then $s,\overline{BC+BA} : s,\overline{BC-BA} :: \grave{t},\frac{1}{2}\overline{a+c} : \frac{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C}}{t,\frac{1}{2}\overline{a+c}}$. (234)

$:: \grave{t}\grave{t},\frac{1}{2}\overline{a+c} : t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C}$. (II. 151)

237. The $\frac{\text{ſine of } \frac{1}{2} \text{ the ſum of legs}}{\text{ſine of } \frac{1}{2} \text{ the diff. of legs}} = \frac{\text{co-tan. of } \frac{1}{2} \text{ the ſum of vert. angles}}{\text{tan. of } \frac{1}{2} \text{ the diff. of the baſe angles}}$.

For $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C}} = \frac{s,\overline{BC+BA}}{s,\overline{BC-BA}}$ (236). And $\frac{t,\frac{1}{2}\overline{A+C}}{t,\frac{1}{2}\overline{A-C}} = \frac{t,\frac{1}{2}\overline{BC+BA}}{t,\frac{1}{2}\overline{BC-BA}}$. (232)

Then $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c} \times t,\frac{1}{2}\overline{A+C}}{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C} \times t,\frac{1}{2}\overline{A-C}} = \frac{s,\overline{BC+BA} \times t,\frac{1}{2}\overline{BC+BA}}{s,\overline{BC-BA} \times t,\frac{1}{2}\overline{BC-BA}}$. (II. 156)

And $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c}}{tt,\frac{1}{2}\overline{A-C}} = \frac{2ss,\frac{1}{2}\overline{BC+BA}}{2ss,\frac{1}{2}\overline{BC-BA}}$ (195, 193). Then $\frac{s,\frac{1}{2}\overline{BC+BA}}{s,\frac{1}{2}\overline{BC-BA}} = \frac{\grave{t},\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{A-C}}$.

238. The $\frac{\text{coſ. of } \frac{1}{2} \text{ ſum of the legs}}{\text{coſ. of } \frac{1}{2} \text{ diff. of the legs}} = \frac{\text{co-t. of } \frac{1}{2} \text{ the ſum of vertical angles}}{\text{tan. of } \frac{1}{2} \text{ the ſum of the baſe angles}}$.

For $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C}} = \frac{s,\overline{BC+BA}}{s,\overline{BC-BA}}$ (236). And $\frac{t,\frac{1}{2}\overline{A-C}}{t,\frac{1}{2}\overline{A+C}} = \frac{\grave{t},\frac{1}{2}\overline{BC+BA}}{\grave{t},\frac{1}{2}\overline{BC-BA}}$. (232)

Then $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c} \times t,\frac{1}{2}\overline{A-C}}{t,\frac{1}{2}\overline{A+C} \times t,\frac{1}{2}\overline{A-C} \times t,\frac{1}{2}\overline{A+C}} = \frac{s,\overline{BC+BA} \times \grave{t},\frac{1}{2}\overline{BC+BA}}{s,\overline{BC-BA} \times t,\frac{1}{2}\overline{BC-BA}}$. (II. 156)

And $\frac{\grave{t}\grave{t},\frac{1}{2}\overline{a+c}}{tt,\frac{1}{2}\overline{A+C}} = \frac{2\grave{s}\grave{s},\frac{1}{2}\overline{BC+BA}}{2\grave{s}\grave{s},\frac{1}{2}\overline{BC-BA}}$ (191, 197). Then $\frac{\grave{s},\frac{1}{2}\overline{BC+BA}}{\grave{s},\frac{1}{2}\overline{BC-BA}} = \frac{t,\frac{1}{2}\overline{a+c}}{t,\frac{1}{2}\overline{A+C}}$.

The two laſt propoſitions ſolve the problem where two ſides, and the included angle, of a ſpheric triangle, are given to find the other angles.

Or where two angles and the included ſide are given, to find the other ſides, uſing the word angles for legs; the given ſide for ſum of vertical angles; the other ſide for baſe angles.

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In art. 237, 238, the concluſions were gained from this principle, namely, that *the ſides of proportional ſquares, are in the ſame proportion as thoſe ſquares.*

239. The co-ſine of an angle, is to Radius ;
As the Radius into co-ſ. of the oppoſite ſide, leſs the product of the co-ſines of the including ſides,
To the product of the ſines of the including ſides.

For $\grave{s},CG = (\overline{\grave{s},AC - AG} =)\,\overline{\grave{s},AC \times \grave{s},AG + s,AC \times s,AG} \div R.$ (216)

And $(\grave{s},CG =)\,\frac{\grave{s},BC \times \grave{s},AG}{\grave{s},AB}$ (227) $= \overline{\grave{s},AC \times \grave{s}AG + s,AC \times s,AG} \div R.$ (II.46)

Therefore $\grave{s},BC \times \grave{s},AG = \frac{\grave{s},AB}{R} \times \grave{s},AC \times \grave{s},AG + \frac{\grave{s},AB}{R} \times s,AC \times s,AG.$

Therefore $\frac{R \times \grave{s},BC - \grave{s},AB \times \grave{s},AC}{R} \times \grave{s},AG = \frac{\grave{s},AB \times s,AC}{R} \times s,AG.$

Then $\frac{R \times \grave{s},BC - \grave{s},AB \times \grave{s},AC}{\grave{s},AB \times s,CA} = \left(\frac{s,AG}{\grave{s},AG}\text{(II. 163)} =\right) \frac{t,AG}{R}$

But $t,AG = \frac{\grave{s},A}{R} \times t,AB$ (131). And $\frac{t,AG}{R} = \left(\frac{\grave{s},A}{RR} \times t,AB =\right) \frac{\grave{s},A}{RR} \times \frac{s,AB \times R}{\grave{s},AB}.$
(187)

Then $\frac{\grave{s},A}{R} \times \frac{s,AB}{\grave{s},AB} = \frac{R \times \grave{s},BC - \grave{s},AB \times \grave{s},AC}{s,AB \times s,AC}$. And $\frac{\grave{s},A}{R} = \frac{R \times \grave{s},BC - \grave{s},AC \times \grave{s},AB}{s,AC \times s,AB}.$

240. Hence $R \times \grave{s},BC = \frac{\grave{s}A \times s,AC \times s,AB}{R} + \grave{s},AC \times \grave{s},AB.$

241. The verſed ſine of an angle, is to the ſquare of Radius :
As the diff. of the verſed ſines of op. ſide, and diff. of including ſides,
To the product of the ſines of the ſides including that angle.

For (239) $\frac{R \times \grave{s},BC - \grave{s},AC \times \grave{s},AB}{s,AB \times s,AC.} = \left(\frac{\grave{s}A}{R} =\right) \frac{R - v,A}{R}.$

Therefore $RR \times \grave{s},BC - R \times \grave{s},AC \times \grave{s},AB = R \times s,AC \times s,AB - s,AC \times s,AB \times v,A.$

Therefore $RR \times \grave{s},BC + s,AC \times s,AB \times v,A = \overline{\grave{s},AC \times \grave{s},AB + s,AC \times \grave{s},AB} \times R.$
$= \grave{s},\overline{AC - AB} \times RR.$ (216)

Then $s,AC \times s,AB \times v,A = (RR \times \grave{s},CD - RR \times \grave{s},BC =)\,\overline{\grave{s},CD - \grave{s},BC} \times RR.$

And $\frac{v,A}{RR} = \left(\frac{\grave{s},CD - \grave{s},BC}{s,AC \times s,AB} =\right) \frac{v,CB - v,CD}{s,AC \times s,AB}.$ (219)

242. Hence $\frac{v,A}{2R}=\frac{v,BC-v,CD}{v,CE-v,CD}$. Or $\frac{1}{2}v,A=\frac{v,BC-v,CD}{v,CE-v,CD}$, when $R=1$.

For $(s,AC\times s,AB=)\frac{1}{2}R\times\overline{v,CE-v,CD}\ (222)=\overline{v,CB-v,CD}\times\frac{RR}{v,A}$. (241)

Then $\frac{v,CB-v,CD}{v,CE-v,CD}=\left(\frac{\frac{1}{2}R\times v,A}{RR}=\right)\frac{v,A}{2R}$.

243. The verſed ſine of the ſup. of an angle, is to the ſquare of Radius;
As the diff. of the verſed ſines of the oppoſite ſide, and ſum of the including ſides,
To the product of the ſines of the ſides including that angle.

For (239) $\frac{R\times\grave{s},BC-\grave{s},AC\times\grave{s},AB}{s,AC\times s,AB}=\left(\frac{\grave{s},A}{R}=\right)\frac{v,A-R}{R}$.

Therefore $RR\times\grave{s},BC-R\times\grave{s},AC\times\grave{s},AB=s,AC\times s,AB\times\grave{v},A-R\times s,AC\times s,AB$.

Therefore $RR\times\grave{s},BC-s,AC\times s,AB\times\grave{v},A=R\times\overline{\grave{s},AC\times\grave{s},AB-s,AC\times s,AB}$
$=(RR\times\grave{s},\overline{AC+AB}=)\,RR\times\grave{s},CE$. (216)

Then $RR\times\grave{s},BC-RR\times\grave{s},CE=s,AC\times s,AB\times\grave{v},A$.

And $\frac{\grave{v},A}{RR}=\left(\frac{\grave{s},BC-\grave{s},CE}{s,AC\times s,AB}=\right)\frac{v,CE-v,BC}{s,AC\times s,AB}$.

244. Hence $\frac{\grave{v},A}{2R}=\frac{v,CE-v,CB}{v,CE-v,CD}$. Or $\frac{1}{2}\grave{v},A=\frac{v,CE-v,CB}{v,CE-v,CD}$, when $R=1$.

For $(s,AC\times s,AB=)\frac{1}{2}R\times\overline{v,CE-v,CD}\ (222)=\overline{v,CE-v,CB}\times\frac{RR}{\grave{v},A}$. (243)

Then $\frac{v,CE-v,CB}{v,CE-v,CD}=\left(\frac{\frac{1}{2}R\times\grave{v},A}{RR}=\right)\frac{\grave{v},A}{2R}$.

245. The ſquare of the ſine of half an angle, is to the ſquare of the Radius;
As $\frac{1}{2}$ Radius into the diff. of the verſed ſines of the ſide oppoſite, and diff. of the ſides including that angle,
To the product of the ſines of the ſides including that angle.

For $(v,A=)\frac{ss,\frac{1}{2}A}{\frac{1}{2}R}\ (222)=\frac{v,CB-v,CD}{s,AC\times s,AB}\times RR$. (241)

Then $\frac{ss,\frac{1}{2}A}{RR}=\frac{v,CB-v,CD}{s,AC\times s,AB}\times\frac{1}{2}R=\frac{\grave{s},CD-\grave{s}CB}{s,AC\times s,AB}\times\frac{1}{2}R$. (219)

246. Hence $\frac{ss,\frac{1}{2}A}{RR}=\frac{s,\frac{1}{2}\overline{CB+CD}\times s,\frac{1}{2}\overline{CB-CD}}{s,AC\times s,AB}$ (181)

$=\frac{s,\frac{1}{2}\overline{CB+AC-AB}\times s,\frac{1}{2}\overline{CB-AC+AB}}{s,AC\times s,AB}$; becauſe $CD=AC-AB$

$=\frac{s,\overline{H-AC}\times s,\overline{H-AB}}{s,AC\times s,AB}$. Putting $2H=AC+AB+BC$.

247. The ſqu. of the co-ſine of half an angle, is to the ſqu. of Radius;
As $\frac{1}{2}$ Radius into the diff. of verſed lines of the ſide oppoſite, and ſum of the included ſides,
To the product of the ſines of the ſides including that angle.

For $(v^{\backprime},A=)\ \frac{s^{\backprime}s^{\backprime},\frac{1}{2}A}{\frac{1}{2}R}$ $(222)=\frac{v,CE-v,CB}{s,AC\times s,AB}\times RR$. (243)

Then $\frac{s^{\backprime}s^{\backprime},\frac{1}{2}A}{RR}=\frac{v,CE-v,CB}{s,AC\times s,AB}\times\frac{1}{2}R=\frac{s^{\backprime},CB-s^{\backprime},CE}{s,AC\times s,AB}\times\frac{1}{2}R$. (219)

248. Hence $\frac{s^{\backprime}s^{\backprime},\frac{1}{2}A}{RR}=\frac{s,\frac{1}{2}\overline{CE+CB}\times s,\frac{1}{2}\overline{CE-CB}}{s,AC\times s,AB}$. (177)

$=\frac{s,\frac{1}{2}\overline{AC+AB+CB}\times s,\frac{1}{2}\overline{AC+AB-CB}}{s,AC\times s,AB}$. For $CE=AC+AB$.

$=\frac{s,H\times s,\overline{H-CB}}{s,AC\times s,AB}$. Putting $2H=AC+AB+BC$.

249. The ſquare of the tan. of half an angle, is to the ſquare of Radius;
As the diff. of ver. ſines of the ſide op. and diff. of including ſides,
To the diff. of ver. ſines of the ſide op. and ſum of including ſides.

For $\frac{v,A}{v^{\backprime},A}=\frac{ss,\frac{1}{2}A}{s^{\backprime}s^{\backprime},\frac{1}{2}A}$ $(222)=\frac{tt,\frac{1}{2}A}{RR}$ $(223)=\frac{v,CB-v,CD}{v,CE-v,CB}$. (241, 243)

250. Hence $\frac{tt,\frac{1}{2}A}{RR}=\frac{s,\frac{1}{2}\overline{CB+CD}\times s,\frac{1}{2}\overline{CB-CD}}{s,\frac{1}{2}\overline{CE+CB}\times s,\frac{1}{2}\overline{CE-CB}}$. (219, 181)

$=\frac{s,\frac{1}{2}\overline{CB+AC-AB}\times s,\frac{1}{2}\overline{CB-AC+AB}}{s,\frac{1}{2}\overline{CB+AC+AB}\times s,\frac{1}{2}\overline{AC+AC-CB}}$

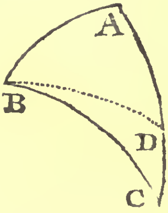
$=\frac{s,\overline{H-AB}\times s,\overline{H-AC}}{s,H\times s,\overline{H-BC}}$.

From the articles in the three laſt pages may be deduced many rules for ſolving the problem, where the three ſides are given to find an angle; and thence, from the three angles given to find a ſide.

251. When two ſides and the included angle are given to find the third ſide: Or when the three ſides are given to find an angle; For theſe particular caſes there have been given compendiums by Sir Jonas Moore, in his Mathematics, Vol. II. page 383: Alſo by Nicholas Facio Duillier, in a ſmall tract of his, which is very ſcarce; and by the learned Dr. Pemberton, in the Philoſophical Tranſactions for the year 1760: The principle employed by each of them is the ſame as in Article 245; which will be here illuſtrated on account of its utility in ſome aſtronomical ſubjects.

In the triangle ABC, where $CD = AC \backsim AB$.

Given AB, AC, $\angle A$; required BC.



$$\text{Now } \frac{ss,\frac{1}{2}\angle A}{R^3} = \frac{\frac{1}{2}s^{\backprime},CB \backsim \frac{1}{2}s^{\backprime},CD}{s,AC \times s,AB}. \qquad (245)$$

$$\text{And } \frac{ss,\frac{1}{2}\angle A \times s,AC \times s,AB}{R^3} = \frac{1}{2}s^{\backprime},CB \backsim \frac{1}{2}s^{\backprime},CD = N$$

Then $2N - s^{\backprime},CD = s^{\backprime},CB$.

Hence the following practical Rules.

252. I. *To twice the log. ſine of half the given angle,*
*Add the log. ſines of the two containing ſides;*
*The ſum, abating three radii in the index, leaves the log. ſine of an arc.*
*From twice the nat. ſine of that arc; take the nat. co-ſine of the diff. of the given ſides,*
*Leaves the nat. co-ſine of the third ſide, or of its ſupplement.*

253. II. But the ſide required may be found without the uſe of natural ſines. Thus

*To twice the log. ſine of half the given angle,*
*Add the log. ſines of the two containing ſides;*
*From half the ſum of theſe logs, ſubtract the log. ſine of half the diff. of the ſides.*
*And the remainder is the log. tangent of an arc;*
*The log. ſine of which arc, ſubtracted from the ſaid half ſum of logs,*
*Leaves the log. ſine of half the required ſide.*

Take the Example uſed in the ſix caſes of oblique ſpheric triangles.

Where AC = 83° 11′, AB = 56° 40′; ∠A = 125° 20′.

Required BC, which was there found to be 114 30.

| | | | | | | |
|---|---|---|---|---|---|---|
| Given ∠ | = | 125° 20′ | | | | |
| its half | = | 62 40 | —— | — | Log. ſine | 9,94858<br>9,94858 |
| One ſide | = | 83 11 | —— | —— | Log. ſine | 9,99692 |
| other ſide | = | 56 40 | —— | — | Log. ſine | 9,92194 |
| Arc found | = | 40 53⅔ | its nat. ſine | 65467 (256) | | 9,81602 |
| | | | its double | 130934 | | |
| diff. ſides | = | 26 30 | the nat. co-ſine | 89480 | | |
| | | 65 30 | the nat. co-ſine | 41454 (257) | | |

The ſide required 114 30

The ſaid Example wrought by the ſecond Rule is as follows;

| | | | | | | |
|---|---|---|---|---|---|---|
| Given ∠ | = | 125 20 | | | | |
| its half | | 62 40 | log. ſine | 9,94858<br>9,94858 | | |
| One ſide | = | 83 11 | log. ſine | 9,99692 | | |
| Other ſide | = | 56 40 | log. ſine | 9,92194 | | |
| Sum logs | = | | | 39,81602 | | |
| half ſum | | | | 19,90801 | half ſum log. | 19,90801 |
| $\frac{1}{2}$ diff. ſides | = | 13 $15\frac{1}{2}$ | | 9,36048 | | |
| Arc | = | 74 10 | log. tan. | 10,54753 | Log. ſine 74 10 | 9,98320 |
| | | | | | Log. ſine of 57 15 | 9,92481 |

The required ſide = 114 30

When the three ſides are given to find an angle;

254. I. *To the nat co-ſ. of the ſide oppoſite the required angle, add the nat. co-ſ. of the diff. of the ſides about that angle; half the ſum is the nat. ſine of an arc. To the log. ſine of that arc, add the arith. comps. of the log. ſines of the ſides about the required angle and alſo the radius.*

*The half of this ſum is the log. ſine of half the angle ſought.*

Or without uſing the natural ſines.

255. II. *To the log. ſine of half the diff. of the ſides about the angle, add the arith. comp. of the log. ſine of half the baſe; the ſum is the log. ſine of an arc. To the log. co-ſine of this arc, add the log. ſine of half the baſe; reject radius from the ſum, and to the double of what will then remain add the arith. comps. of the log. ſines of the containing ſides.*

*Half the ſum is the log. ſine of half the angle.*

EXAM. Let the three ſides be BC = 114° 30′, AC = 80° 11′, AB = 56° 40′. Required the angle A.

By I. Rule

| | | | | | |
|---|---|---|---|---|---|
| Baſe BC = | 114° 30′ | | | | |
| Its ſupl. | 63 30 | nat. co-ſine = | 41469 | (256) | |
| Diff. ſides | 26 31 | nat. co-ſ. = | 89480 | | |
| Sum | | | 130949 | Rad. | 10,00000 |
| Half ſum is the nat. ſine. (217) | | | 65474 | arc 40° 54′ log. ſine | 9,81607 |
| | | | | Ar. co. log. ſine 83 11 | 0,00308 |
| | | | | Ar. co. log. ſine 56 40 | 0,07806 |
| | | | | | 19,89721 |
| | | | | Half the angle ſought 62° 40′ log. ſine | 9,94860 |

By II. Rule.

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{2}$ DC = | 13° $15\frac{1}{2}$′ | Log. ſine | 9,36048 | | |
| $\frac{1}{2}$ BC = | 57 15 | Ar.Co.L.ſin. | 0,07518 | log. ſine | 9,92482 |
| Log. ſine | 15 $49\frac{1}{2}$ | an arc | 9,43566 | log. co-ſ. 15° $49\frac{1}{2}$′ | 9,98322 |
| | | | | | 9,90804 |
| | | | | | 2 |
| | | | | | 19,81608 |
| Ar. Co. Log. ſine AC | | | | | 0,00308 |
| Ar. Co. Log. ſine AB | | | | | 0,07806 |
| Sum | | | | | 19,89722 |
| Half ſum is Log. ſine of | 62° 40′ | | | | 9,94861 |
| Angle ſought is | 125 20 = ∠ A. | | | | |

As the natural ſines of arcs are not contained in this work, and are on ſome occaſions neceſſary, it will be proper to ſhew how they may be found from the Logarithmic tables contained herein.

256. *Firſt. An arc being given, to find its natural ſine to five places of figures.*
Rule. Take out the Log. ſine of the arc, rejecting the Index;
Seek theſe figures among the logarithms of numbers;
The correſponding number is the natural ſine of the given arc; which is to be reckoned as a decimal fraction of the radius, or unity:
Prefixing the decimal comma (,) if the index of the log. ſine was 9;
But if the index was 8; 7; or 6; prefix ,0; ,00; or ,000; by which means the left hand digit of the natural ſine will ſtand in the place of the firſts, ſeconds, thirds, or fourths. (I. 18)

Ex. I. *Required the natural ſine and co-ſine of* 4° 22′?

| | | |
|---|---|---|
| Log. ſines | ſine 8,88161 | Co-ſine 9,99874 |
| Num. or nat. ſines | 0,07614 | 0,99710 |

Ex. II. *Required the natural ſine and co-ſine of* 28° 35′?

| | | |
|---|---|---|
| Log. ſines | ſine 9,67982 | Co-ſine 9,94355 |
| Num. or nat. ſines | 0,47844 | 0,87812 |

If a given log. ſine is found in the table of logs. of numbers, its natural number conſiſting of four places is ſeen at ſight; and its right hand place is 0 when the index of the log. ſine was 9.

But if a given log. ſine is not found to every figure in the tables of log. numbers, its 5th, or right-hand place is thus found.

Take the diff. between the log. num. next greater and leſs, than the given log. ſine; and alſo the diff. between the given log. ſine and its next leſs log. numb.

Then, As 1ſt diff. is to 2d diff. ſo is 10, to the digit for the right hand place.

Thus to 4° 22′, the nat. ſine is 0,07614; and co-ſine is 0,99710.

But to 28° 35′, the log. ſine and co-ſ. does not appear exactly among the log. numb.

And the above-mentioned two differences, for ſine, are 9 and 3; for co-ſ. are 5 and 1.

Then 9 : 3 : : 10 : 3, the 5th place. And 5 : 1 : : 10 : 2, the 5th place.

257. On the contrary. *A natural ſine being given, its correſponding arc may be thus found.*

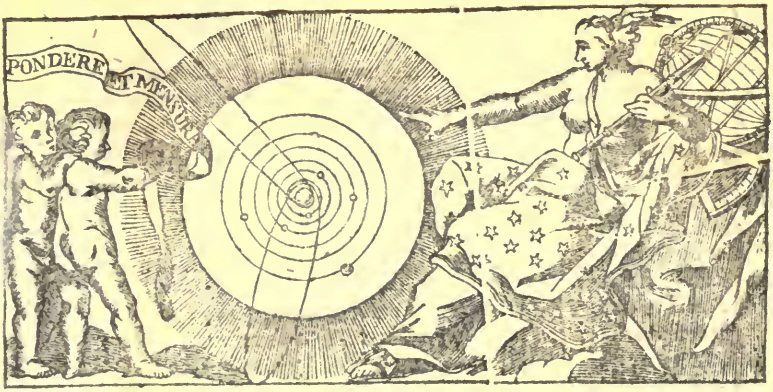
In the tables of num. and logs. enter with the natural ſine as a num. and take out its log.

Seek this log. in the table of log. ſines, and the correſponding degrees and minutes ſhew the arc required.

Prefixing the index 9, 8, 7, 6; according as the left hand digit ſtood in the place of firſts, ſeconds, thirds, or fourths.

What has been ſaid of the nat. and log. ſines of arcs, is alſo applicable to the nat. and log. tangents of arcs.

END OF BOOK IV.



# THE ELEMENTS OF NAVIGATION.

## BOOK V.

## OF ASTRONOMY.

### SECTION I.

### *Of Solar Aſtronomy.*

1. ASTRONOMY is a ſcience which treats of the motions and diſtances of the heavenly bodies, and of the appearances thence ariſing.

There have been great variety of opinions among the philoſophers of preceding ages concerning the ſituation of the great bodies in the univerſe, or of the poſitions of the bodies which appear in the heavens: But the notion now embraced by the moſt judicious Aſtronomers is, that the univerſe is compoſed of an infinite number of ſyſtems, or worlds; in every ſyſtem there are certain bodies moving in free ſpace, and revolving at different diſtances around a SUN, placed in, or near, the center of the ſyſtem; and that theſe *ſuns* and other *bodies* are the ſtars which are ſeen in the heavens.

2. The SOLAR SYSTEM, ſo called by Aſtronomers, is that in which our Earth is placed; and in which the Sun is ſuppoſed to be fixed in or near the center, with ſeveral bodies ſimilar to our Earth revolving round him at different diſtances. This hypotheſis, which is confirmed by all the obſervations hitherto made, is called the COPERNICAN SYSTEM,

from *Nicholas Copernicus*, a Poliſh Philoſopher, who about the year 1500 revived this notion from the oblivion it had been buried in for many ages.

Stars are diſtinguiſhed into two kinds, namely, *fixed* and *wandering.*

3. The Fixed Stars are the ſuns, in the centers of their ſyſtems, ſhining by their own light; and are obſerved to preſerve always the ſame ſituation in reſpect to one another.

4. The fixed ſtars appear of various ſizes, which is doubtleſs occaſioned by their different diſtances; theſe ſizes are uſually diſtinguiſhed into ſix or ſeven claſſes, called Magnitudes: The largeſt, or brighteſt, are ſaid to be of the FIRST MAGNITUDE; thoſe in the next claſs or degree of brightneſs, are called of the SECOND MAGNITUDE; and ſo on to the leaſt, or thoſe juſt diſcernible to the naked eye. But beſides theſe, there is ſcattered throughout the heavens a great number of other ſtars, called Telescopic Stars, becauſe they cannot be ſeen except through a teleſcope. And indeed, it is to the aſſiſtance of that moſt admirable inſtrument, that a great part of the modern Aſtronomy owes its riſe; which will undoubtedly tranſmit with the greateſt honour to the lateſt poſterity the name of Galileo, among whoſe many inventions the teleſcope is ranked.

5. Although the viſible fixed ſtars are probably at very unequal diſtances from the center of the *ſolar ſyſtem*, yet Aſtronomers, for their eaſe in computation, conſider them as equally diſtant from our Sun, forming the ſurface of a ſphere which incloſes our ſyſtem, and is called the Celestial Sphere. This ſuppoſition, with regard to the Solar Syſtem, may be ſtrictly admitted, conſidering the immenſe diſtance even of the neareſt fixed ſtars from the center of the ſyſtem.

6. A Constellation is a number of ſtars lying in the neighbourhood of one another on the ſurface of the *celeſtial ſphere*, which Aſtronomers, for the ſake of remembering them with greater eaſe, ſuppoſe to be circumſcribed by the outlines of ſome *animal*, or other *figure:* by this means the motions of the wandering ſtars are more readily deſcribed and compared.

The number of theſe conſtellations is 80, each containing ſeveral ſtars of different magnitudes. The number of ſtars of each magnitude, and alſo the conſtellation in which they are ranged, are contained in the following table; where it may be obſerved, that the conſtellations are diſtinguiſhed under three heads; namely, in the *zodiac*, and in the northern, and ſouthern hemiſpheres.

## 7. CONSTELLATIONS IN THE ZODIAC.

| Names. | Marks. | Number. | Magnitudes. | | | | | | Names. | Marks. | Number. | Magnitudes. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Northern. | | | I | II | III | IV | V | VI | Southern. | | | I | II | III | IV | V | VI |
| Aries. | ♈ | 46 | 0 | 1 | 1 | 3 | 5 | 36 | Libra. | ♎ | 33 | 0 | 2 | 1 | 8 | 3 | 19 |
| Taurus. | ♉ | 109 | 1 | 1 | 3 | 9 | 24 | 71 | Scorpio. | ♏ | 44 | 1 | 3 | 6 | 14 | 5 | 15 |
| Gemini. | ♊ | 94 | 1 | 2 | 4 | 8 | 13 | 66 | Sagittarius. | ♐ | 48 | 0 | 1 | 5 | 9 | 11 | 22 |
| Cancer. | ♋ | 75 | 0 | 0 | 0 | 6 | 8 | 61 | Capricornus. | ♑ | 58 | 0 | 0 | 2 | 5 | 9 | 42 |
| Leo. | ♌ | 91 | 2 | 2 | 6 | 13 | 11 | 57 | Aquarius. | ♒ | 93 | 0 | 0 | 4 | 7 | 28 | 54 |
| Virgo. | ♍ | 93 | 1 | 0 | 5 | 11 | 24 | 52 | Piſces. | ♓ | 110 | 0 | 0 | 1 | 7 | 28 | 74 |

8. NORTHERN

## 8. Northern Constellations.

| Names. | Numb. | Magnitudes. | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V | VI |
| Little Bear. | 12 | 0 | 2 | 1 | 4 | 3 | 2 |
| Great Bear. | 105 | 0 | 5 | 5 | 16 | 30 | 49 |
| Dragon. | 49 | 0 | 1 | 7 | 8 | 10 | 23 |
| Cepheus. | 40 | 0 | 0 | 3 | 7 | 10 | 20 |
| Greyhounds. | 24 | 0 | 0 | 0 | 1 | 7 | 16 |
| Bootes. | 53 | 1 | 0 | 7 | 10 | 12 | 23 |
| Mons Mænalus. | 11 | 0 | 0 | 0 | 1 | 0 | 10 |
| Berenice's Hair. | 24 | 0 | 0 | 0 | 6 | 8 | 10 |
| Charles's Heart. | 3 | 0 | 1 | 0 | 0 | 0 | 2 |
| Northern Crown. | 11 | 0 | 1 | 0 | 6 | 3 | 1 |
| Hercules. | 92 | 0 | 0 | 12 | 12 | 28 | 40 |
| Cerberus. | 9 | 0 | 0 | 0 | 3 | 1 | 5 |
| Harp. | 24 | 1 | 0 | 3 | 2 | 8 | 10 |
| Swan. | 73 | 0 | 1 | 5 | 15 | 20 | 32 |
| Fox. | 29 | 0 | 0 | 0 | 6 | 11 | 12 |
| Gooſe. | 10 | 0 | 0 | 0 | 0 | 2 | 8 |
| Lizard. | 12 | 0 | 0 | 0 | 3 | 5 | 4 |
| Caſſiopea. | 52 | 0 | 0 | 5 | 7 | 7 | 33 |

| Names. | Numb. | Magnitudes. | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V. | VI |
| Camelopardalus. | 23 | 0 | 0 | 0 | 5 | 7 | 11 |
| Serpent. | 50 | 0 | 1 | 7 | 6 | 5 | 31 |
| Serpentarius. | 67 | 0 | 1 | 6 | 12 | 17 | 31 |
| Sobieſki's Shield. | 8 | 0 | 0 | 0 | 2 | 3 | 3 |
| Eagle. | 29 | 1 | 0 | 5 | 1 | 4 | 18 |
| Antinous. | 34 | 0 | 0 | 5 | 2 | 7 | 20 |
| Dolphin. | 18 | 0 | 0 | 6 | 0 | 2 | 10 |
| Colt. | 12 | 0 | 0 | 0 | 4 | 1 | 7 |
| Arrow. | 13 | 0 | 0 | 0 | 4 | 1 | 8 |
| Andromeda. | 66 | 0 | 3 | 2 | 10 | 16 | 35 |
| Perſeus. | 67 | 1 | 1 | 5 | 10 | 14 | 36 |
| Pegaſus. | 81 | 0 | 3 | 4 | 9 | 11 | 54 |
| Auriga. | 46 | 1 | 1 | 1 | 9 | 9 | 25 |
| Lynx. | 55 | 0 | 0 | 1 | 8 | 21 | 25 |
| Little Lion. | 20 | 0 | 0 | 1 | 6 | 5 | 8 |
| Great Triangle. | 10 | 0 | 0 | 0 | 3 | 1 | 6 |
| Little Triangle. | 5 | 0 | 0 | 0 | 0 | 0 | 5 |
| Muſca. | 6 | 0 | 0 | 1 | 2 | 2 | 1 |

## 9. Southern Constellations.

| Names. | Numb. | Magnitudes. | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V | VI |
| Whale. | 80 | 0 | 2 | 8 | 13 | 10 | 47 |
| Eridanus. | 72 | 1 | 0 | 10 | 24 | 19 | 18 |
| Phenix. | 13 | 0 | 1 | 5 | 5 | 0 | 2 |
| American Gooſe. | 9 | 0 | 0 | 4 | 2 | 3 | 0 |
| Orion. | 93 | 2 | 4 | 3 | 19 | 15 | 50 |
| Monoceros. | 32 | 0 | 0 | 1 | 10 | 10 | 11 |
| Little Dog. | 14 | 1 | 0 | 1 | 0 | 2 | 10 |
| Hydra. | 53 | 0 | 1 | 3 | 14 | 13 | 22 |
| Sextans Uraniæ. | 4 | 0 | 0 | 0 | 0 | 0 | 4 |
| Cup. | 11 | 0 | 0 | 0 | 8 | 2 | 1 |
| Crow. | 8 | 0 | 0 | 2 | 2 | 2 | 3 |
| Centaur. | 36 | 2 | 6 | 6 | 14 | 8 | 0 |
| Wolf. | 36 | 0 | 0 | 3 | 6 | 18 | 9 |
| Altar. | 9 | 0 | 0 | 1 | 6 | 1 | 1 |
| Southern Triangle. | 5 | 0 | 1 | 2 | 0 | 2 | 0 |

| Names. | Numb. | Magnitudes. | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | II | III | IV | V | VI |
| Peacock. | 14 | 0 | 1 | 3 | 5 | 4 | 1 |
| Southern Crown. | 12 | 0 | 0 | 0 | 1 | 3 | 8 |
| Crane. | 14 | 0 | 2 | 1 | 2 | 9 | 0 |
| Southern Fiſh. | 15 | 1 | 0 | 2 | 9 | 2 | 1 |
| Hare. | 25 | 0 | 0 | 4 | 9 | 4 | 8 |
| Noah's Dove. | 10 | 0 | 2 | 0 | 1 | 6 | 0 |
| Charles's Oak. | 13 | 0 | 1 | 2 | 6 | 4 | 0 |
| Ship Argo. | 48 | 1 | 6 | 11 | 13 | 14 | 3 |
| Great Dog. | 29 | 1 | 5 | 1 | 4 | 10 | 8 |
| Bee. | 4 | 0 | 0 | 0 | 2 | 2 | 0 |
| Swallow. | 11 | 0 | 0 | 0 | 4 | 3 | 4 |
| Indus. | 12 | 0 | 0 | 0 | 4 | 6 | 2 |
| Chamelion. | 10 | 0 | 0 | 0 | 0 | 9 | 1 |
| Flying Fiſh. | 7 | 0 | 0 | 0 | 0 | 6 | 1 |
| Sword Fiſh. | 7 | 0 | 0 | 2 | 2 | 1 | 2 |

| | Stars | I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|---|
| Conſtellations in the zodiac 12, contain | 894 | 6 | 12 | 38 | 100 | 169 | 569 |
| Northern conſtellations 36, contain | 243 | 5 | 21 | 92 | 200 | 291 | 634 |
| Southern conſtellations 32, contain | 706 | 9 | 32 | 75 | 185 | 188 | 217 |
| Number of ſtars in the 80 conſtellations | 2843 | 20 | 65 | 205 | 485 | 648 | 1420 |

As theſe ſtars are found not to alter their ſituation in reſpect to one another, they ſerve Aſtronomers as fixed points, by which the motions of other bodies may be compared; and therefore their relative poſitions have been ſought after with great care for many ages, and catalogues of their places have from time to time been publiſhed by thoſe, who have been at the pains to make the obſervations. Among theſe catalogues, the moſt copious, and, as generally eſteemed, the beſt, is that called the *Hiſtoria Celeſtis* of our countryman FLAMSTEED.

10. The poſitions of the ſtars being obtained, their relative places may be delineated on a ſphere or plane; and thus are the maps or charts of the heavens made, and the conſtellations drawn incloſing their reſpective ſtars. There are two maps, uſually called Celeſtial hemiſpheres, which are prefixed to this book; by the help of which a perſon may readily become acquainted with the poſitions and names of ſome of the principal fixed ſtars, thus:

On a clear night, let theſe prints be laid ſo as to correſpond to the north and ſouth parts of the heavens; then the obſerver looking on the ſtars, and then on the hemiſpheres, will with a little practice know ſome of the ſtars in the heavens, the like poſitions and names of which he has obſerved on the prints.

11. The WANDERING STARS are thoſe bodies within our ſyſtem, or celeſtial ſphere, which revolve round the Sun; they appear luminous or bright, only by reflecting the light they receive from the Sun; and are of three kinds, namely, *primary planets*, *ſecondary planets*, and *comets*.

12. The PRIMARY PLANETS are thoſe bodies, which in revolving round the Sun reſpect him only as the center of their courſes; the motions of which are regularly performed in tracks, or paths, that are found by obſervations to be nearly circular and concentric to one another.

13. A SECONDARY PLANET, commonly called a SATELLITE or MOON, is a body, which, while it is carried round the Sun, does alſo revolve round a primary planet, which it reſpects as its center.

14. COMETS, vulgarly called *blazing ſtars*, are bodies which alſo revolve round the Sun; probably in as regular order as the planets, but in much longer periods of time, from what is hitherto known of them. They are in number many more than all the planets, and their tracks or courſes paſs among the paths of the planets in a great variety of directions.

15. The ORBIT of a planet or comet is that track or path along which it moves.

There are ſix *primary planets*; and reckoned in order from the Sun, their names and marks are, MERCURY ☿, VENUS ♀, the EARTH ♁ or ⊖, MARS ♂, JUPITER ♃, SATURN ♄.

*Mars*, *Jupiter*, and *Saturn*, are called SUPERIOR PLANETS, as their orbits include that of the *Earth*: but *Venus* and *Mercury*, the orbits of which are contained within the *Earth*'s, are called INFERIOR PLANETS.

16. It has been diſcovered by the help of teleſcopes, that there are ten *ſecondary planets*; the Earth being attended by one, called the Moon, Jupiter by four, and Saturn by five.

Saturn is alſo obſerved to have a kind of circle, called his RING, which ſurrounds the planet at ſome diſtance from his ſurface: and Jupiter has certain appearances, which ſeem like zones or girdles round him; and theſe are called JUPITER'S BELTS.

Every primary planet is ſuppoſed to have two motions, namely, *annual* and *diurnal*.

17. The ANNUAL MOTION of a planet is that whereby the planet is carried in its orbit round the Sun; which in every one is found by obſervation to be from weſt to eaſt.

This motion is diſcovered by the planets changing their places in the celeſtial ſphere; upon the ſurface of which they appear to move among the fixed ſtars; and in certain times to return to the ſame ſtars from which they were ſeen to depart; and ſo on continually.

18. The DIURNAL MOTION of a planet is that by which it turns or ſpins about its axis, and is alſo from weſt to eaſt.

This motion is diſcovered by the ſpots that are ſeen by teleſcopes on the ſurfaces of the planets. The ſpots appear firſt on the eaſtern margin, or ſide of the planet, and gradually move from thence acroſs it, till they diſappear on the weſtern ſide, or *limb*; after a certain time they appear again on the eaſtern ſide, and ſo on.

19. Each planet is obſerved always to paſs through the conſtellations *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, *Piſces*; and it alſo appears, that every one has a track peculiar to itſelf; hence the paths of the ſix planets form among the ſtars a kind of road, which is called the ZODIAC, the middle path of which, called the ECLIPTIC, is the orbit deſcribed by the Earth, with which the orbits of the other planets are compared.

As the ecliptic runs through twelve conſtellations, it is ſuppoſed to be divided into twelve equal parts, of 30 degrees each, called ſigns, having the ſame names with the twelve conſtellations which they run through.

20. The EQUINOCTIAL POINTS are thoſe two points of the Ecliptic, oppoſite to one another, through which the Earth paſſes in its annual motion, when the length of the day and night is equal in all parts on the Earth. One of theſe points, called the VERNAL EQUINOX, anſwers nearly to the 20th of March; and the other, called the AUTUMNAL EQUINOX, nearly to the 22d of September.

21. The *Plane of the Ecliptic* is ſuppoſed to divide the celeſtial ſphere into two equal parts, called the *northern* and *ſouthern celeſtial hemiſpheres*; and any body in either of theſe hemiſpheres is ſaid to have north or ſouth latitude, according to the hemiſphere it is in: So that the LATITUDE of a celeſtial object is its neareſt diſtance from the ecliptic, taken on the ſphere.

The Planes of the other five orbits are obſerved to lie partly in the northern, and partly in the ſouthern hemiſphere; ſo that every one cuts the ecliptic in two oppoſite points, called NODES. One called the ASCENDING NODE, marked thus, ☊, is that through which the planet paſſes when it moves out of the ſouthern into the northern hemiſphere; and the other called the DESCENDING NODE, marked thus, ☋, is that through which the planet muſt paſs in going out of the northern into the ſouthern hemiſphere.

The right line joining the two Nodes of any planet, is called the LINE OF THE NODES.

22. The names of moſt of the conſtellations were given by the ancient Aſtronomers, who reckoned that ſtar in Aries, now marked γ, (according

to *Bayer*'s maps) to be the firſt point in the ecliptic, this ſtar being next the Sun when he entered the *Vernal Equinox*; and at that time each conſtellation was in the ſign by which it was called. But obſervations ſhew, that the point marked in the heavens by the *Vernal Equinox* has been conſtantly going backward by a ſmall quantity every year, from which cauſe the ſtars appear to have advanced forward as much; ſo that the conſtellation *Aries* is now removed almoſt into the ſign *Taurus*, the ſaid firſt ſtar γ having got almoſt 30 degrees forwards from the equinox; which difference is called the PRECESSION OF THE EQUINOXES, and the yearly alteration is about 50 ſeconds of a degree, or about a degree in 72 years.

23. It was ſaid in art. 12, that the planets revolved round the Sun in orbits *nearly* circular and concentric; for the ſeveral phænomena ariſing from their motions ſhew they are not ſtrictly ſo; and the only curve they can move in, to reconcile all the various appearances, is found to be an Ellipſis: So that the orbits of the primary planets and comets are Ellipſes of different curvatures, having one common focus, in which the Sun is fixed: But every ſecondary planet reſpects the primary planet round which it revolves, as the focus of its elliptic motion. For as no other ſuppoſitions can ſolve all the appearances that are obſerved in the motions of the planets, and as theſe agree with the ſtricteſt phyſical and mathematical reaſoning, therefore they are now received as elementary principles.

24. The line of the nodes of every planet paſſes through the Sun: For as the motion of every planet is in a plane paſſing through the Sun, conſequently the interſections of theſe planes, that is, the lines of the nodes, muſt alſo paſs through the Sun.

25. All the planets, in their revolutions, are ſometimes nearer to, ſometimes farther from the Sun: This is the conſequence of the Sun not being placed in the center of each orbit, and of their being ellipſes.

26. The APHELION, or SUPERIOR APSIS, is that point of the orbit where the planet is fartheſt from the Sun. The PERIHELION, or INFERIOR APSIS, is that point where it is neareſt to the Sun: And the tranſverſe diameter of the orbit, or the line joining the two apſes, is called the LINE OF THE APSES, or ASPIDES.

27. The planets move faſter as they approach the Sun, or come nearer to the perihelion, and ſlower as they recede from the Sun, or come nearer to the aphelion. This is not only a conſequence from the nature of the planets motions about the Sun, but is confirmed by all good obſervations.

28. If a right line drawn from the Sun through any planet, uſually called the *Radius Vector*, is ſuppoſed to revolve round the Sun with the planet, then this line will deſcribe or paſs through every part of the plane of the orbit; ſo that the *Radius Vector* may be ſaid to deſcribe the area of the orbit.

29. There are two *chief laws* obſerved in the Solar Syſtem, which regulate the motions of all the planets; namely,

I. *The planets deſcribe equal areas in equal times:* That is, in equal portions of time the *Radius Vector* deſcribes equal areas, or portions of the ſpace contained within the planet's orbit.

II. *The ſquares of the periodical times of the planets are as the cubes of their mean diſtances from the Sun:* That is, as the ſquare of the time which a planet A takes to revolve in its orbit, is to the ſquare of the time taken by any other planet B to run through its orbit; ſo is the cube of the mean diſtance of A from the Sun, to the cube of the mean diſtance of B from the Sun.

30. The MEAN DISTANCE of a planet from the Sun is its diſtance from him, when the planet is at either extremity of the conjugate diameter; and it is equal to half the tranſverſe diameter.

31. The foregoing laws are the two famous laws of KEPLER, a great Aſtronomer, who flouriſhed in Germany about the beginning of the 17th century, and who deduced them from a multitude of obſervations: But the firſt who demonſtrated theſe laws was the incomparable Sir ISAAC NEWTON.

By the ſecond law, the relative diſtances of the planets from the Sun are known; and were the real diſtance of any one known, the abſolute diſtances of all the others would be obtained by it.

32. Every thing already ſaid of the planets is found in a great meaſure to be applicable alſo to the comets, as well from the obſervations that have been made of them, as from the phyſical and mathematical conſiderations of their motions.

33. Were the motions of the planets to be obſerved from the Sun, each of them would be ever ſeen to move the ſame way, though with different velocities; thoſe nearer to the Sun running their courſes through the Zodiac in leſs time than thoſe at greater diſtances: And hence it would happen, that ſome of them overtaking the others would in paſſing by them appear to be ſometimes above, ſometimes below, and ſometimes as if they touched one another, according to the parts of the orbits in which thoſe planets happened to be with reſpect to their nodes.

34. When two planets are ſeen together in the ſame ſign equally advanced, they are ſaid to be in CONJUNCTION: But when they are in direct oppoſite parts of the Zodiac, they are ſaid to be in OPPOSITION.

35. As the planes of the orbits are inclined to one another, therefore when two planets happen to be in conjunction at the time they come near a node of one of them, they would be ſeen from the Sun apparently to touch one another; and the fartheſt of thoſe planets from the Sun would ſee the neareſt moving over the face of the Sun like a black ſpot, being then directly between the Sun and the remoter planet; ſo the planet Venus was obſerved from the Earth in the tranſits of the years 1761 and 1769. Alſo, ſhould an oppoſition of two planets happen near a node of one of them, the Sun, being then directly between them, would hide the light of one from the other. Theſe obſcurations, or interceptions of the light of the planets one from the other, are called ECLIPSES.

36. The place that any planet appears to occupy in the celeſtial ſphere, when ſeen by an obſerver ſuppoſed to be in the Sun, is called its *Heliocentric place:* And indeed all celeſtial appearances, as ſeen from the Sun, are called *Heliocentric phænomena.*

37. The following table exhibits at once ſome of the moſt material concluſions that have been deduced from the obſervations hitherto made, the mean diſtance of the Earth from the Sun being reckoned at 1000.

### Table of the Solar System.

| Planets Names. | Marks. | Mean diſt. compared to that of the Earth. | Eccentricity. | Inclination of the orbits to the eclipt. | Times of the periodical revolutions. | Diurnal rotations. | True diameter. | N° of moons. |
|---|---|---|---|---|---|---|---|---|
| Mercury. | ☿ | 387 | 81 | 6° 52′ | 3 m. or 87$^d$ 23$^h$ | uncertain | 0,32 | 0 |
| Venus. | ♀ | 724 | 5 | 3 23 | 8 m. or 224 17 | 23$^h$ 0$^m$0$^s$ | 0,87 | 0 |
| Earth. | ♁ | 1000 | 17 | 0 00 | 1 y. or 365 6 | 23 56 4 | 1,00 | 1 |
| Mars. | ♂ | 1524 | 141 | 1 52 | 2 y. or 686 23 | 24 40 0 | 0,73 | 0 |
| Jupiter. | ♃ | 5201 | 250 | 1 20 | 12 y. or 4332 12 | 9 56 0 | 7,70 | 4 |
| Saturn. | ♄ | 9538 | 543 | 2 20 | 30 y. or 10759 8 | uncertain | 4,19 | 5 |

38. By all the obſervations made on the ſecondary planets, it appears,

1ſt. That the ſatellites revolve round their ſuperior planets from weſt to eaſt, in curve-lined orbits like ellipſes, the primary planet being in the focus, and one of the orbit's diameters directed towards the Sun.

2d. That the planes of the orbits of the ſatellites are inclined to the plane of the orbit of their reſpective planet.

3d. That, like the primary planets, they deſcribe equal areas in equal times; and the ſquares of the times of their revolutions are as the cubes of the mean diſtances from their primary planet.

In every revolution of a moon round its primary planet, there muſt be two conjunctions betwixt the planet, moon, and Sun: namely, once, when the moon is in that part of its orbit neareſt to the Sun; and once, when in that part of its orbit fartheſt from the Sun: And an eclipſe may happen at either conjunction, according as the moon's nodes happen to be poſited at thoſe times. For the plane of a moon's orbit is inclined to that of its primary, and ſo makes two nodes: And whenever the Sun, planet, and moon happen to be at the ſame time in the line of the nodes, there muſt be an eclipſe; which would occur at every conjunction, but for the inclination of the orbit.

One of the conjunctions, namely, that made by the moon's going beyond the primary, from the Sun, is called the SUPERIOR CONJUNCTION; and the other, made by the ſatellite on the ſide of the planet next the Sun, is called the INFERIOR CONJUNCTION.

The following table ſhews the time taken by each ſatellite in its revolution, and alſo its mean diſtance from the primary in ſemidiameters of it.

| 39 | I | II | III | IV | V |
|---|---|---|---|---|---|
| | d h m | d h m | d h m | d h m | d h m |
| Saturn's ſatell.<br>Diſt. from Sat. | 1 21 $18\frac{1}{2}$<br>$8\frac{7}{8}$ ſ.diam. | 2 17 $41\frac{1}{3}$<br>$11\frac{1}{4}$ ſ.diam. | 3 12 $25\frac{1}{5}$<br>15 ſ. diam. | 15 22 $41\frac{1}{4}$<br>36 ſ. diam. | 79 7 48<br>108 ſ.diam. |
| Jupiter's ſatell.<br>Diſt. from Jup. | 1 18 $28\frac{1}{5}$<br>$5\frac{2}{3}$ ſ.diam. | 3 13 $18\frac{5}{6}$<br>9 ſ. diam. | 7 3 $59\frac{2}{3}$<br>$14\frac{1}{3}$ ſ.diam. | 16 18 $5\frac{1}{10}$<br>$25\frac{1}{3}$ ſ. diam. | |
| The Moon | 29 12 $44\frac{1}{20}$ | and is diſt. from the Earth $60\frac{1}{5}$ ſemidiameters. | | | |

## 40. *Of the figure and light of the Planets.*

That the Sun and planets are ſpherical bodies is evident from all the obſervations that have been made of them; and that the Earth is of like figure is not only deduced by analogy, but ſufficiently confirmed by obſervation*. Now although Aſtronomers generally ſay that the planets are ſpherical, yet they do not mean a Geometrical ſphere, but a figure called an oblate ſpheroid, which is ſomething like the figure that a flexible ſphere would be formed into by gently preſſing it at its poles. Obſervations have determined this in Jupiter, and it is known that the Earth is of this figure both from obſervations and actual menſuration.

That the planets muſt have this oblate figure, is evident from this conſideration; that as they are of matter, and violently whirled on their axes, all the parts would endeavour to fly off, like water from a trundled mop; thoſe in the equator moving ſwifteſt, have the greateſt tendency to depart: And although the parts are retained in the ſphere by the ſuperior force of gravity, yet the equatorial diameters will be ſomewhat increaſed, and the polar leſſened.

41. The planets are all opake or dark bodies, and conſequently ſhine only by the light they receive from the Sun: This is known by obſerving, that thoſe bodies are not viſible, when they are in ſuch parts of their orbits as are between the Sun and Earth, or partly ſo. Now, as all the planets ſometimes appear with a ſtrong light, therefore the rays they receive from the Sun muſt convey to them a degree of warmth proportional in ſome meaſure to their diſtance; which proportion is reciprocally as the ſquares of the diſtances; and this muſt be readily inferred from the heat which the inhabitants of the Earth receive from the Sun.

42. As a planet revolves on its axis, every part of its ſurface will be turned towards the Sun, and ſo enjoy its light and heat.

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* Obſervations ſhew, that in eclipſes of the moon the darkened part is bounded by a circular curve; and conſequently the body, which caſts the ſhade, or obſtructs the light, muſt be bounded by a like curve: but as theſe obſcurations are cauſed by different parts of the Earth, conſequently its ſurface muſt be limited by a circular figure, that is, it muſt be globular.

## SECTION II.

## *Of Terreſtrial Aſtronomy.*

43. TERRESTRIAL ASTRONOMY is that which conſiders the motions of the celeſtial bodies as ſeen from the Earth, which is alſo in motion.

The motions deſcribed in the preceding ſection were ſuch, as would appear to an obſerver viewing the heavens from the Sun: But were he placed in one of the planets, ſuppoſe the Earth, and there obſerved the motions of the reſt, the Sun, and other planets, would appear to him to revolve round the Earth as a center; but the Sun would be the only one that moved uniformly the ſame way: For the other planets would ſeem to move ſometimes from weſt to eaſt, and then to ſtand ſtill; then they would ſeem to run from eaſt to weſt; and after ſtanding ſome time, they would again move from weſt to eaſt, and ſo on continually.

44. The place in the celeſtial ſphere that any planet appears in, when ſeen from the center of the Earth, is called its GEOCENTRIC PLACE.

45. The DIRECT MOTION of a planet is that by which it appears to move from weſt to eaſt, and this motion is ſaid to be *according to the order of the ſigns*, or in *conſequentia*. When the planet appears to ſtand ſtill, it is ſaid to be STATIONARY; and when its motion is apparently from eaſt to weſt, it is then called RETROGRADE, or has a motion in *antecedentia*, or contrary to the order of the ſigns. Theſe different appearances follow partly in conſequence of the obſerver being himſelf in motion while he is viewing the motions of the planets, and partly becauſe he is not in the center of the motions which he obſerves.

### 46. *The Phænomena of the Inferior Planets.* See Fig. 1. Plate IV.

Let ABC repreſent an arc of the celeſtial ſphere; EOP the Earth's orbit; LNIG the orbit of an inferior planet, as of *Venus*; and S the Sun: Let the Earth at firſt be ſuppoſed to ſtand ſtill in its orbit at E: Now it is evident that the Sun will appear at the point B, and the planet always within the arc AC. Whilſt the planet moves in its orbit from I through Q to N, it will ſeem to move from B to A in *conſequentia:* But paſſing from N to L, it will ſeem to an eye at E to return back from A to B, or be *retrograde.* While the planet is at, or near, the point N, and moving as it were in a right line towards the Earth, it will for ſome time ſeem to ſtand ſtill near A, and it is then ſaid to be *ſtationary.*

47. When the planet is in that part of its orbit N or G, which is contiguous to the tangent EA or EC, it will then appear at A or C, its greateſt diſtance from the Sun, and is ſaid to have then the greateſt ELONGATION: This elongation is meaſured by the angle SEG. The more diſtant a planet is from the Sun, the greater will its angle of elonga-

tion be; that of Venus is about 48 degrees, and that of Mercury about 28 degrees.

In the ſpace of a revolution, the two inferior planets will, with reſpect to the Earth, undergo two conjunctions; one when it is beyond the Sun at I, the other when it is at L between the Sun and the Earth; the former is called the *ſuperior*, and the latter the *inferior conjunction*.

48. Whilſt Venus goes from her ſuperior conjunction, ſhe appears in the arc BA always to the eaſtward of the Sun, and therefore ſets ſome time after the Sun, and is called the *evening ſtar*. But during the time ſhe is going from her inferior to her ſuperior conjunction, ſhe will be ſeen ſomewhere in the arc BC to the weſtward of the Sun, and ſo will ſet before him in the evening, and riſe before him in the morning; hence ſhe is called the *morning ſtar*.

Hitherto the planet only has been ſuppoſed to move while the Earth ſtood ſtill; but when both move, the foregoing phænomena will be much the ſame, only the planet will be more direct in the fartheſt part of the orbit, and leſs retrograde in the neareſt; the former ariſing from the ſum of their motions, and the latter from the difference.

### 49. *Of the Phænomena of the Superior Planets.*

The *direct*, *ſtationary*, and *retrograde* appearances of the ſuperior planets are explained much after the ſame manner as thoſe of the inferior ones, but with theſe differences.

1ſt. The retrograde motions of the ſuperior planets happen oftener the ſlower their motions are, as the retrogradations of the inferior planets happen oftener the ſwifter their angular motions are: Becauſe the retrograde motions of the ſuperior planets depend upon the motions of the Earth, but thoſe of the inferior on their own angular motions. A ſuperior planet is retrograde once in each revolution of the Earth; an inferior one, in every one of its own revolutions.

2d. The ſuperior planets do not always accompany the Sun as the inferior do, but are often in oppoſition to him; which neceſſarily follows from the orbit of the Earth being included in the orbits of the ſuperior planets.

### 50. *Of the apparent Motion of the Sun.*

As a ſpectator in the Sun would ſee the Earth revolve through the ſigns in the ecliptic; ſo to a ſpectator in the Earth the Sun apparently revolves the ſame way, but is always in the oppoſite point of the ecliptic: (For it is well known to every one, eſpecially to thoſe who uſe the ſea, that fixed objects appear to change their place by the motion of the obſerver:) So that the heliocentric place of the Earth, and the geocentric place of the Sun, are always in direct oppoſite points of the ecliptic.

Now although it is the motion of the Earth that really cauſes a great variety in the apparent motions of the other planets, yet as the motion of the Sun being known gives that of the Earth, therefore Aſtronomers ſpeak of the motion of the Sun; and in their computations uſe the quantities of thoſe motions as if they were real.

51. Beſide the various appearances that ariſe from the annual motion of the Earth, there are many reſulting from its diurnal motion: For that the Earth has a daily motion round its axis muſt neceſſarily be inferred from the moſt ſtrict reaſoning on the motions of the planets: and the notion, that bodies ſo immenſely diſtant as the ſtars are, really revolve round the earth in 24 hours, is now treated as a great abſurdity by every one who has rightly conſidered theſe things: However, as the motions are apparent, and the ſpeaking of them as real is cuſtomary and no way affects the concluſions; therefore Aſtronomers treat of thoſe motions as they appear.

52. Any ſphere revolving as on an axis, muſt have two points on its ſurface at the extremities of its axis, that do not revolve at all; theſe points, with reſpect to the Earth, are called its *poles*.

53. By the Earth's rotation on its axis from weſt to eaſt in a day, the ſurface of the celeſtial ſphere appears to move from eaſt to weſt in the ſame time; and all the celeſtial objects appear to deſcribe circles in the heavens, which are greater or leſs according as they are farther from, or nearer to, the apparent centers of thoſe motions: For there are two points in the heavens which are apparently fixed, and the nearer any ſtars are to theſe points, the ſlower are their motions. Theſe points are called the CELESTIAL POLES; the right line joining them is called the AXIS OF THE SPHERE, and paſſes through the poles of the Earth; the circle in the heavens, equally diſtant from the poles of the celeſtial ſphere, is called the EQUINOCTIAL; the correſponding circle on the Earth is called the EQUATOR, which is equally diſtant from both the poles of the Earth.

54. As the Sun's rays falling on any ſphere enlighten one half of its ſurface; therefore one half of the Earth is always illuminated at once, and conſequently the enlightened part is bounded by a great circle, which may be called the TERMINATOR, from its property of terminating, or bounding, the verges of light and darkneſs. Now, by the rotation of the Earth on its axis once in 24 hours, there will be a conſtant ſucceſſion of light on all parts of its ſurface as they are turned towards the Sun, and of darkneſs in thoſe parts as they move out of his rays; and hence ariſe the viciſſitudes of DAY and NIGHT.

55. If the plane of the equator coincided with the plane of the ecliptic, and the axis of the Earth ſtood perpendicular to it, the terminator would always paſs through the poles of the Earth, and there would be a conſtant equality of day and night in every part of its ſurface, except at the two poles, where there would be conſtant day. But the contrary of this is known to every one, and obſervations ſhew, that the Earth's axis is inclined to the plane of the ecliptic in an angle of about $66\frac{1}{2}$ degrees; therefore the poles of the ecliptic and equator are about $23\frac{1}{2}$ degrees diſtant from one another; conſequently the ecliptic and equinoctial, which in the heavens interſect one another in the oppoſite points of Aries and Libra, make at thoſe interſections angles of about $23\frac{1}{2}$ degrees (IV. 33): This angle is called the OBLIQUITY OF THE ECLIPTIC.

The axis of the Earth being thus inclined to the plane of the ecliptic, and moving parallel to itſelf in all points of the ANNUAL ORBIT, or ecliptic, is the occaſion of the inequality of days and nights, and of the different ſeaſons of the year; which two phænomena are explained as follows.

56. It muſt be obſerved, that the Sun will appear to be vertical to that part of the Earth, which is cut by a ſtraight line joining the centers of the Sun and Earth.

57. Now when the Earth is at ♑, Fig. 3. Plate IV. the Sun appearing then in ♋ will be vertical to that point of the terreſtrial ecliptic, it lying in the right line joining the centers of the Sun and Earth. And this point being in the Earth's northern hemiſphere, all thoſe who live there will enjoy ſummer, or the hotteſt time of the year, the ſolar rays falling more copiouſly, and more perpendicularly, upon their hemiſphere at that time.

58. At the ſame time the inhabitants of the ſouthern hemiſphere will have winter, the rays of the ſun falling more obliquely, and in leſs quantity, on them, and conſequently affording them leſs heat.

59. Again, the inhabitants of the northern hemiſphere will have their days longer than their nights, in proportion as they are more diſtant from the equator; while thoſe who live under the equator will have an equal ſhare of day and night all the year round. For in this poſition the terminator, which is always at right angles to the plane of the ecliptic, will paſs 23½ degrees beyond the north pole, and conſequently will cut all the circles parallel to the equator which it meets with into two unequal parts: thoſe that are in north latitude will have the greater portions of thoſe parallels in the enlightened hemiſphere: but the terminator being a great circle, will cut the equator into two equal parts; therefore half the equator is always illuminated.

60. Hence it neceſſarily follows, that thoſe who live under the equator will have their days and nights equal: thoſe who live within the limits of 23½ degrees round the north pole, will have no night; and the inhabitants between this limit and the northern neighbourhood of the equator will have their nights ſhorter than their days. In the mean time thoſe who live in the ſouthern hemiſphere will have their nights longer than their days, in proportion as they approach nearer to the ſouth pole; and the regions contained within the limits of 23½ degrees round the ſouth pole will have no day.

61. Suppoſe the Earth now to move in its orbit from ♑ through the ſigns ♒, ♓ to ♈, the Sun will ſeem to run through the ſigns ♌, ♍ to ♎; and this will be the place of the Sun in autumn.

While the Earth is in ♈, the days and nights will be equal in both hemiſpheres, and the ſeaſon is a medium between ſummer and winter: For at that time the Sun will appear vertical to the equator, becauſe a right line joining the centers of the Sun and Earth will then cut the ſurface of the Earth in the equator; ſo that the terminator, the plane of which is always at right angles to the ſaid line, will paſs through the poles; conſequently, all the Earth will then have an equal ſhare of day and night. And becauſe the rays of the Sun then fall perpendiculary upon the axis of the Earth, it will then follow, that they muſt fall with an equal obliquity, and with equal number, upon either hemiſphere; therefore they muſt enjoy an equal degree of heat and cold.

Now ſuppoſe the Earth to move from ♈ to ♋, the Sun will ſeem to move from ♎ to ♑, where it will be in its neareſt approach to the ſouth pole; and at this time of the year it will be winter in the northern hemiſphere. For to this hemiſphere the like phænomena will now happen, which did before to the ſouthern, when the Earth was in ♑; and by a parity of reaſon, when the Earth has got as far as ♎, and the Sun is apparently in ♈, the northern hemiſphere will enjoy ſpring, and the ſouthern will have autumn.

62. The four points of the ecliptic, in which the Earth has been conſidered in ſummer, autumn, winter, and ſpring, are called the four CARDINAL POINTS; ♑ and ♋ are called SOLSTITIAL POINTS; ♎ and ♈, EQUINOCTIAL POINTS.

63. The firſt point of Cancer is called the SUMMER SOLSTICE; becauſe when the Sun enters it, which is about the 21ſt of June, he has then got to the greateſt extent northwards, and being about to return towards the equator, he ſeems for a day or two to be at a ſtand. And for the ſame reaſon, the firſt point of Capricorn, which the Sun enters about the 21ſt of December, is called the WINTER SOLSTICE, with reſpect to the northern hemiſphere.

64. The firſt points of Aries and Libra are called the VERNAL and AUTUMNAL EQUINOCTIAL POINTS, from the equality of days and nights all over the ſurface of the Earth, when the Sun enters thoſe points.

### 65. *Of the Riſing and Setting of the Stars.*

There is only one half of the celeſtial ſphere viſible at one time to any obſerver on the ſurface of the Earth, the other half being hid by the Earth itſelf. Now the apparent plane on which the obſerver ſtands, ſeems to be extended to the heavens, and there marks out a circle that divides the viſible from the inviſible hemiſphere; this circle is called the HORIZON, above which all the celeſtial motions are ſeen. When this horizon is a great circle of the celeſtial ſphere, it is called the RATIONAL HORIZON: but when by the particular ſituation of the obſerver, he ſees more or leſs than half the celeſtial ſphere, then the circle bounding his view is called the SENSIBLE HORIZON.

The horizon is one of the moſt uſeful circles in Aſtronomy; for to this circle, which is the only apparent one, almoſt all the celeſtial motions are referred. It is the common termination of day and night; it marks out the times of the riſing and ſetting of the Sun and ſtars, and many other particulars, of which hereafter.

### 66. *Of Parallaxes.*

The PARALLAX of any object is the difference between the places that object is referred to in the celeſtial ſphere, when ſeen at the ſame time from two different places within that ſphere: Or, it is the angle under which any two places in the inferior orbits are ſeen from a ſuperior planet, or even from the fixed ſtars: But the parallaxes which are moſt uſed by Aſtronomers are thoſe which ariſe from ſeeing the object from the centers of the Earth and Sun; from the Surface and center of the Earth; and from all three compounded.

67. The difference between the heliocentric and geocentric place of a planet, is called the *parallax of the annual orbit* (namely, that of the Earth); that is, the angle at any planet, ſubtended by the diſtance between the Sun and Earth, is called the parallax of the Earth's, or annual orbit.

68. The difference in the two longitudes is called the parallax of longitude; and that of the two latitudes is called the parallax of latitude.

69. In the SYZIGIES, that is, in the *oppoſitions* or *conjunctions*, the Sun and planet being equally advanced in the ſame ſign, or in like places in oppoſite ſigns, the parallax of latitude is then greateſt.

70. And when the planet is in its QUADRATURES, that is, when it is 90 degrees diſtant from the Sun, the parallax of longitude is then the greateſt.

71. To explain the parallaxes which reſpect the Earth only. Fig. 2. Plate IV.

Let HSW repreſent the Earth, where T is the center; OR part of the Moon's orbit, P*rg* part of a planet's orbit, and Z*a*A part of a great circle in the celeſtial ſphere. Now to a ſpectator at S upon the ſurface of the Earth, let the Moon appear in G, that is in the ſenſible horizon of S, and it will be referred to A; but if viewed from the center T, it will be referred to the point D, which is its true place.

The arc AD will be the Moon's parallax; the angle SGT the parallactic angle: Or the parallax is expreſſed by the angle under which the ſemidiameter TS of the Earth is ſeen from the Moon.

If the parallax is conſidered with reſpect to different planets, it will be greater or leſs as thoſe objects are more or leſs diſtant from the Earth. Thus the parallax AD of G is greater than the parallax A*d* of *g*. If it is conſidered with reſpect to the ſame planet, it is evident that the horizontal parallax (or the parallax when the object is in the horizon) is greateſt of all; and diminiſhes gradually as the body riſes above the horizon, until it comes to the zenith, where the parallax vaniſhes, or becomes equal to nothing. Thus AD and A*d*, the horizontal parallaxes of G and *g*, are greater than *a*B and *ab*, the parallaxes of R and *r*; and the objects O or P, ſeen from S or T, appear in the ſame place Z, or the zenith.

72. By knowing the parallax of any celeſtial object, its diſtance from the center of the Earth may be eaſily obtained by Trigonometry. Thus, if the diſtance of G from T is ſought; in the triangle STG, the ſide ST being known, and the angle SGT determined by obſervation, the ſide TG is thence known.

The parallax of the Moon may be determined by two perſons obſerving her from different ſtations at the ſame time, ſhe being vertical to the one, and horizontal to the other: and it is generally concluded to be about 57 minutes of a degree; conſequently her mean diſtance TG is about 60 ſemidiameters of the Earth, or 60 times TS.

But the parallax moſt wanted is that of the Sun, by which his abſolute diſtance from the Earth would be known; and thence the abſolute diſtances of all the other planets would be obtained from their relative diſtances found by the ſecond *Keplerian Law.*

Before the year 1761 ſome Aſtronomers reckoned the Sun's parallax at 12½ ſeconds, others at 10; theſe different parallaxes gave very different diſtance between the Sun and the Earth; the former making the diſtances near 8270 diameters of the Earth, and the latter 10313 diameters.

But in the years 1761 and 1769 the planet Venus paſſed between the Earth and the Sun, and was ſeen like a black ſpot moving over the face of the Sun. Theſe phenomena (which had not happened in more than 100 years before) were obſerved by many Aſtronomers from different parts of the Earth, and the reſult of their obſervations make the Sun's mean parallax about 8⅔ ſeconds, and hence the mean diſtance between the Sun and Earth comes very nearly to 11900 diameters of the Earth: And from what was ſhewn many years ago by the excellent Dr. Halley, if theſe obſervations were made with the accuracy he ſuppoſed, the diſtance between the Sun and the Earth might be obtained to leſs than a 500th part of the whole diſtance.

## 73. *Of the Meaſure of the Earth.*

The relative diſtances of the planets are diſcovered by the 2d *Keplerian Law*, and their relative magnitudes are gathered from the angles which they appear under (when viewed with very accurate inſtruments) compounded with their diſtances. Now as theſe diſtances and magnitudes can by means of the parallaxes be compared with the diameter of the Earth, conſequently this diameter being accurately known would ſerve as a meaſure with which the magnitudes and diſtances of all the other planets might be compared.

To find the meaſure of the Earth is a problem of ſuch importance in Aſtronomy, that it has been attempted by ſome of the moſt conſiderable men in almoſt all the preceding ages. But its ſolution was not brought to any degree of accuracy till the year 1635, when it was very nearly aſcertained by our countryman Richard Norwood, an eminent mathematician at that time. The principle he proceeded upon was this, that as 360 degrees were contained in every great circle, both of the celeſtial ſphere and of the Earth, and as theſe circles are conſidered as concentric to the center of the Earth; therefore, were the meaſure of a degree known on a great circle of the Earth, correſponding to a degree of a great circle of the heavens, then, by analogy, the whole circumference of a great circle of the Earth would be known in that meaſure, and conſequently its diameter would be obtained. (II. 197)

Norwood ſolved this problem in the following manner: He choſe two diſtant places which were know to lie nearly north and ſouth one of the other, as London and York; and by a method like that of Traverſe ſailing (explained in Book VII.) he found their difference of latitude, or, the diſtance between the parallels of latitude paſſing through thoſe places; or, which is the ſame thing, the length of that arc of the terreſtrial meridian. He alſo with a good inſtrument found the diſtance between the zeniths of thoſe places, and conſequently he thence knew the quantity of the celeſtial arc anſwering to the meaſured terreſtrial one. Then ſaying, As that celeſtial arc is to a great circle of the celeſtial ſphere, or 360 degrees; ſo is the arc of the terreſtial great circle meaſured in feet, to the circumference of a great circle of the Earth in feet meaſure.

And thus he found that about 69½ Engliſh miles anſwered to one degree; hence the circumference of the Earth appears to be 25020 miles, and its diameter about 8000 miles.

By the ſame kind of reaſoning, the diſtances found in art. 72. from the parallaxes, were obtained.

$$\left.\begin{array}{l} \text{For } 12\tfrac{1}{2}'' : 360^\circ :: 1 \text{ ſemi-diam.} : 103680 \\ \quad 10 : 360 :: 1 \text{ ſemi-diam.} : 129600 \\ \quad 8\tfrac{2}{3} : 360 :: 1 \text{ ſemi-diam.} : 149538 \end{array}\right\} \text{the} \left\{\begin{array}{l} \text{Circumference of the} \\ \text{Earth's orbit in ſemi-} \\ \text{diameters of the Earth.} \end{array}\right.$$

$$\left.\begin{array}{l} \text{And } 6{,}283185 : 103680 :: 1 : 16539{,}5 \\ \quad 6{,}283185 : 129600 :: 1 : 20626{,}4 \\ \quad 6{,}283185 : 149538 :: 1 : 23799{,}8 \end{array}\right\} \text{the} \left\{\begin{array}{l} \text{Mean diſt. of the Earth} \\ \text{from the Sun, in ſemi-d.} \\ \text{of the Earth. (II. 197)} \end{array}\right.$$

$$\left.\begin{array}{l} \text{Then } 16539{,}5 \times 4000 = 66158000 \\ \quad 20626{,}4 \times 4000 = 82505600 \\ \quad 23799{,}8 \times 4000 = 95199200 \end{array}\right\} \text{the} \left\{\begin{array}{l} \text{Mean diſtance of the} \\ \text{Earth from the Sun, in} \\ \text{miles.} \end{array}\right.$$

## 74. *Of the Moon.*

The Moon revolves in her orbit from weſt to eaſt round the Earth, and is carried perpetually with it through the annual orbit round the Sun, making in the ſpace of one year 13 *periodical*, and 12 *ſynodical revolutions.*

75. A PERIODICAL MONTH, or REVOLUTION, is the time the Moon takes up in revolving from one point of her orbit to the ſame point again, and conſiſts of 27 d. 7 h. 43 m.

76. A SYNODICAL MONTH, or REVOLUTION, is the time the Moon ſpends in paſſing from one conjunction with the Sun to another, which is 29 d. 12 h. 44 m.; being 2 d. 5 h. 1 m. longer than the *Periodical Month.* For whilſt the Moon is paſſing from her former conjunction with the Sun round to it again; the Earth has proceeded forwards in its annual courſe, as it were leaving the Moon behind it; ſo that, in order to complete her next conjunction with the Sun, ſhe muſt not only come round to her former point again, but alſo go beyond it.

77. Beſides this monthly motion of the Moon round the Earth, ſhe has alſo a motion round her axis, which is performed exactly in the ſame time with her periodical revolution: Hence it comes to paſs, that the ſame face of the Moon is always turned towards the Earth, her diurnal motion turning juſt as much of her face to us, as her periodical motion turns it from us.

78. Though the ſame ſide of the Moon is ever turned towards us, yet it is not always viſible, but ſeems daily to put on different appearances,

called

called Phases: For the Moon being an opake body like the reſt of the planets, borrows its light from the Sun, having always one hemiſphere enlightened by the ſolar rays.

When the enlightened hemiſphere is wholly turned from the Earth, as at her change or time of new-moon, the planet then being betwixt us and the Sun, the Moon's whole enlightened face, or *diſk*, muſt needs be inviſible to the Earth. When ſhe paſſes from this ſtate, and turns ſome little portion of the illuminated half to us, ſhe muſt appear horned, the Cusps or points being turned from the Sun towards the eaſt. When the Moon is in her quadratures, or at 90 degrees from the Sun, then half the illuminated face becomes viſible: She afterwards continues to ſhew more than half the enlightened diſk, until ſhe comes in oppoſition to the Sun or time of full-moon, when the whole of the illuminated orb is preſented to us; from whence receding, ſhe muſt put on the like phaſes as before, but in an inverſe order, the cuſps being now turned towards the weſt.

## 79. *Of Solar and lunar Eclipſes.*

Eclipſes of the Sun and Moon can only happen about the times of the conjunctions and oppoſitions: thoſe of the Sun fall out at the conjunctions, when the Moon intercepts the light of the Sun from the Earth; and thoſe of the ~~Sun~~ occur in the oppoſitions, when the Earth getting between the Sun and Moon, the latter loſes her light during the time of that interpoſition.

The cauſe why there is not an eclipſe in every ſyzigie is the inclination of the plane of the Moon's orbit to that of the ecliptic, which is about $5^\circ\ 18'$: for it is certain, that unleſs the Sun, Earth, and Moon, are all in the plane of the ecliptic, or nearly ſo, the ſhadows of the Earth and Moon can never fall on one another, but muſt be directed either above or below. Now they can never be in the ſame plane, and in one right line, except when the Moon is in her nodes, the nodes and Sun's center being in the ſame right line.

80. The ſolar and lunar eclipſes do not happen every year in the ſame places of the zodiac, but in ſucceeding years they fall in places gradually removed backwards, or towards the antecedent ſigns: For ſince the nodes are found to go continually backwards, the eclipſes muſt alſo obſerve the ſame order.

81. Eclipſes of the Moon are either *total* or *partial:* the total happen when the node falls in or near the center of the ſhadow: and the partial, when the node happens to be on either ſide the center, within or without the ſhadow. Now the longer the duration of a partial eclipſe is, ſo much the greater is that part of the Moon which enters into the ſhadow of the Earth.

82. Hence it is uſual to conceive the Moon's diameter as divided into 12 parts, called Digits, by which the greatneſs of partial eclipſes is meaſured, they being ſaid to be of ſo many digits as they are parts covered by the Earth's ſhadow: Thus if 5 of the 12 parts are covered, it is called an eclipſe of 5 digits.

83. As the planet Mars is never eclipſed by the Earth, it is plain the ſhadow of the latter does not reach ſo far as the orbit of the former,

but tapers to a point at a leſs diſtance; and conſequently the Earth's ſhadow muſt be a cone, the vertex of which is extended beyond the orbit of the Moon. It naturally follows from hence, that the Sun is a much larger body than the Earth; it is indeed, in diameter, above 100 times that of the Earth.

84. If a perſon was placed juſt at the vertex, or point of this ſhadow, he would ſee nothing of the Sun but a ſmall rim of light round his diſk; and the farther the obſerver was removed from the vertex, the larger would the rim of light appear, and conſequently the fewer rays would be intercepted by the opake body, till at laſt it would appear only as a ſpot in the Sun; in like manner as the planets Venus and Mercury appear when they are ſeen to paſs over the Sun's diſk.

85. What has hitherto been ſaid of the ſhadow of the Earth includes that of the atmoſphere ſurrounding the Earth: for in lunar eclipſes the ſhadow of the atmoſphere is to be conſidered. And hence it is that the Moon is viſible in eclipſes, the ſhadow caſt by the atmoſphere being not near ſo dark as that caſt by the Earth.

86. The Moon always enters the weſtern ſide of the ſhadow with her eaſtern limb, and quits it with her weſtern limb; and in her approach to and receſs from the ſhadow, ſhe muſt paſs through a PENUMBRA, or imperfect ſhade, which is cauſed by the Earth itſelf.

87. In the ſame manner, in which it has been ſhewn that the Moon muſt come into the ſhadow of the atmoſphere, when ſhe is at full and at or near a node, it may alſo be ſhewn, that her ſhadow muſt fall upon the Earth at the time of new Moon, provided ſhe is in or near a node: But the penumbra of the Moon's ſhadow is much more ſenſible in ſolar eclipſes, than that accompanying the ſhadow of the Earth in lunar ones.

88. It is obſerved, that to determinate parts of the Earth ſolar eclipſes are not ſeen ſo oft as lunar ones; which is owing to the ſhadow of the Moon being leſs than that of the Earth: For the Earth's ſhadow often covers all the Moon; but that of the Moon cannot cover all the Earth; and as it ſometimes falls on one part, ſometimes on another, it cauſes ſolar eclipſes, in general, to be more frequent than lunar ones; yet to any determinate place on the Earth there are more eclipſes of the Moon viſible than of the Sun.

What has hitherto been ſaid, may ſuffice to give beginners a general idea of the motions of the bodies in the ſolar ſyſtem, and of ſome of the phenomena thence ariſing; thoſe who deſire to be farther acquainted with particulars, may find them fully treated of in M. de la Caille's Elements of Aſtronomy, publiſhed in Engliſh a few years ſince *; and alſo in the works of Gregory, Keil, and others.

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* Tranſlated by the Author of theſe Elements.

# SECTION III.

## *The Aſtronomy of the Sphere.*

### DEFINITIONS and PRINCIPLES.

89. By the Aſtronomy of the ſphere is meant the finding, from proper things given, the meaſure of certain arcs and angles formed on the ſurfaces of the celeſtial and terreſtrial ſpheres, by the apparent motions of the bodies which are ſeen in the heavens.

The ſurfaces of thoſe ſpheres are ſuppoſed to be concentric to the center of the Earth, and to have correſpondent circles deſcribed on both ſpheres.

90. GREAT CIRCLES are thoſe which divide either ſphere into two equal parts.

LESSER CIRCLES, thoſe which divide the ſphere into unequal parts.

The POLES of a circle are the points on the ſphere equally diſtant from that circle.

An AXIS is a right line ſuppoſed to connect the poles.

The CELESTIAL AXIS is that right line about which the heavens ſeem to revolve.

The NORTH and SOUTH POLES of the world are thoſe two points where the axis cuts the celeſtial ſphere.

91. The EQUINOCTIAL or EQUATOR, is the great circle of the ſphere equally diſtant from the poles of the world.

92. MERIDIANS, or HOUR CIRCLES, or CIRCLES of RIGHT ASCENSION, or CIRCLES OF TERRESTRIAL LONGITUDE, are great circles perpendicular to the equator, and paſſing through the poles of the world.

93. The ECLIPTIC is a great circle inclined to the equator in an angle of abour 23½°, and cutting it in two points diametrically oppoſite.

The ecliptic is ſuppoſed to be divided into 12 equal parts, called SIGNS, beginning from one of its interſections with the equator; each ſign containing 30 degrees, named and noted thus:

| | | | | | |
|---|---|---|---|---|---|
| *Aries* | *Taurus* | *Gemini* | *Cancer* | *Leo* | *Virgo* |
| ♈ | ♉ | ♊ | ♋ | ♌ | ♍ |
| *Libra* | *Scorpio* | *Sagittarius* | *Capricornus* | *Aquarius* | *Piſces* |
| ♎ | ♏ | ♐ | ♑ | ♒ | ♓ |

The firſt ſix are called *northern*, and the latter ſix *ſouthern* ſigns.

94. The CARDINAL POINTS of the ecliptic are the four firſt points of the ſigns ♈, ♋, ♎, ♑; thoſe of ♈ and ♎ are called EQUINOCTIAL POINTS, and thoſe of ♋ and ♑ are called SOLSTITIAL POINTS.

95. The EQUINOCTIAL COLURE is a meridian paſſing through the *equinoctial points*; and the SOLSTITIAL COLURE is another meridian paſſing through the *ſolſtitial points*. The coloures cut one another at right angles in the poles of the world.

96. CIR

96. CIRCLES OF CELESTIAL LONGITUDE are great circles perpendicular to the ecliptic.

97. The LATITUDE of any point in the heavens is an arc of a circle of longitude intercepted between that point and the ecliptic, and is called north or ſouth latitude, as the point is on the north or ſouth ſide of the ecliptic.

98. PARALLELS OF CELESTIAL LATITUDE are ſmall circles parallel to the ecliptic.

99. The LONGITUDE of any object in the heavens is an arc of the ecliptic intercepted between the firſt point of Aries and a circle of longitude paſſing through that point.

100. The RIGHT ASCENSION of any object is an arc of the equator, contained between the firſt point of Aries and a meridian paſſing through that point: Or, it is the angle formed by the equinoctial colure, and the meridian paſſing over that point.

101. The DECLINATION of any object is an arc of a meridian contained between that point and the equinoctial: If the point is on the north ſide of the equinoctial, it is called *north declination*; but if on the ſouth ſide, it is called *ſouth declination.*

102. The OBLIQUITY OF THE ECLIPTIC is the angle made by the interſection of the equator and ecliptic, and is meaſured by the Sun's greateſt declination; which, according to modern obſervations, is about 23° 28′.

103. PARALLELS OF DECLINATION are ſmall circles parallel to the equinoctial. The TROPIC OF CANCER is a parallel of declination at 23° 28′ diſtant from the equinoctial in the northern hemiſphere; and the TROPIC OF CAPRICORN is the parallel of declination as far diſtant in the ſouthern hemiſphere.

104. THE ARCTIC POLAR CIRCLE is a parallel of declination at 23° 28′ diſtant from the north pole; and the ANTARCTIC POLAR CIRCLE is the parallel of declination as far diſtant from the ſouth pole.

105. The ZENITH is the point of the heavens directly over a place; and the NADIR is the point directly underneath.

106. The HORIZON is that great circle of the ſphere which is equally diſtant from the *zenith* and *nadir* of any place, and divides the ſphere into the upper and lower hemiſpheres.

107. The RISING of a celeſtial object is when its center appears in the eaſtern part of the horizon; and its SETTING is when its center diſappears in the weſtern quarter of the horizon.

108. AZIMUTH, or VERTICAL CIRCLES, are great circles perpendicular to the horizon, paſſing through its poles, which are the *zenith* and *nadir*.

109. The PRIME VERTICAL is that vertical circle which paſſes through the eaſt and weſt points of the horizon, and is at right angles to the *meridian of the place*, which is a vertical circle paſſing through the north and ſouth points of the horizon.

110. As the meridian of a place is called the *twelve o'clock hour circle*, ſo the hour circle at right angles to the meridian is called the *ſix o'clock hour circle.*

111. The AZIMUTH of any celeſtial object is an angle at the zenith formed by the meridian of any place, and a vertical circle paſſing through

that object when it is above or below the horizon: And it is meaſured by the arc of the horizon intercepted between thoſe vertical circles.

112. The AMPLITUDE of any object in the heavens is uſually taken as an arc of the horizon contained between the eaſtern point of it, and the center of the object at its riſing, or between the weſtern point of it and the center of the object at its ſetting; or it may be taken as an angle at the zenith, included between the meridian of a place and a vertical circle paſſing through the object at its riſing or ſetting.

113. The ALTITUDE of any object in the heavens is an arc of a vertical circle intercepted between the center of that object and the horizon.

114. The ZENITH DISTANCE of any object is an arc of a vertical circle contained between the center of that object and the zenith.

The altitude and zenith diſtance are complements one of the other.

115. The MERIDIAN ALTITUDE, or MERIDIAN ZENITH DISTANCE, is the altitude or zenith diſtance when the object is on the meridian of the place.

116. The CULMINATING of any celeſtial object, is the time it *tranſts*, or comes to the Meridian. And the MEDIUM COELI, or MID-HEAVEN, to any place, is that degree of the ecliptic, or part of the heavens, over the meridian of that place, at any time. Or the MID-HEAVEN is the diſtance of the meridian from the firſt point of Aries, reckoned on the equinoctial.

117. The Nonageſimal degree is the 90th degree of the Ecliptic, reckoned from its interſection with the eaſtern point of the horizon, at any given time.

Conſequently the altitude or height of the nonageſimal degree above the horizon is equal to the diſtance of the poles of the Ecliptic and Horizon; and is the meaſure of the angle which the ecliptic makes with the horizon.

118. ALMICANTHERS, or PARALLELS OF ALTITUDE, are ſmall circles parallel to the horizon.

119. A PARALLEL SPHERE is that poſition of the ſphere in which the circles, apparently deſcribed by the diurnal rotation, are parallel to the horizon; which can happen only at the poles.

120. A RIGHT SPHERE is that in which the diurnal motions are at right angles to the horizon: Thus it appears in all places under the equator.

121. An OBLIQUE SPHERE has all the diurnal motions oblique to the horizon: And thus the motions appear to all parts of the Earth, except under the poles and equator.

122. DIURNAL ARCS are thoſe parts of the parallels of declination of celeſtial objects which are apparently deſcribed between the times of the riſing and ſetting of thoſe objects: And NOCTURNAL ARCS are the parts of thoſe parallels apparently deſcribed from the time of ſetting to the time of riſing.

123. SEMI-DIURNAL and SEMI-NOCTURNAL ARCS, or the halves of diurnal and nocturnal arcs, are the parts of the parallels intercepted between the meridian and the horizon. The correſponding part of the equator anſwering to the ſemi-diurnal arc, gives the times between noon and the riſing or ſetting; and the equatorial part anſwering to the ſemi-

nocturnal arc, ſhews the time between midnight and the time of ſetting or riſing.

124. The Oblique Ascension of any object in the heavens, is an arc of the equinoctial intercepted between the firſt point of Aries and the eaſtern part of the horizon when that object is riſing; and the Oblique Descension is an arc of the equinoctial intercepted between the firſt point of Aries and the weſtern part of the horizon at its ſetting.

125. The Ascensional Difference belonging to any celeſtial object is an arc of the equinoctial intercepted between the horizon and the hour-circle which the object is on when it riſes or ſets; or it is the difference between the right and oblique aſcenſion of that object. In the Sun, it is the time that he riſes or ſets before or after the hour of ſix.

126. The Latitude of any place on the Earth is an arc of a terreſtrial meridian contained between that place and the equator; or it is an arc of a celeſtial meridian intercepted between the zenith of the place and the equinoctial; being north or ſouth, according to the ſide of the equator it is on.

127. The Longitude of any place on the Earth is an arc of the equator contained between the meridian of that place and the meridian which is choſen for the firſt, where the reckoning of longitude begins: Or, it is the angle at the pole formed by the firſt meridian and that of the place.

128. Refraction, in an aſtronomical ſenſe, is the difference between the true and apparent altitudes of celeſtial objects; they appearing more elevated above the horizon than they really are, on account of the denſity of the Earth's atmoſphere, or air and vapours ſurrounding it.

129. The Twilight is that medium between light and darkneſs, which happens in the morning before ſun-riſe, and in the evening after ſun-ſet.

This is occaſioned by the atmoſphere's refracting the ſolar rays upon any place, although the Sun is below the horizon of that place, and by obſervation it is found to begin and end when the Sun is about 18° below the horizon.

130. The Crepusculum is a ſmall circle parallel to the horizon at 18° below it, where the twilight begins and ends.

131. The Latitude of a Place is expreſſed by an arc of the meridian, ſhewing the diſtance between the zenith of that place and the equinoctial; or, by an arc of the meridian, ſhewing the height of the pole above the horizon.

For under the pole, or in the latitude of 90 degrees, the pole is in the zenith, or is 90 degrees above the horizon; ſo that, in this caſe, the horizon coincides with the equinoctial.

And as many degrees as the obſerver goes from the pole towards the equator, ſo many degrees does his horizon go below the equator on one ſide, and approach the pole on the other ſide.

Therefore the pole approaches the horizon juſt as much as the zenith approaches the equator; that is, the height of the pole above the horizon, is equal to the diſtance of the zenith from the equinoctial, which is equivalent to the diſtance of the obſerver from the equator, or is equal to the latitude.

132. Astronomical Tables in general contain numbers ſhewing either the meaſure of the diſtances of the heavenly bodies from certain limits which are uſed to repreſent remarkable *times* and *places*; or, the times when thoſe bodies had, or will have, given poſitions relative to thoſe limits.

Some of the chief aſtronomical tables are,

Solar and lunar tables for finding the places of thoſe luminaries at given times.

Tables for finding the places of the other planets.

Stellar tables for finding the places of the ſtars.

Tables ſhewing the Sun's place, declination, and right aſcenſion for given times.

Tables of refractions for correcting obſervations on altitudes.

Tables of the equation of time; or the difference between the times ſhewn by a ſun-dial and a well-regulated clock.

*&c.*

The aſtronomical tables chiefly wanted in this work are placed at the end of this book; and are preceded by an account of their conſtruction and uſe.

133. As the Earth makes one revolution on its axis in a common day of 24 hours; therefore every point of the equator will deſcribe the circle of 360 degrees in 24 hours; and conſequently, if 360 degrees give 24 hours, any other number of degrees will give its proportional hours: And if 24 hours give 360 degrees, any other number of hours will give its proportional number of degrees.

And hence are derived methods for converting arcs of circles into meaſures of time, and meaſures of time into arcs of circles.

### *To reduce degrees, minutes, &c. to time.*

Multiply by 24, and divide by 360; or multiply by 4, and divide by 60: *Or*, Divide the given degrees by 15 for hours; multiply the remainder by 4 for minutes, adding to the product 1 minute for every 15′ of a degree; the overplus minutes of a degree, multiplied by 4, give ſeconds of time, *&c.*

*Or thus:* Let the quotient of the given degrees by 60 ſtand for the firſt name; the remaining degrees for the ſecond name; and the other given names in order following: Then this number multiplied by 4, will give the hours, minutes, ſeconds, &c. in order.

### *To reduce time into degrees.*

Multiply the given hours by 15 gives degrees, to which add 1° for every 4 minutes of time; for every overplus minute reckon 15′ of a degree; and for every ſecond of time take 15″ of a degree.

*Or thus:* Divide the time by 4, carrying by ſixties, the quotient will be in order, ſixties of degrees, degrees, minutes, ſeconds, &c.: Then ſixties of degrees and degrees being reduced, will give the degrees, &c. required.

Exam. I. *Reduce* 69° 20′, 45″, *to its correſponding time.*

15) 69° 20′ 45″ ($4^h$ $37^m$ $23^s$

9×4+1=37

5×4+3=23

Exam. II. *Reduce* $4^h$, $37^m$, $23^s$, *to its correſponding degrees.*

$4^h$ $37^m$ $23^s$
15

| | | | |
|---|---|---|---|
| 60° | 0′ | 0″ | for 4 hours. |
| 9 | 15 | 0 | for 37 minutes. |
| | 5 | 45 | for 23 ſeconds. |
| 69° | 20′ | 45″ | |

Exam. III. *Reduce* 237°, 44′, 37″, *to its equivalent time.*

60) 237°

3 57 44 37
4

Anſwer $15^h$ $50^m$ $58^s$ $28^t$

Exam. IV. *Reduce* $15^h$,$50^m$,$58^s$, $28^t$, *to its equivalent degrees.*

4) $15^h$ $50^m$ $58^s$ $28^t$

$3^s$ 57° 44′ 37″

Or 237° 44′ 37″ Anſwer.

As the Sun is conſtantly changing his place, the tables of his right aſcenſion ſhew for every day at noon (when he comes to the meridian of the place for which the tables are made) what part of the equator is intercepted between that meridian and the equinoctial point ♈. The tables for the ſtars ſhew the equatorial arcs contained between the point ♈ and the ſection of circles of right aſcenſion, paſſing through thoſe ſtars: The meaſures of the arcs of right aſcenſion are reduced to time.

There are few days when one or more ſtars do not come to the meridian with the Sun, and then they have the ſame right aſcenſion with him: Alſo, at ſome time of the year, the Sun muſt have the ſame right aſcenſion which any propoſed ſtar has; though at other times he may have a leſs, and ſo precedes, or comes to the meridian before that ſtar; or a greater, and ſo follows the ſtar, and comes to the meridian later. And hence is derived the following method.

## Of Finding the Culminating of the Stars.

134. *To find the time when any ſtar in the table will be on the meridian.*

Rule. Subtract the ſun's right aſcenſion for the propoſed day from the right aſcenſion of the given ſtar; the difference will be the time of the ſtar's culminating nearly. Say as $24^h$ is to the daily change of the ſun's right aſcenſion, ſo is the time of culminating, nearly, to a fourth number; which being ſubtracted from the time of culminating nearly, will give the true time of the ſtar's culminating. If this time be leſs than $12^h$ it happens in the afternoon; but if more than twelve hours, the exceſs above $12^h$ will ſhew the time next morning.

N. B. $24^h$ muſt be added to the ſtar's right aſcenſion, if the ſun's right aſcenſion be greateſt.

EXAM. I. *At what time will the ſtar Arcturus come to the meridian of London on the 1ſt of September*, 1780?

| | | | |
|---|---|---|---|
| Right aſcen. of Arcturus | $14^h$ | $5'$ | $42''$ |
| Sun's right aſcenſion | 10 | 44 | 34 |
| Time of culmin. nearly | 3 | 21 | 8 |
| And $3^h\ 21\frac{1}{4}^m$ give | | | 30 |
| True time of ſtar's culm. | 3 | 20 | 38 |

EX. II. *On the 26th of Feb.* 1780, *at what hour will the ſtar Virgin's Spike be on the meridian of London?*

| | | | |
|---|---|---|---|
| Virg. Spike's right aſc. | $13^h$ | $13'$ | $38''$ |
| Sun's right aſcenſion | 22 | 37 | 10 |
| Time of culm. nearly | 14 | 36 | 28 |
| And $14^h$, $36\frac{5}{8}^m$ give | | 2 | 18 |
| True time of ſtar's culm. | 14 | 34 | 10 |

If the time of the ſtar's culminating be wanted for any other meridian than that of Greenwich, or London, add the longitude in time to the time of culminating nearly, if the longitude be weſt, or take their difference if it be eaſt, and uſe that ſum or difference inſtead of the time of culminating nearly: obſerving, only, in the latter caſe, that if the longitude in time be greater than the time of culminating nearly, that the min. and ſec. reſulting from the proportion, muſt be added to the time of culminating nearly, inſtead of being ſubtracted from it.

EXAM. *On the 26th of February*, 1784, *what time will Syrius be on the meridian of a place which is in longitude* 166° 30′ *E. of London?*

| | | | |
|---|---|---|---|
| Rt. aſcen. Syrius, 1780 | $6^h$ | $35'$ | $28''$ |
| Preceſſion for 4 years | + | | 11 |
| Time of culm. nearly | 7 | 58 | 36 |
| Long. in time | 11 | 6 | 00 |
| Difference | 3 | 7 | 24 |

| | | | |
|---|---|---|---|
| } Rt. aſc. of Syrius, 1784 | $6^h$ | $35'$ | $39''$ |
| Sun's right aſcen. | 22 | 37 | 10 |
| Time of culm. nearly | 7 | 58 | 29 |
| And $3^h\ 7^m$, 4 give + | | | 29 |
| True time of ſtar's culm. | 7 | 58 | 58 |

*To find if any ſtar in the table will be on, or near the meridian at a given time, reckoned from the preceding noon.*

RULE. To the given time add the Sun's right aſcenſion for that time; the ſum (rejecting 24 hours, if above) is the right aſcenſion of the mid-heaven; which being ſought among thoſe of the ſtars, will ſhew what ſtar will be on, or near the meridian at the time propoſed.

EXAM. I. *What ſtar will be on the meridian of London about* 10 *o'clock at night on the 25th January*, 1784?

| | | |
|---|---|---|
| Given time 10 hours P. M. | $10^h$ | $0^m$ |
| Sun's right aſcenſion at noon | 20 | 31 |
| And for 10 hours more | | 2 |
| Sum (abating 24 hours) anſwers to Syrius. | 6 | 33 |

EX. II. *On the 10th May*, 1784, *what ſtar will be on the merid. of Lond. about 30 min. after 4 in the morning?*

| | | |
|---|---|---|
| Given time | $16^h$ | $30'$ |
| Sun's right aſcenſion at noon | 3 | 12 |
| And for 16 hours more | | 3 |
| Right aſcenſion mid-heaven anſwers nearly to Altair. | 19 | 45 |

SECTION

## SECTION IV.

## *Of the Projection of the Sphere.*

### 135. PROBLEM I.

*To project the ſphere upon the plane of the ſolſtitial colure, or upon the plane of the meridian of any place, thoſe planes being ſuppoſed to coincide.*

For this projection, the eye is ſuppoſed to be in the firſt point of Aries, or the common interſection of the equator, ecliptic, and equinoctial colure; that being the pole of the plane of projection, or primitive circle. Pl. IV. Fig. 4.

1ſt. With the chord of 60 degrees deſcribe a circle PESQ to repreſent the ſolſtitial colure, the center of which ♈ is its pole. (IV. 62)

2d. A diameter EQ will be the equator, and another PS at right angles to it will ſhew the equinoctial colure (IV. 60), or the axis of the world, the extremities of which P, S, will be the north and ſouth poles.

3d. *For the parallels of declination.* On the primitive circle, beginning at E and Q, apply the chords of the given degrees of declination, ſuppoſe every 10 degrees, and alſo the diſtances of the *tropics* and *polar circles* from the equator, namely, $23\frac{1}{2}^\circ$ and $66\frac{1}{2}^\circ$. Then from the center ♈ in the axis PS produced, apply the reſpective ſecants of the complements of the degrees laid on the primitive (IV. 58), and theſe will give the centers of the correſponding parallels of declination; from which centers, with the extents to the ſeveral diviſions in the circumference, deſcribe the ſmall circles 10, 10; 20, 20; *&c.* and theſe will be the parallels of declination required: Among which *a* ♋, *b* ♑, are the tropics of Cancer and Capricorn; and *cc*, *dd*, the arctic and antarctic polar circles.

4th. *For the circles of right aſcenſion, or hour circles.* In the diameter EQ produced lay off from the center ♈ both ways the tangents of 15°, 30°, 45°, 60°, 75°, reſpectively, and they will give the centers of circles to be deſcribed through P and S, and cutting the equator in the points repreſenting the 24 hours; the ſolſtitial colure being the 12 o'clock, and the equinoctial colure, PS, the ſix o'clock hour circles. And in like manner may any other of this kind of circles be drawn. (IV. 75)

5th. *The ecliptic* ♋ ♑ is drawn, making with the equator an angle of $23\frac{1}{2}^\circ$; the poles of which *c*, *d*, are the interſections of the polar circles with the ſolſtitial colure.

6th. *Parallels of celeſtial latitude* are drawn parallel to the ecliptic, in the ſame manner as the circles of declination are drawn parallel to the equator.

7th. *Circles of celeſtial longitude* are deſcribed through *c*, *d*, the poles of the ecliptic, in the ſame manner as the circles of right aſcenſion were deſcribed through P, S, the poles of the equator; and thus were the diviſions of the ecliptic found that are marked with the ſigns.

8th. *The horizon* is repreſented by drawing a diameter HR, making an angle with the axis PS, equal to the latitude of the place; and the poles of the horizon Z, N, the zenith and nadir, are at 90° diſt. from the circle HR.

9th. *Azimuth*, or *vertical circles*, making any angle with the meridian, are deſcribed like circles of right aſcenſion: Thus ZN is the prime vertical, and ZAN is another azimuth, 45° from the ſouth.

10th. *Almicanthers*, or *parallels of altitude*, are in this projection drawn parallel to the horizon, in like manner as the circles of declination were drawn parallel to the equator.

## 136. PROBLEM II.

*To project the ſphere upon the plane of the horizon.*

In this projection, the eye is ſuppoſed in the nadir, one of the poles of the horizon, or plane of projection. Plate IV. Fig. 5.

1ſt. *The horizon* is repreſented by the primitive circle, where the upper XII is the north, the lower XII the ſouth, E the weſt; and Q the eaſt points.

2d. *The azimuth circles* are repreſented by diameters drawn through Z, the center or pole of the horizon: Thus the diameter XII, XII is for the meridian, and EZQ for the prime vertical; and other azimuth circles, forming any angle with the meridian, are readily drawn by laying off their diſtances in the primitive from the north or ſouth points.

3d. *Parallels of altitude* are concentric to the primitive, and are deſcribed about the pole Z with the half tangent of their diſtance from it: Thus the ſmall circle, the diameter of which is *ab*, is a parallel of altitude 10° above the horizon, or at 80° diſtant form its pole Z.

4th. The diſtance of the *equinoctial* from the zenith is equal to the latitude of the place, and therefore this circle makes with the horizon an angle, which is meaſured by the complement of the latitude; then ſetting off from the center Z in Z XII continued, the tangent of 50° (the latitude in this example being 40°), it will give the center of the circle EAQ, repreſenting the equinoctial; and the half tangent of 50°, ſet the ſame way from Z, will give P, the pole of the world.

5th. *The ſix o'clock hour circle* paſſes through the poles of the world, making with the horizon an angle equal to the meaſure of the latitude; therefore taking in the meridian, from Z towards A, the tangent of the latitude 40°, it gives G, the center of the ſix o'clock hour circle EPQ.

6th. *The hour circles* paſs through the poles of the world, and make with one another angles of 15 degrees: Therefore (IV. 55) in a line DE, drawn through G, at right angles to the meridian, ſet off on both ſides of G the tangents 15°, 30°, 45°, 60°, 75°, to the radius PG, and they will give the centers of the ſeveral hour circles paſſing through P, cutting the horizon and equinoctial in the hour points.

7th. *The polar circles, tropics, and other circles of declination*, are deſcribed parallel to the equinoctial, about its pole P, at given diſtances from it, either by finding the centers of ſuch parallels, as ſhewn in B. IV. 66; or by ſetting off on each ſide of Z the half tangents of their greateſt and leaſt diſtances from Z; then the middles of thoſe intervals are the centers ſought. Thus; the arctic circle is diſtant from P 23½°; then to, and from ZP=50°, add and take 23½°; there remain 73½° and 26½°; the half tangents of theſe ſet off from Z give *p* and *q*; then a circle deſcribed on the diameter *pq* is the arctic circle.

In like manner will the centers of the tropic of Cancer *c* ♋ *c*, and of Capricorn *d* ♑ *d*, be obtained.

8th. *The northern portion of the ecliptic* ♈ ♋ ♎ is deſcribed from a center diſtant from Z towards P, the tangent of 73½°, = ∠ the ecliptic makes with the horizon.

9th. *Circles of longitude* ♈ *p* ♎, *p* ♉, *p* ♊, *p* ♌, *p* ♍, are deſcribed thro' *p*, the pole of the ecliptic, from centers in the line BFC; in like manner as the hour circles were deſcribed through P, the pole of the equator.

10th. *Circles of celeſtial latitude*, VIII *q* IX, are deſcribed about *p*, as the circles of declination were deſcribed about P, the pole of the equinoctial.

## 137. PROBLEM III.

*To project the ſphere upon the plane of the equator.*

In this projection the eye is ſuppoſed to be in one of the poles of the equator, ſuppoſe in the ſouth pole, and projecting the north hemiſphere. Plate IV. Fig. 6.

1ſt. *The equator* is repreſented by the primitive circle, the center and pole of which is P.

2d. *The hour circles* are expreſſed by diameters making angles of 15° with one another; of which XII P XII is the meridian, or ſolſtitial colure, and VI P VI the 6 o'clock circle, or equinoctial colure.

3d. *Circles of declination* are circles parallel and concentric to the equator, deſcribed from its center, with radii equal to the half tangents of their ſeveral diſtances from the pole P, or half co-tangents of their degrees of declination: Thus *pq* the arctic circle, and *a* ♋ the tropic of Cancer, are deſcribed with the half tangents of 23½° and 66½° reſpectively; and ſo of the others.

4th. *The ecliptic* making an angle of 23½° with the equator; the tangent of theſe degrees laid from P towards *a* will give the center for deſcribing the ecliptic ♈ ♋ ♎, the pole of which *p* is in the polar circle.

5th. *Circles of longitude* are deſcribed through *p*, the pole of the ecliptic, in like manner as the hour circles were deſcribed through P, the pole of the equator in the laſt problem; and thus were the diviſions ♉, ♊, ♌, ♍, obtained.

6th. *Circles of celeſtial latitude* are projected in the ſame manner as the circles of declination in the laſt problem.

7th. *The horizon of any place*, ſuppoſe of London, being inclined to the equator in an angle equal to the co-latitude, 38° 28′; the tangent of this laid from P towards ♋, and the half tangent laid from P to Z, will give the center, and Z the pole, of the horizon HOR.

8th. *The prime vertical* HZR making an angle with the equator equal to 51° 32′, the latitude of the place, its center is found by laying the tangent of 51° 32′ from P towards O.

9th. *Azimuth circles*, making given angles with the meridian ZO, are thus deſcribed: In a line drawn through the center of the prime vertical, at right angles to the meridian, take diſtances from that center, equal to the tangents of the propoſed azimuth angles, the ſemidiameter of the prime vertical being the radius, thoſe diſtances give the centers ſought; and thus was the azimuth circle ZA deſcribed.

10th. *Parallels of altitude* are deſcribed about Z, the pole of the horizon, at the diſtances of the co-altitudes, in the ſame manner as the circles of declination were deſcribed about P, the pole of the equator in the laſt problem; and thus was the ſmall circle V *h* VII deſcribed at 10° diſtance from the horizon, or 80° diſtant from its pole Z.

## 138. PROBLEM IV.

*To project the ſphere upon the plane of the ecliptic.*

The eye is here ſuppoſed to be in one of the poles of the ecliptic, and thence viewing the northern hemiſphere. Plate IV. Fig. 7.

1ſt. *The ecliptic* is here repreſented by the primitive circle, the center of which *p* is its pole.

2d. *Circles of longitude* are here repreſented by diameters; thoſe that make angles of 30° with one another, being drawn through the diviſions marked with the ſigns of the zodiac.

3d. *Parallels of celeſtial latitude* are circles deſcribed about *p*, concentric to the ecliptic; ſuch is the ſmall circle, the diameter of which is *ab*, repreſenting the parallel of 10° of latitude.

4th. *The equator* making an angle with the ecliptic of $23\frac{1}{2}^\circ$; therefore the tangent of this inclination laid from *p* towards ♋ will give the center of the equator ♈ XII ♎; and the half tangent of $23\frac{1}{2}^\circ$ laid from *p* the ſame way, gives P for the pole of the equator.

5th. *The equinoctial colure*, which here makes the *ſix o'clock circle*, makes an angle with the ecliptic of $66\frac{1}{2}^\circ$; therefore the tangent of $66\frac{1}{2}^\circ$ laid from *p* towards ♑, gives the center of the 6 o'clock circle ♈ P ♎.

6th. *Hour circles* paſſing through P, and making angles of 15° with one another, are deſcribed from centers, found in a right line paſſing through the center of ♈ P ♎, and drawn at right angles to the ſolſtitial colure ♈ *p* ♑; by laying off in that line the tangents of 15°, 30°, 45°, 60°, 75°, reckoned from the center of ♈ P ♎, on both ſides, the ſemi-diameter of this circle being the radius. Theſe hour circles cut the equator in the hour points.

7th. *Parallels of declination*, ſuch as the tropic of Cancer, and the arctic circle, the diameters of which are 12, 12, and *pq*, are deſcribed by laying off from *p* the half tangents of their greateſt and leaſt diſtances: Thus *q* being diſtant from *p* 47°, makes $pq = \frac{1}{2}$ tangent of 47°, the middle of *pq* will be the center of the polar circle.

8th. *The horizon* HOR is to make an angle with the ecliptic equal to the difference between the co-latitude and the obliquity of the ecliptic, when P is projected to the north of *p*; otherwiſe that angle is equal to the ſum of thoſe quantities. And for London, where the ſaid difference $=(38^\circ\, 28' - 23^\circ\, 28' =)$ $15^\circ\, 00'$, the tangent of $15^\circ\, 00'$, gives the center of HOR, and the half tangent gives Z the zenith.

9th. *The prime vertical* HZR is deſcribed by laying from *p*, towards O, the co-tangent of *p*Z for a center.

10th. *Azimuth circles* are deſcribed through Z, making given angles with the meridian ZO, by finding their centers in a line drawn through the center of HZR, in the manner deſcribed for the hour circles, Prob. II.

11th. *Parallels of altitude* are repreſented by deſcribing ſmall circles parallel to the horizon HOR, at given diſtances from it; or, which comes to the ſame, deſcribing ſmall circles about the pole Z, at diſtances equal to the complements of the given altitudes: And thus the circle *cdc* was deſcribed for a parallel of 33° of altitude.

## SECTION V.

## *Problems of the Sphere.*

### 139. PROBLEM V. Pl. V.

Given the Sun's longitude, and the obliquity of the ecliptic; Required the Sun's right aſcenſion and declination.

Exam. *Let the obliquity of the ecliptic, or the Sun's greateſt declination, be* 23° 28′, *and the Sun's place* 13° 16′ *in Taurus: Required the reſt.*

### Construction.

In the primitive circle PESQ, repreſenting the ſolſtitial colure, the center of which is ♈, draw a diameter EQ for the equator, and at right angles to EQ draw a diameter PS for the equinoctial colure: Make E ♋ = 23° 28′, and draw a diameter ♋ ♑ for the ecliptic, in which (IV. 71.) take ♈ ☉ = 43° 16′ for the Sun's diſtance from the point ♈: Through P ☉ S deſcribe a circle of right aſcenſion.

Computation. See Book IV. art. 130, 131.

In the right angled ſpheric triangle ♈ ☉ B.

Given Sun's longitude ♈ ☉ = 43° 16′ } Req. right aſcen. ♈ B.
Obliquity of the Eclip. ∠ ☉ ♈ B = 23 28 } declin. B ☉.

| *To find the declination.* | | | | *To find the right aſcenſion.* | | | |
|---|---|---|---|---|---|---|---|
| As Radius | = | R | 10,00000 | As Radius | = | R | 10,00000 |
| To ſ. Sun's lon. | = | 43° 16′ | 9,83594 | To t. Sun's lon. | = | 43° 16′ | 9,97371 |
| So ſ. ob. eclip. | = | 23 28 | 9,60012 | So co-ſ. obl. ecl. | = | 23 28 | 9,96251 |
| To ſ. Sun's decl. | = | 15 50 | 9,43606 | To t. rt. aſcen. | = | 40 48 | 9,93622 |

140. While the Sun is moving from ♈ to ♋, or is in the firſt quadrant of the ecliptic, the given longitude is the hypothenuſe in the triangle ♈ ☉ B, the declination B ☉ is north, and ♈ B is the right aſcenſion.

When the Sun has paſt the ſolſtice ♋, and is deſcending towards ♎, he is then ſaid to be in the ſecond quadrant, and his longitude or diſtance from ♈ being taken from 180°, the remainder ♎ ☉ becomes the hypothenuſe, and the declination is ſtill north; but the arc B ♎ found for the right aſcenſion is only the ſupplement, and muſt therefore be taken from 180°.

The Sun having paſt the point ♎, and deſcending towards ♑ has got into the third quadrant; the longitude then, reckoned from ♈, will be greater than 180°: In this caſe the exceſs above 180°, or the diſtance the Sun is removed from ♎, will be the hypothenuſe ♎ ☉; the declination will be ſouth; and the arc ♎ A, found for the right aſcenſion, muſt be added to 180°, to give the right aſcenſion eſtimated from ♈.

When the Sun has paſt the ſolſtice ♑, and is aſcending towards ♈, he is then in the fourth quadrant; therefore the longitude is greater than 270°, and muſt be taken from 360°, to give the hypothenuſe ♎ ☉. Here the declination is ſouth, and the right aſcenſion ♎ A, found by the proportion, muſt be taken from 360°, to give the right aſcenſion from ♈.

At equal diſtances from the equinoctial points ♈ or ♎, the Sun will have equal quantities of declination; but will be of different names, according as it is on the north or ſouth ſides of the equinoctial.

## 141. PROBLEM VI. Pl. V.

Given the obliquity of the ecliptic, and the Sun's declination;
Required the Sun's longitude and right aſcenſion.

EXAM. *The obliquity of the ecliptic, being 23° 28′, what is the Sun's longitude and right aſcenſion when he has 20° 43′ of north declination?*

### CONSTRUCTION.

Having deſcribed the ſolſtitial colure, and drawn the equator EQ, the axis PS, and the ecliptic ♋ ♑, as before; make E*n*, Q*n*, equal to the given declination, and (3d 133) deſcribe the parallel of declination *nn*, its interſection with the ecliptic gives ☉ the Sun's place; through P, ☉, S, deſcribe the circle of right aſcenſion P ☉ S.

### COMPUTATION.

In the right-angled ſpheric triangle ♈ ☉ B.

Given the ob. eclip. ∠ ☉ ♈ B = 23° 28′ } Required Sun's long. ♈ ☉.
the Sun's decl. ☉ B = 20 43 } rt. aſcen. ♈ B.

| *To find the Sun's longitude.* | | | *To find the Sun's right aſcenſion.* | | |
|---|---|---|---|---|---|
| As ſ. obliq. eclip. | = 23° 28′ | 0,39988 | As Radius | = R | 10,00000 |
| To ſin Sun's decl. | = 20 43 | 9.54869 | To co-t. obl. eclip. | = 23° 28′ | 10,36239 |
| So radius | = R | 10,00000 | So tan. Sun's dec. | = 20 43 | 9,57772 |
| To ſin. ☉ longit. | = 62 40 | 9,94857 | To ſin. rt. aſcen. | = 60 36 | 9,94011 |

Therefore the Sun is in ♊ 2° 40′, or in ♋ 27° 20′, according as the time of the year is before or after the ſummer ſolſtice.

## 142. PROBLEM VII. Pl. V.

Given the obliquity of the ecliptic, and the Sun's right aſcenſion;
Required the Sun's longitude and declination.

EXAM. *When the Sun's right aſcenſion is 60° 31′, what is the longitude and preſent declination, the obliq. of the eclip. being 23° 28′?*

### CONSTRUCTION.

The ſolſtitial colure, equator, axis, and ecliptic being deſcribed as before, make ♈ B = given right aſcenſion (4th 133), and deſcribe the circle PBS, cutting the ecliptic in ☉ the Sun's place.

### COMPUTATION.

In the right-angled ſpheric triangle ♈ ☉ B; the leg ♈ B and ∠ ☉ ♈ B being known, the hypoth. ♈ ☉, and other leg ☉ B, are found as in art. 137, 138. Book IV.

| | |
|---|---|
| As Rad. : co-ſ. ob. eclip. : : co-t. rt. aſ. : co-t. ☉ long. | As Rad. : tan. obl. eclip. : : ſin. rt. aſ. : tan. decl. |
| As Rad. : co-ſ. 23° 28′ : : co-t. 60° 31′ : co-t. 62° 35′ | As Rad. : tan. 23° 28′ : : ſin. 60° 31′ : tan. 20° 42′ |

Three other problems may be formed out of the four things concerned, or obliquity of the ecliptic, declination, longitude, and right aſcenſion: But theſe being of little more importance than as an exerciſe for right-angled ſpheric triangles, they are therefore omitted.

## 143. PROBLEM VIII. Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the Sun's altitude and azimuth at 6 o'clock.

EXAM. *At London, in lat.* 51° 32′ *N., on the longeſt day, when the Sun's declination is* 23° 28′*: Required the Sun's altitude and azimuth at* 6 *o'clock in the morning or evening.*

### CONSTRUCTION.

Deſcribe the meridian, draw the horizon HR, and prime vertical ZN; make RP = latitude 51° 32′ N.; draw the 6 o'clock hour circle PS, the equator EQ, the 23° 28′ N. parallel of declination *n m*, cutting the 6 o'clock hour circle PS in ☉; and through Z, ☉, N, deſcribe the azimuth circle Z ☉ N, cutting the horizon in A; then the things given and required fall in either of the triangles Z ☉ P or ♈ ☉ A, they being ſupplemental triangles one to the other.

### COMPUTATION.

In the ſpheric triangle Z ☉ P, right-angled at P.
Given the co-latit. ZP = 38° 28′ } Required the co-altitude Z ☉.
the co-decl. ☉P = 66 32 } the azimuth ∠ ☉ ZP.
Or in the ſpheric triangle ♈ A ☉, right-angled at A.
Given the latit. A ♈ ☉ = 51° 32′ } Required the altitude A☉.
the decl. ♈ ☉ = 23 28 } the co-azimuth ♈A.

*To find the altitude* A☉.

| | | | |
|---|---|---|---|
| As Radius | = | R | 10,00000 |
| To ſin. decl. | = | 23° 28′ | 9,60012 |
| So ſin. lat. | = | 51 32 | 9,89375 |
| To ſin. alt. | = | 18 10 | 9,49387 |

*To find the azimuth* AR.

| | | | |
|---|---|---|---|
| As Radius | = | R | 10,00000 |
| To co-ſ. lat. | = | 51° 32′ | 9,79383 |
| So tan. decl. | = | 23 28 | 9,63761 |
| To co-t. azim. | = | 74 53 | 9,43144 |

For the arc AR meaſures the ∠RZA, the azimuth. (IV. 9)

144. On the ſhorteſt day at London, the parallel of S. declination cuts the 6 o'clock hour circle below the horizon; and as the triangles ♈A☉, ♈a☉, are congruous, the depreſſion below the horizon, on the ſhorteſt day at 6 o'clock, will be equal to the altitude at the ſame hour on the longeſt day; and the azimuth will alſo be equal, if eſtimated from the ſouth.

So that on the 21ſt of June, at London, the Sun will bear N. 74° 53′ E. at 6 o'clock in the morning, and N. 74° 53′ W. at 6 in the evening; but on the 21ſt of December, at the ſame hours, it will bear S. 74° 53′ E., and S. 74° 53′ W.

From a due conſideration of this Problem it is evident, that as the declination increaſes, the altitude increaſes and the azimuth leſſens; and the contrary happens while the declination is diminiſhing: So that on the days of the equinoxes, on which the Sun has no declination, the altitude at 6 o'clock will be nothing, or the Sun will be in the horizon; and the azimuth being then 90 degrees, the Sun will be due eaſt in the morning, and weſt in the evening; that is, on the days of the equinoxes the Sun riſes and ſets at ſix, in the eaſt and weſt points of the horizon.

## 145. PROBLEM IX. Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the altitude and hour when the Sun is due eaſt or weſt.

Exam. *At London, in latitude 51° 32′ N., what is the Sun's altitude, and the hour when he is due eaſt or weſt, on the longeſt day, or when the declination is 23° 28′ N.?*

### Construction.

Deſcribe the primitive circle to repreſent the meridian of London, draw the horizon HR, and the prime vertical ZN; make RP = 51° 32′, the given latitude, draw the 6 o'clock hour circle PS, the equator EQ, the parallel of declination *nm* (3d 135), cutting the prime vertical in ☉, and through P☉S deſcribe (II. 72) the hour circle P☉S, cutting the equator in A.

Here the things concerned in the Problem fall in either of the triangles PZ☉ or ♈A☉.

### Computation.

In the ſpheric triangle PZ☉, right-angled at Z.

Given the co-latit. PZ = 38° 28′ } Required the co-altitude Z☉
the co-decl. P☉ = 66 32 } the hour fr. noon ∠ZP☉.

Or in the ſpherical triangle ♈A☉, right angled at A.

Given the latit. ∠A♈☉ = 51° 32′ } Required the altitude ♈☉.
the decl. A☉ = 23 28 } the hour after 6 ♈A.

*To find the altitude ♈☉.*

| | | |
|---|---|---|
| As ſ. lat. ∠A♈☉ | = 51° 32′ | 0,10625 |
| To ſin. decl. A☉ | = 23 28 | 9,60012 |
| So Radius | = R | 10,00000 |
| To ſin. alt. ♈☉ | = 30 34 | 9,70637 |

*To find the hour after 6.*

| | | |
|---|---|---|
| As Radius | = R | 10,00000 |
| To co-t. lat. A♈☉ | = 51° 32′ | 9,90009 |
| So tan. decl. A☉ | = 23 28 | 9,63761 |
| To ſ. h. fr. 6. A♈ | = 20 11 | 9,53770 |

Which 20° 11′ converted into time (132), gives 1 h. 20 m. 44 s. for the time after 6 in the morning, and before 6 in the evening, when the Sun will appear due eaſt or weſt; which will be at 7 h. 20 m. 44 s. in the morning, and 4 h. 39 m. 16 s. in the afternoon.

Or, the compl. of 20° 11′, *viz.* 69° 49′ put into time, which gives 4 h. 39 m. 16 s., ſhews the time before and after noon, when the Sun will be due eaſt or weſt.

146. This Problem worked for the ſhorteſt day, namely in the △ ♈*a*☉, which is congruous to ♈A☉, would give the Sun's depreſſion at the time when he was eaſt or weſt, which would be before 6 in the morning, and after 6 in the evening, by as much as was found above, *viz.* 1 h. 20 m. 44 s.

By this Problem it appears, that when the latitude of the place, and the Sun's declination, have the ſame name, then, the greater the declination and latitude, the greater the altitude and time from 6: and having contrary names, the ſame things happen; but with this difference, that in the former caſe the days lengthen on account of the increaſe of the latitude and declination; whereas in the latter caſe the days ſhorten on that account.

## 147. PROBLEM X. Pl. V.

Given the latitude of a place and the Sun's declination;
Required his amplitude and aſcenſional difference.

Exam. *At London, lat.* 51° 32′ *N. on the* 21*ſt of June, being the longeſt day, when the Sun's declination is* 23° 28′ *N. How far from the north does the Sun riſe and ſet, at what time, and what is the length of the day and night?*

### Construction.

Let the primitive circle repreſent the meridian of the place, and the diameter HR the horizon; from R, the north point, take RP = 51° 32′ for the latitude, draw the axis, or 6 o'clock hour circle PS, and at right angles to it draw the equator EQ; make E*n*, Q*m* = 23° 28′, the declination, and (3d 135) deſcribe the parallel of declination *n m*, cutting the horizon in ⊙, the place of the Sun at its riſing and ſetting; through which deſcribe (II. 72) the hour-circle P⊙S.

### Computation.

Now as the arc QR = co-latitude, meaſures the ∠Q♈R.
In the ſpheric triangle ♈⊙A, right-angled at A.

| | | |
|---|---|---|
| Given Sun's decl. A⊙ = 23° 28′ | Required | the amplitude ♈⊙ |
| co-latit. ∠A♈⊙ = 38 28 | | the aſcen. diff. ♈A. |

*To find the amplitude* ♈⊙.

| | | |
|---|---|---|
| As ſin. A♈⊙, co-l. | = 38° 28′ | 0,20617 |
| To ſin. decl. A⊙ | = 23 28 | 9,60012 |
| So Radius | = R | 10,00000 |
| To ſin. amp. ♈⊙ | = 39 48 | 9,80629 |

This 39° 48′ is the amplitude reckoned from the eaſt or weſt points of the horizon: But its complement 50° 12′ ſhews how far from the north the Sun riſes or ſets on the longeſt day at London.

*To find the aſcenſional difference* ♈A.

| | | |
|---|---|---|
| As Radius | = R | 10,00000 |
| To t. lat. ∠P♈⊙ | = 51° 32′ | 10,09991 |
| So tan. decl. A⊙ | = 23 28 | 9,63761 |
| To ſ. aſ. diff. ♈A | = 33 07 | 9,73752 |

Which 33° 07′ converted into time (132) gives 2 h. 12 m. 28 s. for the time which the Sun riſes before, and ſets after, the hour of ſix on the longeſt day.

Suppoſe *rs* to be a parallel of declination as far ſouth, as *m n* is north; then the hour circle P⊙S, paſſing through ⊙ the place of the ſun at its riſing or ſetting, will form a triangle ♈⊙B = △♈⊙A, where the amplitude is to the ſouthward of the eaſt and weſt points.

148. Hence it is evident, *that when the latitude and declination have the ſame name, the Sun riſes before, and ſets after* 6: *But when they are of contrary names, the Sun riſes after, and ſets before* 6.

149. And as the Sun deſcribes the parallel of declination *n m* in 24 hours, being at *n* when it is noon, and at *m* when it is midnight; therefore the time in paſſing from *m* to ⊙, or the time of riſing being doubled, gives the length of the night; and the time of ſetting being doubled, muſt give the length of the day.

| | h | m | s |
|---|---|---|---|
| Then to, and from | 6 | 0 | 0 |
| Add and ſubtract the aſcen. diff. | 2 | 12 | 28 |
| Sum, gives ⊙ ſetting | 8 | 12 | 28 |
| Diff. gives ⊙ riſing | 3 | 47 | 32 |
| Length of day is | 16 | 24 | 56 |
| Length of night is | 7 | 35 | 04 |

But when it is the ſhorteſt day at London, which is, when the Sun has 23° 28′ ſouth declination; then the lengths of the day and night change places; the day being 7 h. 35 m. 04 s. long, and the night 16 h. 24 m. 56 s.

150. When the latitude and declination have the ſame name, the difference between the right aſcenſion and the aſcenſional difference, is the oblique aſcenſion; and their ſum is the oblique deſcenſion..

But when they are of contrary names, their ſum is the oblique aſcenſion, and their difference is the oblique deſcenſion.

151. When the declination is equal to the co-latitude of any place (which can only happen to places within the polar circles), than the parallel of declination will not cut the horizon, and conſequently the Sun will not ſet in thoſe places during the time his declination exceeds the co-latitude: And the ſame may be ſaid of all thoſe ſtars, the polar diſtance of which is leſs than the latitude of the place; or, which is the ſame thing, that have declinations leſs than the co-latitude, for thoſe ſtars will never deſcend below the horizon of that place. But this is to be underſtood only when the Sun or ſtars are in the ſame hemiſphere with the given place; for when the Sun or ſtars are in a contrary hemiſphere to any place, the co-latitude of which does not exceed the declination of thoſe celeſtial objects, then they will never riſe above the horizon of that place, and conſequently are never viſible there.

## 152. PROBLEM XI. Pl. V.

Given the latitude of a place, the Sun's declination and altitude; Required the hour from noon, and the Sun's azimuth.

Exam. *In the latitude of* 51° 32′ *N. the Sun's altitude was obſerved to be* 46° 20′, *when his declination was* 23° 28′ *N. What was the Sun's azimuth, and the hour when the obſervation was made?*

### Construction.

Let the primitive circle zrnh repreſent the meridian of London, hr the horizon, zn the prime vertical; make rp = 51° 32′ the height of the pole at London; draw the axis ps, and the equator eq; lay off the declination e*n*, q*m*, 23° 28′ N. the altitude h*r*, r*s*, 46° 20′; and (IV. 68) deſcribe the parallel of declination *n m*, and the parallel of altitude *rs*, cutting one another in ⊙, the place of the Sun at that time; through z, ⊙, n, deſcribe an azimuth circle z⊙n, and through p, ⊙, s, deſcribe an hour circle p⊙s: Then the angles ⊙zp, ⊙pz, being meaſured (IV. 72), will give the azimuth and hour from noon required.

### Computation.

In the oblique-angled ſpheric triangle p⊙z.

| Given | | |
|---|---|---|
| Given the co-latitude - | zp = 38° 28′ | Required the azim. ∠ ⊙zp |
| the co-alt. or zen. diſt. z⊙ | = 43 40 | and the h. fr. noon ∠ ⊙pz. |
| the co-dec. or pol. diſt. ⊙p | = 66 32 | See art. 167. Book IV. |

*To find the azimuth* ∠ ⊙zp.

Here z⊙ = 43° 40′
zp = 38 28
z⊙ − zp = 5 12 = d
p⊙ = 66 32
71 44 | 35° 52′
2)
61 20 | 30 40

| | | | |
|---|---|---|---|
| Then | Co-ar. ſin. co-lat. | = 38° 28′ | 0,20617 |
| | Co-ar. ſin. co-alt. | = 43 40 | 0.16086 |
| | Sin. ½ ſum co-decl. & d | = 35 52 | 9,76782 |
| | Sin. ½ diff. co-decl. & d | = 30 40 | 9,70761 |
| | The ſum of the four logs. | | 19,84246 |
| | The ½ ſum gives 56° 31½′ | | 9,92123 |

Which doubled, gives 113° 03′ for the azimuth ſought, reckoning from the north.

*To find the hour from noon,* ∠ ⊙pz.

Here p⊙ = 66° 32′
pz = 38 28
p⊙ − pz = 28 4 = d
⊙z = 43 40
71 44 | 35° 52′
2)
15 36 | 7 48

| | | | |
|---|---|---|---|
| Then | Co-ar. ſin. co-decl. | = 66° 32′ | 0,03749 |
| | Co-ar. ſin. co-lat. | = 38 28 | 0,20617 |
| | Sin. ½ ſum co-alt. & d | = 35 52 | 9,76782 |
| | Sin. ½ diff. co-alt. & d | = 7 48 | 9,13263 |
| | The ſum of the four logs. | | 19,14411 |
| | The ½ ſum gives 21° 55′ | | 9,57206 |

This doubled, gives 43° 50′ for the meaſure of the hour from noon, which is 2 h. 55 m. 20 s.

Hence it appears, that the obſervation was made either at 9 h. 4 m. 40 s. in the morning, or at 2 h. 55 m. 20 s. in the afternoon.

The azimuth being firſt found, the hour from noon might have been found by the proportion between oppoſite ſides and angles.

Had the declination and latitude been of contrary names, the ſame kind of operation would have been uſed to find the things required, only the ſide ⊙ P would have been obtuſe; by adding the declinat. to 90°, inſtead of ſubtracting it, as in the caſe of the lat. and decl. having like names.

## 153. PROBLEM XII. Pl. V.

Given the latitude of the place, and the Sun's declination;
Required the time when the twilight begins and ends.

Exam. *At what time does the twilight begin and end at London, when the Sun's declination is* 15° 12′ *N. the latitude of the place being* 51° 32′ *N.*

### CONSTRUCTION.

Let the circle ZRNH repreſent the meridian of the place, HR the horizon, ZN the prime vertical, and *ts* the Crepuſculum, or ſmall circle parallel to the horizon deſcribed at 18 degrees below it (IV. 68); lay off the latitude RP, draw the axis PS, the equator EQ, and deſcribe the parallel of declination *n m*, and where *n m* cuts *t s* in ⊙, is the Sun's place at the time of the beginning or end of the twilight; through ⊙ deſcribe (II. 72) the vertical circle Z⊙N, and the hour circle P⊙S; then the ∠ ZP⊙ being meaſured (IV. 72) will give the time before or after noon as required.

### COMPUTATION.

In the oblique-angled ſpheric triangle Z⊙P.

Given the co-lat. ZP = 38° 28′
the polar diſt. P⊙ = 74 48
the zenith diſt. Z⊙ = 108 00

Req. the hour from noon = ∠ ZP⊙. The manner of ſolution is the ſame as in laſt Problem.

Here P⊙ = 74° 48′
PZ = 38 28
P⊙ — PZ = 36 20 = D
⊙Z = 108 00
144 20 | 72° 10′
2) 71 40 | 35 50

| | | |
|---|---|---|
| Then Co-ar. ſin. polar diſt. | = 74° 48′ | 0,01547 |
| Co-ar. ſin. co-latit. | = 38 28 | 0,20617 |
| Sine ½ ſum. of zen. d. & D | = 72 10 | 9,97861 |
| Sine ½ diff. of zen. d. & D | = 35 50 | 9,76747 |
| The ſum of theſe four logs. | | 19,96772 |
| The half ſum gives 74° 28½′ | | 9,98386 |

Which doubled, gives 148° 57′ for ∠ ZP⊙.

And 148° 57′ reduced to time gives 9 h. 55 m. 48 s. either before or after noon; that is, the twilight begins at 2 h. 04 m. 12 s. in the morning, and ends at 9 h. 55 m. 48 s. in the evening on the given day, at London.

154. When the declination becomes greater than the difference between the co-latitude and 18 degrees, then the parallel of declination *n m* will not cut the parallel *t s* 18 degrees below the horizon, and conſequently at that time there will be no night at that place, but the twilight will continue from Sun-ſetting to Sun-riſing; and on this account it is, that from the 22d of May to the 21ſt of July nearly, there is no total darkneſs at London, the Sun's declination during that interval being greater than 20° 28′, which is the difference between 18° and 38° 28′, the complement of the latitude.

155. PRO-

## 155. PROBLEM XIII. Pl. V.

Given the time of the year, the latitude of a place, and the altitude of a known fixed ſtar;
Required the hour of the night when the obſervation was made.

EXAM. *Some time in the night, on the 1ſt of September* 1780, *ſuppoſe the ſtar Arcturus, the declination of which is* 20° 30′ *N. ſhould be obſerved at London to be* 27° 12′ *above the horizon: At what hour would the obſervation be made?*

### CONSTRUCTION.

Deſcribe the meridian of the place, draw the horizon HR, the zenith and nadir of which are Z and N, and deſcribe the parallel of altitude *rs* at 27° 12′ above the horizon; take P the north pole 51° 32′ above the horizon for the latitude of the place, and S the ſouth pole as much below the horizon; draw the equator EQ, and deſcribe (3d 135) the ſtar's parallel of declination *n m*; and where this parallel *n m* cuts the former *r s* in ✳, is the poſition of the ſtar at the time of obſervation; deſcribe (II. 72) the vertical circle Z ✳ N, and the hour circle P ✳ S, and the angle ZP ✳ being meaſured (IV. 72) gives the hour from, or to, the time of the ſtar's culminating.

### COMPUTATION.

In the oblique-angled ſpheric triangle P ✳ Z.

| | | | |
|---|---|---|---|
| Given the co-latitude PZ | = 38° | 28′ | Required ∠ ZP ✳, or the hour from culminating. |
| the co-altitude Z ✳ | = 62 | 48 | |
| the polar diſt. ✳ P | = 69 | 30 | |

Here P✳ = 69° 30′
PZ = 38 28
P✳ — PZ = 31 2 = D
P✳ = 62 48
93 50 | 46° 55
2)
31 46 | 15 53

| | | | |
|---|---|---|---|
| Then Co. ar. ſin. co-lat. | = 38° | 28′ | 0,20617 |
| Co. ar. ſin. pol. diſt. | = 69 | 30 | 0,02841 |
| Sin. ½ ſum zen. diſt. & D | = 46 | 55 | 9,86354 |
| Sin. ½ diff. zen. diſt. & D | = 15 | 53 | 9,43724 |
| The ſum of the four logs. | | | 19,53536 |
| The ½ ſum gives 35° 51′ | | | 9,76768 |

Which doubled, gives 71° 42′ = ∠ ZP✳.

This 71° 42′ turned into time (132) gives 4 h. 46 m. 48 s. for the time which has elapſed ſince the ſtar was on the meridian.

Now, at the time of obſervation, September 1ſt, 1780. (133)

| | | | | |
|---|---|---|---|---|
| The right aſcenſion of Arcturus was | $14^h$ | 5′ | 42″ | |
| The right aſcenſion of the ſun at noon † | 10 | 44 | 34 | |
| Time of culminating nearly | 3 | 21 | 08 | |
| And $24^h$ is to 3′ 37′ as $3^h$ 21′¼ is to | | | 30 | |
| The ſtar ſouths, or culminates at | 3 | 20 | 38 | |
| The time that the ſtar has paſſed the meridian | 4 | 46 | 48 | |
| The ſum is the hour of the night | 8 | 07 | 26 | P. M. |

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† Aſtronomical tables at the end of Book V.

And whether to ſubtract or add will always be known by the ſtar's being in the eaſtern part of the horizon, or aſcending; or by being in the weſtern part of the horizon, or deſcending.

## 156. PROBLEM XIV. Pl. V.

Given the obliquity of the ecliptic, and a ſtar's right aſcenſion and declination;

Required its latitude and longitude.

Exam. *What is the latitude and longitude of a ſtar, its right aſcenſion being* 16 *h.* 14 *m., its declination* 25° 51′ *N., and the obliquity of the ecliptic* 23° 28′ *?*

### CONSTRUCTION.

Let the primitive circle repreſent the ſolſtitial colure, in which draw the equator EQ, mark its poles P, S, and deſcribe (3d 135) the parallel of the ſtar's declination $n\,m$.

The right aſcenſion 16 h. 14 m. = 243° 30′, which being 63° 30′ above 180°, falls in the third quadrant; therefore make (IV. 75.) ♈$a$ = 63° 30′, deſcribe (4th 135) the circle of right aſcenſion, cutting the parallel $n\,m$ in ✱, the point of the heavens repreſenting the ſtar.

Make E♋ = 23° 28′, the obliquity of the ecliptic, draw the ecliptic ♋ ♑, find its poles $p$, $q$, and through $p$, ✱, $q$ deſcribe a circle of longitude; then the arc $p$ ✱ meaſured (IV. 70) will give the co-latitude, and the ∠ P$p$ ✱ will ſhew the longitude.

### COMPUTATION.

In the oblique-angled ſpheric triangle $p$ P ✱.

Given the obliq. ecliptic $p$P = 23° 28′
the co-declination P✱ = 64 09
the right aſcen. ∠ $p$P✱ = 243 30

Required the co-lat. $p$ ✱.
and the longit. ∠ P$p$ ✱.
See art. 150, 151. B. IV.

*To find the latitude.*

| | | | | | |
|---|---|---|---|---|---|
| As Radius | = R | 10,00000 | As co-ſ. 4th arc | = 61° 34′ | 0,32227 |
| To co-ſ. rt. aſc. | = 26° 30′ | 9,95179 | To co-ſ. 5th arc | = 38 06 | 9,89594 |
| So tan. co-decl. | = 64 09 | 10,31471 | So ſine decl. | = 25 51 | 9,63950 |
| To tan. 4th arc | = 61 34 | 10,26650 | To ſin. lat. | = 46 06½ | 9 85771 |
| Obl. eclipt. | = 23 28 | | | | |
| Fifth arc | = 38 06 | | | | |

Here the ſtar's latitude is 46° 06½′ N. becauſe the declinat. is N., and greater than the obliquity of the ecliptic.

*To find the longitude.*

| | | |
|---|---|---|
| As ſine 5th arc | = 38° 06′ | 0 20969 |
| To ſine 4th arc | = 61 34 | 9,94417 |
| So tan. rt. aſc. | = 26 30 | 9,69774 |
| To tan. longit. | = 234 36 | 9,85160 |

Here the longitude 144° 36′ being added to 90°, gives 234° 36′ for the ſtar's longitude, reckoning from the firſt point of Aries.

For the right aſcenſion being in the third quadrant, the ſtar is there alſo. Now in 234° 36′, are 7 ſigns 24° 36′; that is the ſtar's place, or longitude is 24° 36′ in the 8th ſign, or 24° 36′ in ♏.

By the preceſſion of the equinoxes, the fixed ſtars, although they always keep the ſame latitudes, yet are continually altering their longitude, right aſcenſion, and declination; the alteration in longitude is uniformly 50 ſeconds and 3-10ths yearly (22), but that of the right aſcenſion and declination is conſtantly varying: So that many ſtars, which once had north declination, come to have ſouth; while others change from S. to N. declination.

### 157. PROBLEM XV. Pl. V.

Given the right aſcenſions and declinations of two fixed ſtars;
Required their diſtance.

Exam. *What is the diſtance between the fixed ſtars, Betelgueſe in the eaſt ſhoulder of Orion, and Aldebaran in Taurus; the former having* 7° 21′ *N. declinat. with* 5 *h.* 43 *m.* 16 *s. right aſcenſion; and the other* 16° 03′ *N. declinat. with* 4 *h.* 23 *m.* 20 *s. of right aſcenſion.*

#### Construction.

As Aldebaran precedes Betelgueſe in right aſcenſion, let the primitive circle repreſent the circle of right aſcenſion paſſing through Betelgueſe; deſcribe the circle of right aſcenſion PAS, making with PBS an angle of 19° 59′, (IV. 75) equal to the difference between the given right aſcenſions.

Deſcribe the parallels of declination B*m*, *rs*, at the given diſtances 16° 03′ N. and 7° 21′ N. (3d 133); and the interſections A, of Aldebaran's declination, and B, of Betelgueſe's, with their reſpective circles of right aſcenſion, will be the poſitions of thoſe ſtars from one another: Then draw a great circle BAC, through B and A, and the intercepted arc BA (meaſured by art. 70. Book IV.) ſhews the diſtance of thoſe two ſtars.

#### Computation. (IV. 151)

In the oblique-angled ſpheric triangle PAB.
Given the co-decl. of Ald. PB=73° 57′
the co-dec. of Betelgueſe PA=82 39
diff. of right aſcen. ∠APB=19 39
} Required their diſt. AB.

| | | |
|---|---|---|
| Radius | =R | 10,00000 |
| To co-ſ. diff rt. aſ. | 19° 59 | 9,97303 |
| As co-t. Betelg. dec. | 7 21 | 10,88944 |
| To tang. 4th arc | 82 11 | 10,86247 |
| Aldeb. co-dec. | 73 57 | |
| Remains 5th arc. | 8 14 | |

| | | |
|---|---|---|
| Co-ſ. 4th arc | 82° 11′ | 0,86645 |
| To co-ſ. 5th arc | 8 14 | 9,99550 |
| As ſin. Betelg. dec. | 7 21 | 9,10697 |
| To co-ſ. diſt. | 21 25 | 9,96892 |

The ſame reſult would have come out, had the declination of Aldebaran been uſed in the proportions.

### 158. PROBLEM XVI. Pl. V.

Given the latitudes and longitudes of two known fixed ſtars;
Required their diſtance.

Exam. *Aliath, in the Great Bear,* *Lon.* ♍ 5° 49′ *Lat.* 54° 18′ *N.*
*Arcturus, in Bootes,* *Lon.* ♎ 21 10 *Lat.* 30 54 *N.*

The conſtruction of this Problem is like that of the laſt; only inſtead of circles of right aſcenſion read circles of longitude, and uſe parallels of latitude inſtead of parallels of declination.

The computation is alſo like that of the laſt, there being given two co-latitudes and the included angle, which is the difference between the given longitudes: Thus Arcturus's long. is $6^s$ 21° 10′, and Aliath's is $5^s$ 5° 49′; their difference is $1^s$ 15° 21′, or 45° 21′.

Hence the diſtance will be found to be 39° 45′.

## 159. PROBLEM XVII. Pl. V.

Given the latitude and longitude of a fixed ſtar, and alſo the obliquity of the ecliptic;

Required the right aſcenſion and declination of that ſtar.

EXAM. *Suppoſe the latitude of a ſtar is 7° 09′ N. its longitude ♈ 29° 01′: What is the right aſcenſion and declination of that ſtar, the obliquity of the ecliptic being 23° 28′?*

The conſtruction of this Problem is much like that of Prob. XIV.; only here the interſection of a parallel of latitude *cb* with a circle of longitude *p*A*q*, will give the place of the ſtar.

The computation is alſo as in Prob. XIV; for here are given P*p* = 23° 28′, *p*A = 82° 51′, and the ∠P*p*A = 60° 59′, the longitude from the firſt point of ♋, to find PA the co-declination, and ∠*p*PA the right aſcenſion.

The declination of the ſtar will be found to be 17° 49′ N. (IV. 151)

And the right aſcenſion will be 24° 19′. (IV. 150)

## 160. PROBLEM XVIII. Pl. V.

Given the meridional altitude of any celeſtial object, ſuppoſe a comet, its diſtance from a known ſtar, and the latitude of the place;

Required the declination and right aſcenſion of that comet.

EXAM. *Suppoſe a comet was obſerved on the meridian at London, when its altitude was 51° 55′, and its diſtance from the ſtar Arcturus was 59° 47′: What was the declination and right aſcenſion of the comet at that time?*

### CONSTRUCTION.

In the primitive circle, repreſenting the meridian of the place, draw the horizon HR and prime vertical ZN; lay off the given latitude RP = 51° 32′, draw the axis PS, the equator EQ, and (3d 135) *n m* Arcturus's parallel of declination = 20° 21′. From the ſouth point of the horizon lay off the given altitude of the comet = 51° 55′ from H to O: About the point O as a pole, at the given diſtance between the comet and Arcturus, deſcribe (IV. 68) a ſmall circle *a a* cutting the parallel *n m* in ✳, the poſition of Arcturus at that time: Deſcribe the circle of right aſcenſion P ✳ S, and a great circle through O and ✳.

### COMPUTATION.

Since HE = 38° 28′, the co-lat. and the alt. HO = 51° 55′, then EO = (HO − HE =) 13° 27′, is the decl. ſought; which is north, as the altitude exceeds the co-lat.; conſequently the polar diſtance OP = 76° 33′.

Then in the triangle P✱O are given the three ſides to find the ∠OP ✱, the difference between the right aſcenſions of the comet and Arcturus. And (IV. 154) the ∠OP ✱ will be found = 62° 24′ = 4h. 9 m. 36 s. which is the difference of their right aſcenſions: Now if Arcturus had paſſed the meridian, the right aſcenſion of the comet was 18 h. 15 m. 16 s. but if Arcturus had not paſſed the meridian, the right aſcenſion of the comet was 9 h. 56 m. 4 s.; it being, in the former caſe, equal to the ſum, and in the latter to the diff. of Arcturus's right aſcen. and the ∠OP ✱.

### 161. PROBLEM XIX. Pl. V.

Given the latitude of a place, the Sun's declination and Azimuth;
Required his altitude and the time of the obſervation.

EXAM. *In the latitude of* 13° 30′ *N. and when the Sun has* 23° 28′ *N. declination: What is the Sun's altitude and time of the day, when he is ſeen on the ENE azimuth circle?*

### CONSTRUCTION.

Let the primitive circle repreſent the meridian of the place, in which HR repreſents the horizon, and ZN the prime vertical; make RP equal to the latitude, draw the 6 o'clock hour circle PS, the equator EQ, and (3d 153) the parallel of 23° 28′ of declination *n m*: The tangent of 67° 30′ being laid from the center *a* towards H, gives the center of the vertical circle ZDN, which cuts the parallel *n m* in the points A and B; and ſhews that at two diſtant times in the forenoon the Sun will have the azimuth propoſed: Through the point A and B deſcribe the hour circles PAS, PBS (II. 72); the angles ZPA, ZPB, ſhewing the times from noon, may be meaſured by art. 72. Book IV.; and the altitudes DA, DB, by art. 70. Book IV.

### COMPUTATION. See art. 146. Book IV.

In the ſpheric triangle PZA, or PZB, there are known,

The co-lat. PZ = 76° 30′; the co-decl. PA, or PB = 66° 32′; the azim. ∠PZD = 67° 30′.

To find ZA, or ZB, and the ∠ZPA, or ∠ZPB.

As Radius : co-ſ. azimuth :: co-t. lat. : to tan. of a 4th arc M = 57° 54′.

And as ſin. lat. : ſin. decl. :: co-ſ. M to co-ſ. of a 5th arc N = 24° 56′.

Then M + N, or 57° 54′ + 24° 56′ = 82° 50′ = ZA, is the comp. of leaſt alt.

And M − N, or 57° 54′ − 24° 56′ = 32° 58′ = ZB, is the comp. of the gr. alt.

Therefore, when the Sun has 7° 10′, or 57° 02′, of alt. he is on the given azimuth.

Again, As co-ſ. dec. : ſin. azim. :: co-ſ. leaſt alt. : ſin. hour fr. noon 87° 55′.

And as co-ſ dec. : ſin. azim. :: co-ſ. greater alt. : ſin. h. fr. noon 33° 14′.

But 87° 55′ = $5^h\ 51^m\ 40^s$; and 33° 14′ = $2^h\ 12^m\ 56^s$, the reſpect. times fr. noon.

Conſequently the Sun will be ſeen on the ENE azimuth at 6h. 08m. 20 s. and again at 9 h. 47 m. 4 s. both in the morning: Alſo, in the afternoon he will be on the WNW azimuth at 2 h. 12 m. 56 s. and at 5 h. 51 m 40 s.

162. Now to find at what time, and at what altitude, the greateſt azimuth will happen at that place on the ſaid day;

As the azimuth circle in this caſe is to touch the parallel *n m*, therefore the greateſt diſtance of the azimuth from the equator will be 23° 28′; and as their poles muſt lie at the ſame diſtance (IV. 33) therefore a ſmall circle *n*, deſcribed about the poles, at the diſtance of 23° 28′ (IV. 66), its interſection *p* with the horizon, is the pole of the circle ZCN; then deſcribe an hour circle PCS through *p*.

In the ſpheric triangle ZPC right angled at C. (IV. 34)

Given the co-lat. PZ = 76° 30′, and the co-decl. PC = 66° 32′.

Required the greateſt azim. ∠PZC = 70° 37′, the diſt. ZC = 54° 08′, and the hour from noon = 56° 26′ = 3 h. 45 m. 44 s.

So that the azimuth is altered only 3° 7′ in 2 h. 6 m.; and conſequently the variation of the compaſs may be obſerved with more certainty in the torrid zone than elſewhere.

## 163. PROBLEM XX. Pl. XIV. Fig. 1.

*In the latitude of* 20° 00′ *N. ſtands a horizontal dial, the gnomon of which is perpendicular to the plane of the horizon: It is required to know at what hour in the afternoon on the longeſt day, the ſhadow of that gnomon ſhall ſtand ſtill; and how many degrees ſhall the ſhadow run back.*

CONSTRUCTION. Let the circle ZRNH be the meridian of the place, HR the horizon, the poles of which are Z, N; RP = 20°, the latitude P and S the north and ſouth poles: About P deſcribe the tropic of Cancer *aa*, cutting the horizon in L; about S, a ſmall circle being deſcribed at the diſt. of 23° 28′, the complement of the diſt. of *aa* from P, its interſection *p* with the horizon, is the pole of the azimuth circle which will touch the parallel *aa* in ⊙, the place of the Sun when he has the greateſt azimuth that day: Through Z, ⊙, N, deſcribe a vertical circle cutting the horizon in K; and through ⊙ and L deſcribe the hour circles P⊙S, PLS.

Then will the ∠ZP⊙ be equivalent to the hour when the ſhadow will ſtand ſtill; and KL, the difference between the meaſures of the azimuth and amplitude, will ſhew how much the ſhadow will run back.

COMPUTATION. In the right-angled ſpheric triangle PRL.

| | | | |
|---|---|---|---|
| Given | ∠R | = | 90° 00′ |
| lat. | = RP | = | 20 00 |
| co-decl. | = PL | = | 66 32 |
| Requir. ampl. | = RL | = | 64 56 |

In rt. angl. ſpheric triangle P⊙Z.

| | | | |
|---|---|---|---|
| Given | ∠⊙ | = | 90° 00′ |
| co-lat. | = ZP | = | 70 00 |
| co-dec. | = P⊙ | = | 66 32 |
| Required hour | = ∠ZP⊙ | = | 33 06 |
| azim. | = ∠⊙ZP | = | 77 28 |

Then 33° 02 = 2 h. 12 m. 08 s.: And 77° 28′ — 64° 56′ = 12° 32′.

So that the ſhadow will ſtand ſtill at 2 h. 12 m. 08 s. and will run back 12° 32′.

## 164. PROBLEM XXI. Pl. XIV. Fig. 2.

*A comet, the declination of which was* 47° 00′ *N. was obſerved to be diſtant from a ſtar, to the eaſtward of it,* 49° 00′; *the ſtar's declination was* 36° 00′ *N. and its right aſcenſion* 45° 00′: *What was the latitude and longitude of that comet?*

### CONSTRUCTION.

On the plane of the ſolſtitial colure, where P and E are the poles of the equator and ecliptic, put the ſtar at A by its right aſcenſion and declination: About P and A deſcribe ſmall circles, at the diſtances of the comet from thoſe points, their interſection ⊙ gives the place of the comet; deſcribe great circles through A⊙, P⊙, E⊙, then E⊙ will be the co-latitude, and ∠PE⊙ the co-longitude; and their meaſures may be obtained from articles 70 and 72 of Book IV.

COMPUTATION. In the triangle AP⊙.
Given the ✳'s co-dec. PA=54° 00′
comet's co-dec. P⊙=43 00
their diſtance A⊙=49 00

Req.com.rt.aſ.fr.✳ ∠AP⊙=65 48
Then 65° 48′—45° 00=20° 48′.
And 90° 00′—20° 48′=69° 12′=⊙PE

In the triangle EP⊙.
Given comet's co-decl. P⊙=43° 00′
obliq. of ecliptic PE=23 28
comet's rt. aſ. ∠EP⊙=69 12

Req. comet's co-lat. E⊙=39 54
and co-longit. ∠PE⊙=83 42
Which being taken from 90°, leaves ♈ 6° 18′ for the long. required.

## 165. PROBLEM XXII. Pl. XIV. Fig. 3.

*At London, on the* 10*th of December* 1780, *at what time of the night will the ſtars Aldevaran and Rigel be on the ſame azimuth circle?*

Aldeb. decl.=16° 03′ N.; right aſc.=4 h. 23 m. 19 s. Rigel's decl. =8° 28′ S.; right aſc.=5 h. 03 m. 58 s.

Their difference of right aſcenſions is 40 m. 39 s. or 10° 10′.

CONSTRUCTION. On the plane of the equinoctial put Aldebaran at A, and Rigel at B, by their right aſcenſions and declinations, then a great circle through B and A will be the azimuth they are on at the time ſought; and the parallel of London's lat. deſcribed about P, will cut the azimuth circle BA in ZZ the zenith, through which draw the meridians PZ, PZ.

Here the neareſt interſection to the ſtars is taken for the zenith, for as the ſtars are both above the horizon, the greateſt zenith diſtance is leſs than 90 degrees.

COMPUTATION. In the △ APB.
Given B's co-decl. PB= 98° 28′
A's co-decl. PA= 73 57
diff. rt. aſc. ∠APB= 10 10

Required the ∠BAP=156 58

In the triangle APZ.
Given A's co-decl. PA=73° 57′
the co-lat. PZ=38 28
the ſuppl. of BAP or ∠PAZ=23 02

Req. ∠APZ = 142° 35′, or=24 01

Now the ſtar Aldebaran comes to the meridian at 11 h. 8 m. 27 s. in the evening; which leſſened by 1 h. 36 m. 4 s. (24° 01′) gives 9 h. 32 m. 23 s. for the time in the evening when thoſe ſtars will be on the ſame azimuth.

## 166. PROBLEM XXIII. Pl. XIV. Fig. 4.

*At what time in the evening will the ſtars Betelgueſe and Pollux have one common altitude above the horizon of London, on the* 10*th of December* 1780?

Betelgueſe right aſcen.=5 h. 43 m. 16 s.; decl.=7° 21′ N. Pollux right aſcen.=7 h. 31 m. 51 s.; decl.=28° 32′ N.

Their difference of right aſcenſion is 1 h. 48 m. 35 s.=27° 09′.

CONSTRUCTION. On a primitive circle, where any point P repreſents the pole of the equinoctial, put the ſtar Pollux at B, and Betelgueſe at A, by their declination and difference of right aſcenſions; through A, B, deſcribe a great circle; through C, the middle of AB, deſcribe a great circle at right angles to AB, and cutting PA in D; then a ſmall circle deſcribed about P, at the diſtance of 38° 28′, the co-lat. will cut the circle CD in Z the zenith.

For the ſtars A and B having a common altitude, are equally diſtant from Z.

Computation. In the triangle APB.

| | | | |
|---|---|---|---|
| Given A's co-decl. | PA = 82° 39′ | Required the | ∠BAP = 47° 01′ |
| B's co-decl. | PB = 61 28 | | AB = 33 14 |
| diff. rt. aſc. | ∠ APB = 27 09 | Its half | AC = 16 37 |

In the triangle ACD.

| | | | |
|---|---|---|---|
| Given the | ∠C = 90° 00′ | Required the | ∠D = 45° 30′ |
| | ∠A = 47 01 | | AD = 23 38 |
| | AC = 16 37 | Theref. PA—AD = PD | = 59 01 |

In the triangle PDZ.

| | | | |
|---|---|---|---|
| Given the co-lat. | PZ = 38° 28′ | Required | ∠ZPD = 48° 56 |
| | PD = 59 01 | Or it is | = 75° 46 |
| | ∠PDZ = 45 30 | | |

Now the ſtar Betelgueſe comes to the meridian at 12h. 28m. 8s. that is between twelve and one o'clock in the morning (133); from which take 3 h. 15 m. 44 s. as the ſtars are to the eaſt of the meridian, and it leaves 9 h. 12 m. 24 s. in the evening, for the time when thoſe ſtars have the ſame altitude.

### 167. PROBLEM XXIV. Pl. XIV. Fig. 5.

*Wanting to know the latitude and longitude of a comet* C, *its diſtance from two known ſtars* A *and* B *were obſerved, and are as follows:*

A*'s lat.* = 49° 12′ *N. Lon.* = 16° 39′ ♓; *diſtance from* C = 49° 05′.
B*'s lat.* = 30 05 *N. Lon.* = 2 48 ♊; *diſtance from* C = 45 57.
*Hence the place of the comet* C *is required.*

Construction. On the plane of the ſolſtitial coloure, where E is the pole of the ecliptic, put the ſtars A and B by their latitudes and longitudes, and deſcribe a great circle through A and B; then ſmall circles deſcribed about A and B as poles, at the reſpective diſtances of the comet, their interſection C will give its place; deſcribe great circles through A, C; B, C; E, C; and E C will be the co-latitude, and from ∠ AEC will be obtained the longitude.

| | | | |
|---|---|---|---|
| Computation. In the △ ABE. | | In the triangle ABC. | |
| Given A's co-lat. | AE = 40° 48′ | Given the diſtance | AB = 59° 01′ |
| B's co-lat. | BE = 59 55 | diſtance | AC = 49 05 |
| diff. longit. | ∠ AEB = 76 09 | diſtance | BC = 45 57 |
| Required | AB = 59 01 | Required the | ∠BAC = 56 27 |
| and | ∠EAB = 78 31 | Then ∠EAB + ∠BAC = ∠EAC | = 134° 58 |

In the triangle CEA.

| | | | |
|---|---|---|---|
| Given A's co-lat. | AE = 40° 48′ | Required the co-lat. | EC = 81° 33′ |
| diſtance | AC = 49 05 | diff. long. | AEC = 32 43 |
| and the | ∠EAC = 134 58 | Hence lat. is 8° 27′ N. lon. 19° 22′ in ♈ | |

### 168. PROBLEM XXV. Pl. XIV. Fig. 6.

*The diſtance of the ſtar* C *being obſerved from two ſtars* A *and* B, *the latitude and diſtance of which are known, and alſo the longitude of one of them; thence to find the lat. and long. of* C.

Suppoſe A's lat. to be 5° 30′ N.; its diſt. from C = 39° 40′: B's lat. 9° 57′ N. its long. Taurus, 18° 16′, and diſt. from C 10° $7\frac{1}{2}$′: And the diſtance of AB 44° 43′: Required the long. and lat. of C.

CONSTRUCTION. On a circle of longitude, where E is the pole of the ecliptic, put the ſtar B by its lat.; about the points B and E deſcribe circles at the diſtances of A from thoſe points, their interſection gives the place of A: alſo circles deſcribed from A and B, at the diſtances of C reſpectively from them, their interſection is the place of C: Then deſcribe the great circles EA, EC; AC, AB; BC; and EC will be the co-lat. and ∠BEC the longitude, of C from B.

COMPUTATION. In the Δ AEB
Given A's co-lat. - AE = 84° 30′
B's co-lat. BE = 80 03
the diſtance AB = 44 43
Required the ∠ABE = 92 14

In the triangle ABC.
Given the diſtance AB = 44° 43′
diſtance AC = 39 40
diſtance BC = 10 07½
Required the ∠ABC = 55 22

In the triangle BEC.
Given B's co-lat. BE = 80° 03′
the diſtance BC = 10 07½
(∠ABE — ∠ABC =) ∠CBE = 36 52
Required C's co-lat. CE = 72° 01′
C's lon. fr. B, ∠BEC = 6 22
And its abſolute long. ♉ 11 54

## 169. PROBLEM XXVI. Pl. XIV. Fig. 7.

*From the altitudes of two known fixed ſtars, and the altitude of a planet when in the ſame azimuth with one of theſe ſtars; to find the place of the planet.*

EXAMPLE. Obſerved the Moon and Cor Leonis in the ſame azimuth, when the Moon's zenith diſtance was 36° 37′.

Cor Leonis's zen. diſt. = 45° 00′; decl. 13° 02′ N.; rt. aſ. $9^h\ 56^m\ 39^s$.
Cor Hydra's zen. diſt. = 49 16; decl. 7 43 S.; rt. aſ. $9^h\ 16^m\ 47^s$.

CONSTRUCTION. On the plane of the equinoctial, the pole of which is P, draw the colures, and in the ſolſtitial, take E for the pole of the ecliptic; put the given ſtars at B and A by their declinations and right aſcenſions: About B and A as poles, with their reſpective zenith diſtances, deſcribe circles cutting in Z the zenith; through Z and B deſcribe an azimuth circle, and making Z ☽ equal to the ☽'s zenith diſtance, it gives her place: Then deſcribe the great circles ZA, AB, E☽; and the arc E☽ will be the co-latitude, and ∠PE☽ the longitude from the firſt point of ♋.

COMPUTATION. In the Δ ABP.
Given A's co-decl. PA = 97° 43′
B's co decl. PB = 76 58
diff. rt. aſ. ∠BPA = 9 58
Required the ∠ABP = 153 57½
the ſide AB = 22 59½

In the triangle AZB.
Given A's zen. diſt. ZA = 49° 16′
B's zen. diſt. ZB = 45 00
the ſide AB = 22 59½
Required the ∠ABZ = 89 38½
Then ∠ ABP — ABZ = ∠ ZBP = 64° 19′ = ∠ ☽BP.

In the triangle B ☽ P.
Given B's co-decl. BP = 76° 58′
(BZ — Z ☽ =) ſide B ☽ = 8 23
∠PB☽ = 64 19
Requ. ☽'s co-decl. P☽ = 73 26½
☽'s rt. aſ. fr B, ∠BP ☽ = 7 53
Then 90° + ∠ ♎ PB + ∠ BP ☽ = ∠EP☽ = 128° 43′.

In the triangle P ☽ E.
Given obl. of eclip. PE = 23° 28′
☽'s co-decl. P☽ = 73 26½
☽'s co-rt. aſ. ∠EP☽ = 128 43
Requir. ☽'s co-lat. ☽ E = 88 41
☽'s lon. fr. ♋, ∠PE☽ = 48 25
And its abſolute long. is ♌ 18 25

## SECTION VI.

## *Of various methods to find the Latitude.*

The uſual way at ſea to find the latitude is from the Sun's meridional altitude and declination; the manner of doing this will be particularly ſhewn in Book IX. But as it frequently happens at ſea, that the meridian altitude cannot be taken, therefore the mariner ſhould be furniſhed with other means to come at the knowledge of this moſt uſeful article. To help him in this point, and as a farther exerciſe in the Aſtronomy of the Sphere, the following problems are collected together.

### 170. PROBLEM XXVII. Pl. V.

Given the Sun's declination and his amplitude;
Required the latitude of the place.

EXAM. *Being in a place where the compaſs had no variation, on a day when the Sun's declination was* 15° 12′ *N., I obſerved him to riſe* 62° 30′ *from the north towards the eaſt: Required the latitude of that place.*

CONSTRUCTION.

Having deſcribed the primitive circle, drawn the horizon HR, and (IV. 71) taken RO = 62° 30′; then about O as a pole deſcribe (IV. 66) a ſmall circle, at the diſtance of 74° 48′ = co-decl., cutting the primitive in P, the place of the north pole: Draw the axis PS, the equator EQ, and the circle POS, cutting the equator in A.

COMPUTATION.

In the ſpheric triangle ♈OA right angled at A.

| | |
|---|---|
| Given the co-amp. ♈O = 27° 30′ | Then ſ. ♈O : rad. : : ſ. AO : ſ. ∠A♈O. |
| the declin. AO = 15 12 | Hence the latitude will be 55° |
| Required the co-latitude A♈O. | 24′ N. |

### 171. PROBLEM XXVIII. Pl. V.

Given the Sun's declination, and his aſcenſional difference;
Required the latitude of the place.

EXAM. *When the Sun had* 20° 01′ *of declination S., he was obſerved to ſet at* 4 *h.* 30 *m.: Required the latitude of the place.*

As the aſcenſional difference is the time that the Sun riſes or ſets before or after 6 o'clock; therefore 6 h.—4 h. 30 m. = 1 h. 30 m. = 22° 30′ = aſcenſional difference.

CONSTRUCTION. In the primitive circle repreſenting the meridian of the place, draw the equator EQ, the axis PS, the parallel of declination *ro*, 20° 01′ S.: Make ♈B = 22° 30′, the aſcenſional difference; deſcribe the circle of right aſcenſion PBS, cutting *ro* in O; then a diameter HR through O will be the horizon, and RP the lat. ſought.

COMPUTATION. In the ſpheric triangle ♈BO, right angled at B.

| | |
|---|---|
| Given the aſc. diff. ♈B = 22° 30′ | Then rad. : cot. OB : : ſ. ♈B : cot. ∠O♈B. |
| the declin. OB = 20 01 | Or rad. : cot. dec. : : ſin. aſ. diff. : tan. lat. |
| Required the co-lat. ∠O♈B. | Hence the lat. will be 46° 25′ N., |

being contrary to the decl. when the aſc. diff. falls between noon and ſix.

172. PRO-

## 172. PROBLEM XXIX. Pl. V.

Given the Sun's declination, and altitude at ſix o'clock;
Required the latitude of the place.

Exam. *Being at ſea, on a day when the Sun's declination was 20° 04′ N. his altitude at ſix o'clock in the evening was 18° 45′: What was the latitude of the place of obſervation?*

Construction. Having deſcribed the meridian, drawn the horizon HR, the prime vertical ZN, and the parallel *st* of 18° 45′ of altitude; from the center ♈, with the half tangent of the declination = 20° 04′, cut the parallel *st* in O: Through O draw the axis PS, and the azimuth circle ZON (II. 72), and the meaſure of RP will give the latitude ſought.

Computation. In the ſpheric triangle ♈AO, right-angled at A.

| | |
|---|---|
| Given the decl. ♈O = 20° 04′ N. | Then ſin. ♈O : rad. :: ſ. AO : ſ. ∠O♈A: |
| the altit. AO = 18 45 | Or ſin. decl. : rad. : : ſin. alt. : ſin. lat. |
| Required the lat. = ∠O♈A | Which is 69° 32′ N., as the decl. is N. |

## 173. PROBLEM XXX. Pl. V.

Given the Sun's declination, and his altitude when due eaſt or weſt;
Required the latitude of the place.

Exam. *In a place where the compaſs had no variation, the Sun was obſerved to be due eaſt when his declination was 16° 38′ N., and his altitude 20° 12′: What is the latitude of that place?*

Construction. In the meridian HZRN, draw the horizon HR, the prime vertical ZN, and make ♈O = the half tan. of the alt. 20° 12′: About O as a pole, at the diſtance of 73° 22′, the co-decl., deſcribe (IV. 66) a ſmall circle, cutting the meridian in P the elevated pole; draw the axis PS, equator EQ, and through P, O, S, deſcribe an hour circle POS; then the meaſure of PR ſhews the latitude.

Computation. In the ſpheric triangle ♈AO, right-angled at A.

| | |
|---|---|
| Given the alt. ♈O = 20° 12′ | Then ſin. ♈O : rad. :: ſin. AO : ſin. ∠A♈O. |
| the decl. AO = 16 38 N. | Or ſin. alt. : rad. : : ſin. decl. : ſin. lat. |
| Required the latit. = ∠A♈O. | Which is 56° 00′ N., as the decl. is N. |

But had the declination been S., the other interſection of the parallel circle and meridian muſt have been taken for the elevated pole, and the latitude would be ſouth.

## 174. PROBLEM XXXI. Pl. V.

Given the Sun's altitude and the hour of the day on either equinox;
Required the latitude of the place.

Exam. *On the day the Sun entered the vernal equinox, his alt. was found 22° 56′ at 9 o'clock in the morning. In what lat. was that obſervation made?*

Construction. Deſcribe the meridian, draw the horizon, the prime vertical, and (IV. 68) the parallel *st* of 22° 56′ of altitude; from the center ♈, with the half tan. of 45° = 3 h., the time from 6 o'clock, cut *st* in O, and deſcribe the vertical circle ZON, cutting the horizon in B.

Computation. In the ſpheric triangle ♈BO, right-angled at B.

| | |
|---|---|
| Given the time after 6, ♈O = 15° 00′ | As ſ. ♈O : rad. :: ſ. BO : ſ. ∠B♈O. |
| the altitude BO = 22 56 | Or ſ. time 6 : rad. :: ſ. alt. : co-ſ. l |
| Required the co-latitude B♈O. | Which is 56° 34′. |

175. PRO-

## 175. PROBLEM XXXII. Pl. V.

Given the Sun's altitude, declination and azimuth;
Required the latitude of the place.

Exam. *Being at ſea in a place where the compaſs had no variation, in the afternoon when the Sun was* 42° 30′ *high, his bearing was S.* 57° 45′ *W. and his declination* 22° 30′ *N.: What is the latitude of that place?*

Construction. Draw the meridian, the horizon HR, the prime vertical ZN, and the parallel *s t* at 42° 30′ above the horizon (IV. 68): The tangent of 57° 45′ ſet from A towards R gives the center of the azimuth circle ZON, cutting the parallel of altitude *s t* in O: About O as a pole (IV. 66), at the diſtance of 67° 30′, equal to the co-declination, deſcribe a ſmall circle, cutting the meridian in P, the place of the pole; then the meaſure of RP gives the latitude ſought.

Computation. In the oblique-angled ſpheric triangle ZOP.

$$\left.\begin{array}{lrr}\text{Given the zenith diſt. ZO} = & 47^\circ & 30' \\ \text{the polar diſt. PO} = & 67 & 30 \\ \text{the azimuth} \angle \text{PZO} = & 122 & 15\end{array}\right\}\text{Required the co-latitude PZ.}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| As rad. | = | R | | 10,00000 | As ſin. alt. | = | 42° | 30′ | 0,17032 |
| To co-ſ. azim. | = | 122° | 15′ | 9,72723 | To ſin. decl. | = | 22 | 30 | 9,58284 |
| So co-t. alt. | = | 42 | 30 | 10,03795 | So co-ſ. 4th. | = | 30 | 13 | 9,93658 |
| To tan. 4th. | = | 30 | 13 | 9,76518 | To co-ſ. 5th. | = | 60 | 42 | 9,68974 |

Then the difference between the 5th and 4th arcs, that is 30° 13′ taken from 60° 42′, the remainder 30° 29′ is the co-lat. Therefore 59° 31′ N. is the latitude ſought.

## 176. PROBLEM XXXIII. Pl. V.

Given the Sun's declination, his altitude and the hour of the day;
Required the latitude of the place.

Exam. *Being at ſea, the Sun's altitude was obſerved to be* 37° 20′ *at* 9 *h.* 45 *m. in the morning, his declination at that time being* 22° 30′ *N.: What is the latitude of the place of obſervation?*

Construction. In the meridian PESQ, draw the equator EQ, axis PS, and parallel of declin. *n m*, 22° 30′ diſt. from the equator (3d 135). Set off from the center A towards Q the tang. of 33° 45′ = 2 h. 15 m., the diſtance between the time of obſervation and noon, which gives the center of the hour circle POS, cutting the parallel *n m* in O: about the point O as a pole, deſcribe (IV. 66) at the diſt. of 52° 40′, the zen. diſt., a ſmall circle, cutting the meridian in Z the zenith; through Z O deſcribe an azimuth circle ZON; then the meaſure of ZE will give the lat. ſought.

Computation. In the oblique-angled triangle ZOP.

$$\left.\begin{array}{llrr}\text{Given the zenith diſtance} & \text{ZO} & =52^\circ & 40' \\ \text{the polar diſtance} & \text{PO} & =67 & 30 \\ \text{the hour from noon} & \angle \text{ZPO} & =33 & 45\end{array}\right\}\text{Required the co-lat. ZP.}$$

As rad. : co-ſ. hour A. M. : : co-t. decl. : tan. 4th arc = 63° 31′.
As ſin. decl. : ſin. alt. : : co-ſ. 4th arc : co-ſ. 5th arc = 45 02.
Their difference is the co-latitude 18° 29′. Therefore the lat. is 71° 31′ N.

## 177. PROBLEM XXXIV. Pl. V.

Given the altitude of one of two known fixed ſtars, when they have the ſame azimuth;
Required the latitude of the place.

EXAM. *Being at ſea in an unknown latitude, I obſerved the ſtar Schedar in Caſſiopeia, and Almaach in Andromeda, to have the ſame azimuth, when the altitude of Schedar was 37° 15′: What is the latitude of that place?*

CONSTRUCTION. Let the primitive circle repreſent the equator, the pole of which is P, and any point ♈ the place where the right aſcenſion begins, from whence lay off ♈*a*=27° 37′ for Almaach's right aſcenſion, and ♈*b*=7° 2′ for Schedar's; draw the circles of right aſcenſion P*a*, P*b*: Deſcribe (3d 137) Almaach's and Schedar's parallels of declination, cutting P*a*, P*b*, in A, B; A being Almaach, and B Schedar. A great circle paſſing through A and B (IV. 61) will be the azimuth they are on. About B at the diſtance of 52° 45′, Schedar's zenith diſtance, deſcribe (IV. 66) a ſmall circle, cutting the ſaid azimuth circle in Z, the zenith of the place; draw PZ, which meaſured on the half tangents gives the co-latitude of the place of obſervation.

### COMPUTATION.

1ſt. In the oblique-angled ſpheric triangle ABP.

| | | |
|---|---|---|
| Given Almaach's co-dec. | PA=48° 44′ | Required the angle of poſition ABP. |
| Schedar's co-dec. | PB=34 40 | |
| their diff. of r. aſc. ∠ APB | 20 33 | For the ſolution, ſee IV. 165. |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| As rad. | 90° | 00′ | 10,00000 | As ſin. 5th arc | 12° | 11′ | 0,67563 |
| To co-ſ. ∠ APB | 20 | 35 | 9,97135 | To ſin. 4th arc | 46 | 51 | 9,86306 |
| So is tan. AP | 48 | 44 | 10,05676 | So is tan. ∠ APB | 20 | 35 | 9,57466 |
| To tan. 4th arc | 46 | 51 | 10,02811 | To tan. ∠ B | 52 | 23 | 10,11335 |
| The ſide BP | 34 | 40 | | | | | |
| The 5th arc | 12 | 11 | | | | | |

2d. In the oblique ſpheric triangle PBZ.

| | | |
|---|---|---|
| Given Schedar's co-dec. PB | = 34° 40′ | Required the co-lat. PZ. |
| Schedar's co-alt. BZ | = 52 45 | For the ſolution, ſee IV. 165. |
| angle of poſition PBZ | = 52 23 | Here ∠ PBZ=ſup. ∠ PBA. |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| As rad. | = R | | 10,00000 | | | | |
| To co-ſ. ∠ poſit. | = 52° | 23′ | 9,78560 | As co-ſ. 4th arc | = 22° | 53′ | 0,03560 |
| So co-t. Sch. decl. | = 55 | 20 | 9,83984 | To co-ſ. 5th arc | = 29 | 52 | 9,93811 |
| | | | | So ſin. Sch. dec. | = 55 | 20 | 9.91512 |
| To tan. 4th arc | = 22 | 53 | 9,62544 | To ſin. latit. | = 50 | 44 | 9,88883 |
| Which taken from | 52 | 45=BZ | | | | | |
| Leaves 5th arc | 29 | 52 | | | | | |

## 178. PROBLEM XXXV. Pl. V.

Given the difference of time between the riſing of two known ſtars; Required the latitude of the place.

EXAM. *Being at ſea in an unknown place, the ſtar Aldebaran was obſerved to riſe 3 h. 15 m. later than the bright ſtar in Aries: Required the latitude of that place.*

Bright ſtar in ♈ decl. 22° 25′ N.; right aſcen. 1 h. 54 m. 49 s. Aldebaran's decl. 16° 03′ N.; right aſcenſion 4 h. 23 m. 19 s.

### CONSTRUCTION.

Let the primitive circle repreſent the equator, deſcribe (3d 137) the parallels of declination of the two ſtars, that of Aries being 22° 25′ N. and of Aldebaran 16° 03′ N.: Draw P*a* for the circle of right aſcenſion paſſing through the ſtar in ♈, which ſuppoſe in A: From *a* lay off *ab*, =37° 7½′=diff. of right aſcenſions; and *ac*=48° 45′=3 h. 15 m., the diff. of time between their riſing; draw *b*P, *c*P, cutting the parallels of declination in B, C: Through the points B, C, deſcribe (IV. 61) the great circle HOR; draw PO at right-angles to HR; then the meaſure of PO will give the latitude ſought.

### COMPUTATION.

In the oblique-angled ſpheric triangle PBC.

| | |
|---|---|
| Given B's co-decl. PB=73° 57′ | Rad.:co-ſ.∠BPC::t.PB:t.M=73°38′. |
| A's co-decl. PC=67 35 | From M take PC, leaves N = 6 03′. |
| ∠APC—∠APB=∠BPC=11 37½ | As ſin.N : ſ. M : : tan. ∠CPB : t.∠C= |
| Required the angle PCB. | Hence ∠ PCO=61° 54′. [118° 06′. |

In the ſpheric triangle PCO, right-angled at O.

| | |
|---|---|
| Given C's co-decl. PC=67° 35′ | As rad. : ſin. PC : : ſin. ∠ PCO : ſin. |
| and the ∠ PCO=61 54 | [PO=54° 38′. |
| Required the latitude PO. | Therefore the latitude is 54° 38′. |

## 179. PROBLEM XXXVI. Pl. V.

Two known fixed ſtars being obſerved to have the ſame altitude; Required the latitude of the place of obſervation.

EXAM. *In the evening, the ſtars Capella and Procyon were obſerved at the ſame time to have each 38° 00′ of altitude: Required the latitude of the place where that obſervation was taken.*

Capella's decl.=45° 45′ N. right aſcen. 5 h. 0 m. 28 s. Procyon's decl. =5° 47′ N. right aſcen.=7 h. 27 m. 48 s.

### CONSTRUCTION.

On the plane of the equator, repreſented by the primitive circle, deſcribe (3d 137) the parallels of the given declinations; take *ab*=36° 50′. the diff. of the given right aſcenſions; draw P*a*, P*b*, then A repreſents Procyon, and B Capella.

About A and B as poles, deſcribe (IV. 66) arcs of ſmall circles, at the diſtance of 52°, the co-altitude, and their interſection gives Z the zenith of the place: Through the points A, B; Z, A; Z, B; deſcribe great circles (IV. 61) and draw ZP, the meaſure of which will give the co-latitude of the place of obſervation.

### COMPUTATION.

In the oblique angled ſpheric triangle APB.

| | |
|---|---|
| Given A's co-decl. AP=84° 13' | Rad. : coſ. ∠P : : t.BP : t. M=37° 57 |
| B's co-decl. BP=44 15 | And M tak. fr. PA leaves N=46 16 |
| diff. rt. aſc. ∠APB=36 50 | Sin. N : ſin. M : : tan. ∠P : tan. ∠BAP [=32° 31 |
| Required the ſtars diſtance AB. | Coſ.M:coſ.N::coſ.BP:coſ.BA=51 06 |
| And the angle BAP. | |

In the oblique angled ſpheric triangle AZB.

| | |
|---|---|
| Given A's co-alt. ZA=52° 00' | The angle BAZ will be found= 68° 04'. |
| B's co-alt. ZB=52 00 | |
| ſtars diſtance AB=51 06 | Then ∠BAZ—∠BAP=∠PAZ= 35° 33'. |
| Required the angle ZAB. | |

In the ſpheric triangle AZP.

| | |
|---|---|
| Given A's co-alt. ZA=52° 00' | Rad. : coſ. ∠A :: t. ZA : t.M=47° 29' |
| A's co-declin. AP=84 13 | M taken from PA leaves N=36 44 |
| the angle ZAP=35 30 | Coſ.M:coſ.N::coſ.ZA:coſ.PZ=43 06 |
| Required the co-lat. ZP. | Therefore the latitude is 46° 54' N. |

## 180. PROBLEM XXXVII. Pl. V.

Given the altitudes of two known ſtars;
Required the latitude of the place.

EXAM. *The altitude of the Hydra's heart was obſerved to be* 40° 44', *and of the Lion's heart* 45° 00' *: What is the latitude of the place of obſervation?*

Hydra's heart, decl = 7° 43' S.; right aſcen. = 9 h. 16 m. 47 ſ.
Lion's heart, decl. = 13° 02' N.; right aſcen. 9 h. 56 m. 39ſ.

### CONSTRUCTION.

If this problem is conſtructed on the plane of the equator, it will be in every reſpect like the laſt; only the ſmall circles, deſcribed about A and B, are to be unequally diſtant from their reſpective poles A, B.

### COMPUTATION.

Here, as in the laſt, there will be three ſpheric triangles to work in; namely, the triangles APB, ZAB, and ZPB.

In the triangle APB, where AP=97° 43', BP=76° 58', ∠APB=9° 58'.

As rad. : coſ. ∠APB : : tan. BP : tan. M=76° 46'½. Then AP—M=N =20° 57'½.

As ſin. N : ſin. M : : tan. ∠APB : tan. ∠BAP=25° 34'.

And as coſ. M : coſ. N : : coſ. BP : coſ. BA=22 59.

In the triangle BAZ, where AZ=49° 16', BZ=45° 00', AB=22° 59'.

The angle BAZ will be found equal to 68° 56'.

Then ∠BAZ—∠BAP=∠PAZ=43° 22'.

In the triangle APZ, where AP=97° 43, AZ=49° 16', ∠PAZ=43° 22'.

As rad. : coſ. ∠PAZ : : tan. AZ : tan. M=40° 10'. Then AP—M=N=57° 33'.

And as coſ. M : coſ. N : : coſ. AZ : coſ. PZ=62° 39'.

Hence the latitude ſought is 27° 16 N.

## 181. PROBLEM XXXVIII. Pl. V.

Given the Sun's declination, two altitudes, and the time between the obſervations;
Required the latitude of the place.

EXAM. *On a day when the Sun's declination was* 20° 00′ *N., in the forenoon the Sun's altitude was obſerved to be* 18° 30′, *and* 3 *hours after, his altitude was* 44° 00′ : *What was the latitude of the place?*

### CONSTRUCTION.

Let the primitive circle repreſent that hour circle on which the Sun was at the firſt obſervation, EQ being the equator, then A*a*, the parallel of 20° of declination, gives A the Sun's place at firſt; and as CQ is the tangent of 45°, Q will be the center of the hour circle PBS three hours diſtant from the former, its interſection B with the parallel of declination, is the Sun's place at the ſecond obſervation: About A as a pole, at the diſtance of 71° 30′, the firſt zenith diſtance, deſcribe (IV. 66) a ſmall circle; about B, as a pole, at the diſtance of 46° 00′, the ſecond zenith diſtance, deſcribe (IV. 66) another ſmall circle, cutting the former in Z the zenith: Through Z, A; Z, B; A, B; P, Z; deſcribe (IV. 61) great circles; then PZ is the co-latitude required.

### COMPUTATION.

Here are three triangles to work in; namely, ABP, ABZ, BPZ.

In the iſoſceles ſpheric triangle APB.

| | |
|---|---|
| Given AP=70° 00′ | Suppoſe the perpendicular P*b* is drawn. |
| BP=70 00 | Rad. : tan. BP*b* : : coſ. PB : co-t. ∠PBA=81° 56′. |
| ∠APB=45 00 | Rad. : ſin. PB : : ſin. ∠BP*b* : ſin. B*b*=21° 04½′. |
| Req. ∠ABP and AB. | Then B*b* double gives AB=42° 09′. |

In the oblique angled ſpheric triangle ABZ.

| | |
|---|---|
| Given AZ=71° 30′ | Then working with the three ſides, the angle ABZ |
| BZ=46 00 | will be found=114° 11′. |
| AB=42 09 | And ∠ABZ—∠PBA=PBZ=32° 15′. |
| Required ∠ABZ. | |

In the oblique angled ſpheric triangle PBZ.

| | |
|---|---|
| Given PB=70° 00′ | As rad. : coſ. ∠PBZ : : tan. BZ : tan. M=41° 13′. |
| BZ=46 00 | And PB—M=N=28° 47′. |
| ∠PBZ=32 15 | As coſ. M : coſ. N : : coſ. BZ : coſ. PZ=30° 59′. |
| Req. the co-lat. PZ. | Therefore the latitude is 54° 01′ N. |

182. If the Sun's altitude can be taken both before and after noon, when he has equal heights, then the time between theſe two obſervations being biſected, will give the time when the Sun was on the meridian: Now the co-declination, the co-altitude, and the time from noon at either obſervation being known, the latitude may be readily computed in one oblique angled triangle, in which are known two ſides, and an angle oppoſite to one of them to find the other ſide, which is the co-latitude; for which ſee the problem, art. 176.

## 183. PROBLEM XXXIX. Pl. V.

Given the Sun's declination, two altitudes, and the difference of the magnetic azimuths;
Required the latitude of the place.

EXAM. *On the 21ſt of May, the Sun's declination being* 20° 16′ *N., in the morning when the Sun was on the ESE. point of the compaſs, his altitude was* 43° 30′; *and when he bore S.* 20° 30′ *E. his altitude was* 58° 30′: *What is the latitude of the place of obſervation?*

### CONSTRUCTION.

Let the primitive circle repreſent the azimuth circle which the Sun was on at the greater altitude, 58° 30′, A being the Sun's place at that time, HR the horizon, and Z the zenith; draw *aa* a parallel of 43° 30′ of altitude, and (IV. 75) deſcribe a vertical circle, making an angle with AZ, of 47° 00′, the difference of the obſerved azimuths; the place where this cuts the parallel of altitude *aa*, gives B the Sun's place at the firſt obſervation: Then ſmall circles being deſcribed about A and B, as poles at the diſtances of 69° 44′, the co-declination (IV. 66), their interſection will give P the place of the pole: Through A, B; P, A; P, B; and Z, P, deſcribe great circles (IV. 61); then PZ is the co-latitude ſought.

### COMPUTATION.

1ſt. In the ſpheric triangle AZB: Given AZ = A's co-alt. = 31° 30′
BZ = B's co-alt. = 46 30
AZB = diff. of az. = 47 00

Required ∠ ABZ = 45 41
AB = 32 17

2d. In the iſoſceles ſpheric △ APB: Given AP = A's co-decl. = 69 44
BP = B's co-decl. = 69 44
AB = diſtance = 32 17

Required ∠ ABP = 83 52

3d. In the ſpheric triangle ZBP: Given BZ = B's co-alt. = 46 30
PB = B's co-decl. = 69 44
∠ ZBP = 38 11

Requir. ZP = co-lat. = 39 21

Therefore the latitude of the place is 50° 39′ N.

## 184. PROBLEM XL. Pl. V.

Two known ſtars being obſerved on the ſame azimuth, and two other known ſtars being obſerved on another azimuth, and the time between the obſervation being known; to find the latitude of the place.

EXAM. *The ſtars Aldebaran in Taurus, and Rigel in Orion, were obſerved on the ſame azimuth*; *and* 2 *h.* 35 *min. after, the ſtars Caſtor in Gemini and the Hydra's heart were alſo obſerved on another azimuth: What was the latitude of the place of obſervation?*

### CONSTRUCTION.

On the plane of the equator put the ſtars Aldebaran and Rigel at A, B, alſo the ſtars Caſtor and Hydra at *c*, *d*, by means of their right aſcenſions and declinations (2d and 3d of 137): Let the ſtars *c*, *d*, be removed forwards $2^h\ 35^m$, or 38° 45′ to C, D; through A, B, and D, C, deſcribe (IV. 61) great circles interſecting in Z, the zenith of the place; draw the great circles AC and PZ; then the meaſure of PZ gives the co-latitude required.

### COMPUTATION.

| | | | | | | |
|---|---|---|---|---|---|---|
| Aldebaran's declin. | = | 16° 03′ | N. | right aſcen. = | $4^h\ 23^m\ 19^s$ | = 65° 50′. |
| Rigel's | = | 8 28 | S. | = | 5 03 58 | = 75 $59\frac{1}{2}$. |
| Caſtor's | = | 32 21 | N. | = | 7 20 33 | = 110 08. |
| Hydra's heart | = | 7 43 | S. | = | 9 16 47 | = 139 12. |

1ſt. In the triangle PAB, where PB = 98° 28′, PA = 73° 57′, ∠APB = 10° $09\frac{1}{2}'$.
Then the ∠PAB will be 156° 59′, and the ∠PAZ, the ſuppl. = 23° 01′.

2d. In the triangle PCD, where PD = 97° 43′, PC = 57° 39′, ∠CPD = 29° 04′.
Then the ∠PCD will be 140° 09′, and the ∠PCZ, the ſuppl. = 39° 51′.

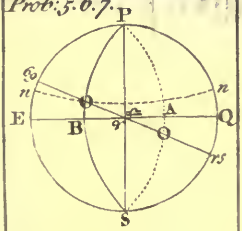
3d. In the triangle APC, where AP = 73° 57′, CP = 57° 39′, ∠APC = 5° $33\frac{1}{2}'$, which is the difference between 2 h. 35 m. and the difference of the right aſcenſions of A and C.
Then the ∠PAC will be 16° 12′, ∠PCA = 161° 30′, and AC = 17° 03′.
Now ∠PAC + ∠PAZ = ∠CAZ = 39° 13′; and ∠PCA — ∠PCZ = ∠ACZ = 121° 39′.

4th. In the triangle ACZ, where AC = 17° 03′; ∠CAZ = 39° 13′; ∠ACZ = 121° 39′.
Then CZ will be found equal to 28° 26′.

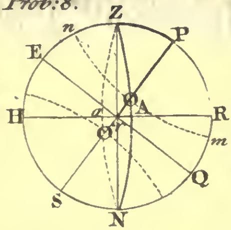
5th. In the triangle CPZ, where CZ = 28° 26′, CP = 57° 39′, ∠PCZ 39° 51′.
Then PZ will be found equal to 38° 49′.
And the latitude of the place of obſervation is 51° 11′. N.

# ASTRONOMY

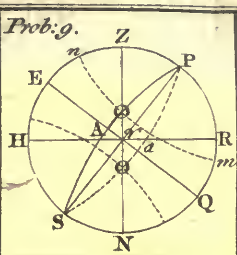
*Prob: 5. 6. 7.*



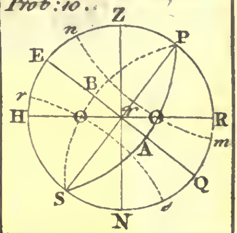
*Prob: 8.*



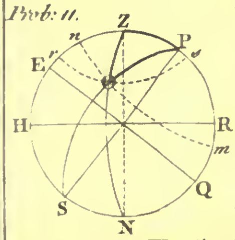
*Prob: 9.*



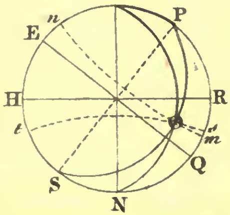
*Prob: 10.*



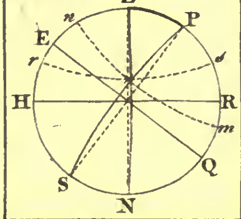
*Prob: 11.*



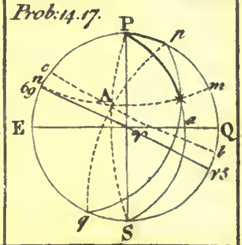
*Prob: 12.*



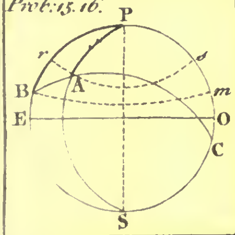
*Prob: 13.*



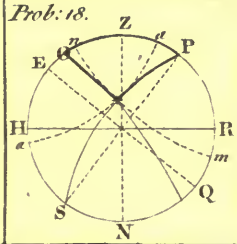
*Prob: 14. 17.*



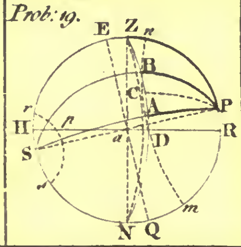
*Prob: 15. 16.*



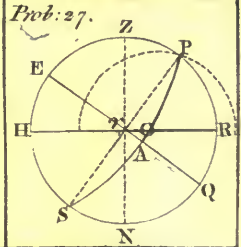
*Prob: 18.*



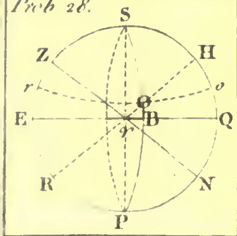
*Prob: 19.*



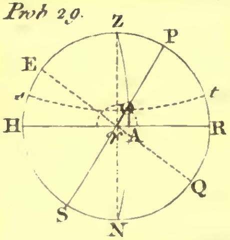
*Prob: 27.*



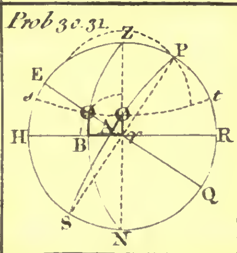
*Prob 28.*



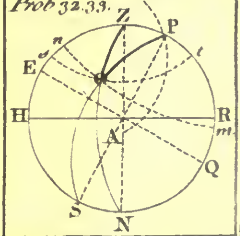
*Prob 29.*



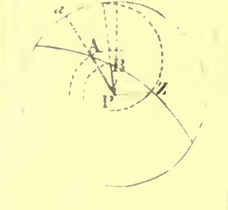
*Prob 30. 31.*



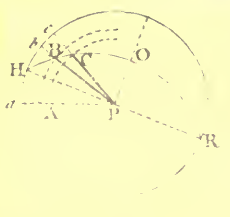
*Prob 32. 33.*



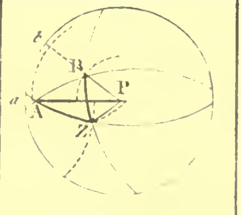
*Prob 34*



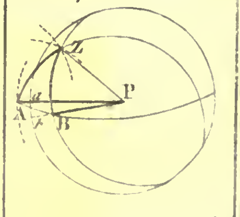
*Prob 35.*



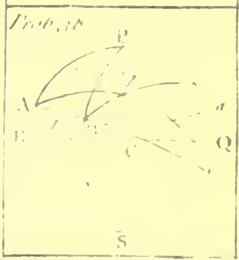
*Prob 36*



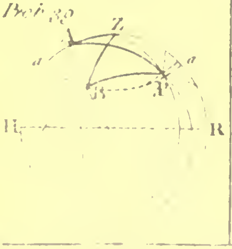
*Prob 37.*



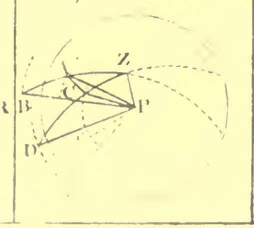
*Prob 38*



*Prob 39*



*Prob 40.*



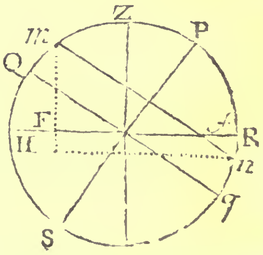
The Problems
belonging to this plate
fall between the
pages 225. 250.

There might be given a great variety of other problems to find the latitude from various circumſtances; but the trouble of ſolving them, as well as ſome of the foregoing ones, is too great to render them of general uſe: And indeed ſome of them were only inſerted as trigonometrical exerciſes for young ſtudents; it being generally allowed that the ſciences are moſt readily learned by working many examples: And on this account it was judged, that the few following queſtions might not only be entertaining to thoſe who have a love for theſe matters; but on ſome occaſions might be uſefully applied at ſea.

## 185. PROBLEM XLI.

*Given the Sun's meridian, or mid-day altitude* $=62^\circ\ 00'$.
*And its mid-night depreſſion, below the horizon,* $=22\ \ 00$.
*Required the latitude of the place, and the Sun's declination.*

SOLUTION. Let the circle HZR be the meridian;
arc H$m$, the meridian alt.; its ſine F$m$;
arc R$n$, the mid-night depreſſion; its ſine $nf$;
$mn$, the parallel of declination;
Q$q$, parallel to $mn$ the equator;
PS, at right angles to Q$q$, the axis, or 6 o'clock circle.



$$\left.\begin{array}{l}\text{Now } HQ+Qm=Hm\\ \text{And } HQ-Qm=Rn\end{array}\right\}\text{For } HQ=Rq\text{; and } Qm=qn.$$

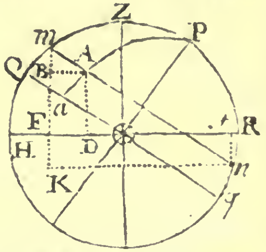
$$\text{Then } HQ=\frac{Hm+Rn}{2}=\text{co-latitude}=42^\circ\ 00'.$$

$$\text{And } Qm=\frac{Hm-Rn}{2}=\text{declination}=20^\circ\ 00'.$$

In the following problems, as it was the method of computation which was chiefly intended for the information of beginners, the conſtruction is ſuppoſed to be done: And the lines and letters, as here deſcribed, are to be underſtood to repreſent the ſame things in each figure.

## 186. PROBLEM XLII.

*Some time in the month of May, 1780, at a place in the weſtern ocean, the Sun's meridian altitude was obſerved to be 62° 00′; and $1^h\ 48^m\ 14^s$ after, the altitude was found to be 54° 30′: Required the latitude of that place, and the Sun's declination.*



Let $m$ and A be the Sun's places at the given times.

$mF$, $AD$, the ſines of the obſerved altitudes.

$mB$ the difference of thoſe right ſines.

$\angle QPa$, the given interval of time,

and $Qa$, the verſed ſine of that interval.

Now $Qa : Qq :: mA : mn$ (II. 182). And $mA : mn :: mB : MK$. (II. 167)
Therefore $Qa : Qq :: mB : mK = mF + nf$.

$$\text{And } mK = \frac{Qq \times mB}{Qa} = \frac{2 \times \text{radius} \times \text{diff. of ſines of alts.}}{\text{verſed ſine of hour from noon}}.$$

But verſed ſine of an arc = twice ſquare of the ſine of half that arc. (IV. 193)

$$\text{Therefore } mK = \frac{2 \times R \times \text{diff. ſines of alts.}}{2ss,\ \frac{1}{2}\text{ hour } \grave{a}\text{ noon *}} = \frac{\text{diff. of ſines of alt.}}{ss,\ \frac{1}{2}\text{ hour } \grave{a}\text{ noon}}.$$ Radius being 1.

Or L, diff. ſines of alts.−2L$s$, $\frac{1}{2}$ hour *à* noon = L, ſum ſines, $mF + nf$.

Here $1^h\ 48^m\ 14^s$ = 27° 3′ 30″, (131) | Alt. 62° 00′ nat. ſine = 0,88295 (IV. 256)
| Alt. 54 30 nat. ſine = 0,81412
And $\frac{1}{2}$ hour *à* noon = 13° 31′ 45″. | Diff. of ſines of alts. = 0,06883

Now diff. ſines alts. = 0,06883; its log. 8,83778 †
$\frac{1}{2}$ hour *à* noon = 13° 31′ 45″; twice its log. ſine 8,73822 ‡
$mK$ = 1,2574 the number to log. 10,09946 ||
$mF$ = 0,8829 the nat. ſine of 62° 00′ the meridian altitude.
$nf$ = 0,3745 the nat. ſine 21° 59′ 36″ the mid-night depr.
Sum 83 59 36, its $\frac{1}{2}$ 41° 59′ 48″ = co-lat.
Diff. 40 00 24 its $\frac{1}{2}$ 20 00 12 = dec.

Latitude 48° 0′ 12″ N. obſervations made on the 19th of May.

---

* The mark *à* is uſed for the word *from*.

† Here 8, is the index; becauſe 6, the left-hand digit of 0,06881, is in the place of 2ds.

‡ The log. ſin. of 13° 31′ is 9,36871; and of 13° 32′ is 9,36924; their diff. is 53; then 60″ : 53 :: 45″ : 40; and 9,36871+40 = 9,36911; its double is 8,73822, rejecting 10 in the doubled index.

In ſubtracting 8,73822 from 8,83765; the index of the minuend is to be increaſed by 10 for a radius; or augment 0, the index of the remainder, by 10.

|| The log. 10,09946, having 10 for its index, ſhews that the left-hand place of its correſponding number ſtands in the place of units.

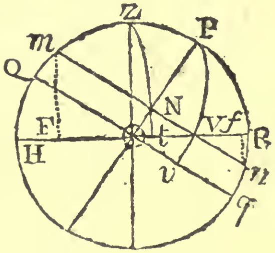
*To find the degrees, minutes, and ſeconds to a given natural right ſine.*

Now $nf$ = 0,3745 its log. is ,57345; which ſought among the log. ſines, falls between thoſe of 21° 59′ and 22° 00′; the difference of their logs. is 31; and the difference between the given log. and that of 21° 59′ is 19; then 31 : 19 :: 60″ : 36″; ſo that $nf$ anſwers to 21° 59′ 36″. (iv. 257.)

187. PRO-

## 187. PROBLEM XLIII.

*Being at ſea, ſome time in July, in North lat. the Sun was obſerved to riſe at 4 h. 24 m. 36 s. A. M.; and in the ſame place, his altitude at noon was 62° 00′: Required the latitude of that place, and day of the month.*



Let *m*, v, *n*, be places of the Sun, at noon, at riſing, and at midnight.

*m*F, *nf*, ſines of mer. alt. and midnight depr.

o*v* the ſine of the aſcenſional difference.

*vq* the verſed ſine of the time of ſetting from midnight.

Q*v* = radius + ſine of the aſcenſional difference.

Now Q*v* : *vq* : : (*m*V : V*n* : :) *m*F : *nf*.

Then $nf = \frac{vq \times m\text{F}}{\text{Q}v}$; or Lſ, *nf* = L‵, Q*v* + Lſ, *m*F + L*v*, ∠VPR.

Or, L‵, Q*v* + Lſ, merid. alt. + 2Lſ, $\frac{1}{2}$ time *à* midnight + L, 2 = Lſ, midnight depreſſion.

Here o*v* = (*s*, 1 h. 35 m. 24 ſ. = *s*, 23° 51′ =) 0,40435; and Q*v* = 1,4043.
Time from midnight = 4 h. 24 m. 36 s. = 66° 09′; its half = 33° 4$\frac{1}{2}$′.

| | | |
|---|---|---|
| Q*v* = 1,4043 | its L‵ | 9,85254 |
| *m*F = *ſ*, 62° | its Lſ, | 9,94593 |
| *vq* = { ſſ, 33° 4$\frac{1}{2}$′ | 2Lſ, | 9,47397 |
| { 2 | its L | 0,30103 |
| *nf* = 21° 59′ 40″ | its Lſ, | 9,57347 |

| | | |
|---|---|---|
| Merid. alt. | = 62° 0′ 0″ | |
| Midnt. depr. | = 21 59 40 | |
| Sum | = 83 59 40; | its $\frac{1}{2}$ = 41° 59′ 50″ |
| Diff. | = 40 0 20; | its $\frac{1}{2}$ = 20 0 10 |
| Latitude | = 48° 0′ 10″ N. | |
| Declination | = 20° 0′ 10″ N, on July 23d, | (185.) |

## 188. PROBLEM XLIV.

*At a place in the northern hemiſphere, ſome time in the month of May, the Sun was obſerved to have 14° 43½′ altitude at 6 h. P. M. and to ſet at 7 h. 35 m. 24 s. : Required the latitude of that place, and day of the month.*

Let $m$, $N$, $V$, $n$, be places of the Sun, at noon, 6 o'clock, ſetting, and midnight. See the fig. to Problem 43.

$mF$, $Nt$, ſines of the altitudes at $m$ and $N$.

$ov$, $vq$, and $Qv$, the ſame as in the laſt problem.

Now $ov : vq :: (Nv : Vn ::)\ Nt : nf$. Then $L, nf = L^{\backprime}, ov + L, Nt + L, vq$.
Alſo $ov : Qv :: (NV : Vm ::)\ Nt : mF$. And $L, mF = L^{\backprime}\ ov + L, Nt + L, Qv$.
Here $ov = s$, 23° 51′ = 0,40434 ; and $Qv = 1{,}4043$.
$qv = v$, 66° 09′ = 2ss, 33° 4½′.

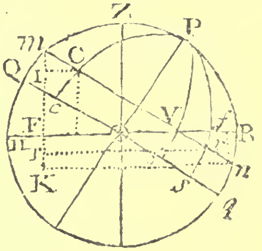
| | | | | | | |
|---|---|---|---|---|---|---|
| Then $ov = s$, 23° 51′ | its $L^{\backprime}s$, | 0,39325 | And $ov = s$, 23° 51′ | its $L^{\backprime}s$, | 0,39325 |
| $Nt = s$, 14° 43½′ | $Ls$, | 9,40514 | $Nt = s$, 14 43½ | its $Ls$, | 9,40514 |
| $qv = \begin{cases} ss, 33° 4½′ \\ 2 \end{cases}$ | $2Ls$, | 9,47397<br>0,30103 | $Qv = 1{,}40\ 43$ | its $L$, | 10,14746 |
| | | | $mF = s$, 62 0 | $Ls$, | 9,94585 |
| $nf\ s$, 22° 0′ | | 9,57339 | | | |

$$\text{Then } \frac{62° + 22°}{2} = 42° \text{ the co-latitude.}$$

$$\text{And } \frac{62 - 22}{2} = 20° \text{ the declination anſwering to May 19th.} \qquad (185.)$$

## 189. PROBLEM XLV.

*At a place in the weſtern ocean, ſome time in July, the Sun's altitude was obſerved to be 36° at 3 h. 51 m. 49 s. P. M. ; and was ſeen to ſet at 7 h. 35 m. 24 s. P. M. : Required the latitude of the place, and day of the month.*



Let $m$, $C$, $V$, $n$, be the Sun's places, at noon, at 3 h. 51 m. 49 ſ. at ſetting, and at midnight.

$mf$, $IF$, the ſines of the altitudes at $m$, $C$ ;

$nf$ the ſine of the midnight depreſſion.

$oc$, $ov$, the ſines of the times from 6 $h$, and $cv$, their ſum.

Now $cv : \mathrm{Q}v :: (\mathrm{CV} : \mathrm{V}m ::)\ \mathrm{IF} : m\mathrm{F}$. Or $\mathrm{L}s, m\mathrm{F} = \mathrm{L}\grave{}, cv + \mathrm{L}s, \mathrm{IF} + \mathrm{L}, \mathrm{Q}v$.
And $cv : qv :: (\mathrm{CV} : \mathrm{V}n ::)\ \mathrm{IF} : nf$. Or $\mathrm{L}s, nf = \mathrm{L}\grave{}, cv + \mathrm{L}s, \mathrm{IF} + \mathrm{L}v, q\mathrm{P}v$
Here $oc = s$, 2h. 8m. 11s. $= s, 32^\circ\ 2'\ 45'' = 0{,}53059$ } $cv = 0{,}93493$;
$ov = s$, 1h. 35m. 24s. $= s, 23\ 51\ 00 = 0{,}40434$ }
$\mathrm{Q}v = 1{,}4043$; $qv = v, 66^\circ 9' = 2ss, 33^\circ 4' 30''$; or $\mathrm{L}, qv = 2\mathrm{L}s, 33^\circ 4\tfrac{1}{2}' + \mathrm{L}2$.

$$\begin{array}{llll}
\text{Then } \mathrm{L}\grave{}, cv = 0{,}93493 & 0{,}02922. & \text{And } \mathrm{L}\grave{}, cv = 0{,}93493 & 0{,}02922 \\
\mathrm{L}s, \mathrm{IF} = 36^\circ\ 00' & 9{,}76922 & \mathrm{L}s, \mathrm{IF} = 36^\circ\ 00' & 9{,}76922 \\
\mathrm{L}, \mathrm{Q}v = 1{,}4043 & 10{,}14746 & \mathrm{L}, qv \begin{cases} ss, 33^\circ\ 4\tfrac{1}{2}' \\ 2 \end{cases} & \begin{array}{l} 9{,}47397 \\ 0{,}30103 \end{array} \\
\hline
\mathrm{L}, m\mathrm{F} = 62^\circ\ 00' & 9{,}94590 & \mathrm{L}s, nf\ 22^\circ\ 00' & 9{,}57344
\end{array}$$

Then $\dfrac{62^\circ + 22^\circ}{2} = 42$ the co-latitude.

And $\dfrac{62 - 22}{2} = 20$ the declination, anſwering to July 23. (185.)

## 190. PROBLEM XLVI.

*Some time in the month of May, at a place in the weſtern ocean, the day broke at* 1 *h.* 45 *m.* 36 *s. A. M.*; *and at* 8 *h.* 8 *m.* 11 *s. A. M. the Sun's altitude was obſerved to be* 36° : *Required the latitude and day of the month.*

Let $m$, $c$, $r$, $n$, be the Sun's places, at noon, at 8 h. 8 m. 11 s., at the beginning of twilight, and at midnight.
$m\mathrm{F}$, $\mathrm{IF}$, the ſines of the altitudes at $mc$.
$\mathrm{FT}$, $\mathrm{FK}$, the ſines of the depreſſions at $r$, $n$.
$oc$, $os$, the ſines of the times from 6 h., and $cs$ their ſum.

Now $cs : \mathrm{Q}s :: (\mathrm{C}r : mr ::)\ \mathrm{IT} : m\mathrm{T}$. And $m\mathrm{T} - \mathrm{FT} = m\mathrm{F}$.
Alſo $cs : qc :: (\mathrm{C}r : \mathrm{C}n ::)\ \mathrm{IT} : \mathrm{IK}$. And $\mathrm{IK} - \mathrm{IF} = nf$.
Here $\mathrm{IF} = s, 36^\circ\ 00' = 0{,}58778$ } $\mathrm{IT} = 0{,}89680$
$\mathrm{FT} = s, 18\ \ 00 = 0{,}30902$ }
$oc = s$, 2h. 8m. 11s. $= s, 32^\circ\ 2'\ 45'' = 0{,}53059$; and $qc = 1{,}5306$
$os = s$, 4h. 14m. 24s. $= s, 63\ 36\ 00 = 0{,}89571$; and $\mathrm{Q}s = 1{,}8957$

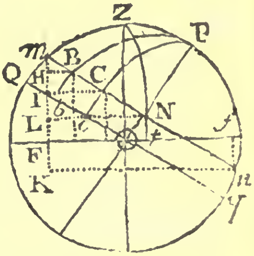
$$\begin{array}{ll|ll|l}
\text{Then } cs & 9{,}84579 & cs & 9{,}84579 & m\mathrm{T} - \mathrm{FT} = m\mathrm{F} = 0{,}8829 \text{ the } s, 62^\circ\ 00' \\
\mathrm{IT} & 9{,}95269 & \mathrm{IT} & 9{,}95269 & \mathrm{IK} - \mathrm{IF} = nf = 0{,}37458 \text{ the } s, 22\ \ 00 \\
\mathrm{Q}s & 10{,}27777 & qc & 10{,}18486 & \text{Hence the lat. } 48^\circ \text{ N.} \\
\hline
m\mathrm{T} & 10{,}07625 & \mathrm{IK} & 9{,}98334 & \text{Decl. } 22^\circ\ 00' \text{ N., anſwering to May 19th.} \\
\hline
m\mathrm{T} = & 1{,}1919 & \mathrm{IK} = & 0{,}96237 &
\end{array}$$

THE PRO-

191.

## PROBLEM XLVII.

*In the month of May, at ſome place in the weſtern ocean, the Sun's altitude at* $6^h$ *A. M. was* 14° 43½′ ; *and at* 8*h.* 8 *m.* 11*s. its altitude was* 36° *: Required the latitude of the place and day of the month.*



Let *m*, C, N, *n*, be the Sun's places at noon, at 8 h. 8 m. 11 s., at 6 h., and at midnight.
*m*F, IF, LF, the ſines of the alts. at *m*, C, N.
*nf* the ſine of the midnight depreſſion.
O*c* the ſine of 2 h. 8 m. 11s.

Now O*c* : OQ : : (NC : N*m* : :) IL : *m*L. Then *m*L + LF = *m*F.
And O*c* : *cq* : : (NC : C*n* : :) IL : IK. Then IK — IF = *nf*.

Hence L,*m*L = L\`,O*c* + rad. + L,IL. And L,IK = L\`,O*c* + L*cq* + L,IL.

Here O*c* = *s*, 2 h. 8 m. 11s. = *s* 30° 2′ 45″ = 0,53059 ; and *cq* = 1,5306.
IF = *s*, 36° 0′ = 0,58778 ; and LF = *s*, 14° 43½′ = 0,25418 ; ſo IL = 0,3336

| | | | | | |
|---|---|---|---|---|---|
| O*c* = 32° 2′ 45″ | its L\`*s*, | 0,27524 | O*c* = 32° 2′ 45″ | its L\`*s*, | 0,27524 |
| IL = 0,3336 | its L. | 9,52323 | II = 0,3336 | its L. | 9,52323 |
| Rad. | | 10,00000 | *cq* = 1,5306 | its L. | 10,18486 |
| *m*L = 0,62874 | | 9,79847 | IK = 0,96234 | | 9,98333 |

Then *m*F = 0,88292 the ſine of 62° 00′ the meridian altitude.
And *nf* = 0,37456 the ſine of 22 00 the midnight depreſſion.
Hence the latitude is 48° N. decl. 20° N. on May 19th. (185).

192.

## PROBLEM XLVIII.

*At a place in the weſtern ocean, in the month of July, the Sun's altitude was found to be* 46° *at* $2^h\ 49^m\ 9^s$ *P. M.* ; *and to be* 36° *high at* $3^h\ 51^m\ 49^s$ *P. M. : Required the latitude of the place and day of the month.*

Let *m*, B, C, *n*, be the Sun's places at noon, at 2 h. 49 m. 9 s., at 3 h. 51 m. 49 s., and at midnight ; and *m*F, HF, IF, the ſines of the alts. at *m*, B, C. O*b*, O*c*, the co-ſines of the time from noon ; or the ſines of the time to 6 o'clock.

Now *bc* : Q*c* : : (BC : *m*C : :) HI : *m*I. Then *m*I + IF = *m*F.
And *bc* : *bq* : : (BC : B*n* : :) HI : HK. Then HK — HF = FK = *nf*.

Here O*b* = *s*, 3 h. 10 m. 51 s. = *s*, 47° 42′ 45″ = 0,73978 } Hence *bc* = 0,20919
O*c* = *s*, 2 h. 8 m. 11 s. = *s*, 32 2 45 = 0,53059 }
HF = *s* 46° = 0,71934 ; IF = *s*, 36° = 0,58778 ; HI = 0,13156 ; *bq* = 1,7398.
Alſo Q*c* = (ver. ſine of 57° 57′ 15″ =) 2*ss*, ½57° 57′ 15″ = 2*ss*, 28° 58′ 37″.

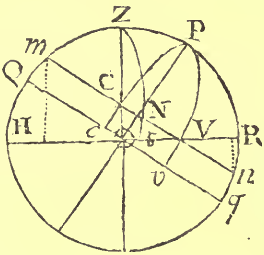
Then

| | | | | |
|---|---|---|---|---|
| Then $bc=$ | 0,20919 | 0,67947 | And $bc=0,20919$ | 0,67947 |
| $\mathrm{HI}=$ | 0,13156 | 9,11912 | $\mathrm{HI}=0,13156$ | 9,11912 |
| $\mathrm{QC}=$ | $ss$,28° 58′ 37″ | 9,37050 | $bq=1,7398$ | 10,24050 |
| | 2 | 0,30103 | | |
| | | | $\mathrm{HK}=1,0942$ | 10,03909 |
| $m\mathrm{I}=$,29520 | | 9,47012 | | |

Then $m\mathrm{F}=0,88298$ the ſine of 62° the mer. alt. } Hence lat. = 48° N. decl.
And $nf=0,37486$ the ſine of 22 the mid. depr. } 20° on July 23.

## 193. PROBLEM XLIX.

*Being at ſea in the weſtern ocean, the Sun was obſerved to have 27° 24′ of altitude when due W.; and to have 14° 43½′ alt. at 6 h. P. M.: Required the latitude of that place, and the Sun's declination.*



Let C, N, be the Sun's places at W., and at 6 h.
OC, N$t$, the ſines of their altitudes.
C$c$ = ON, the arcs of declination.
$\angle \mathrm{CO}c = \angle \mathrm{NO}t$ = latitude.

In $\triangle \mathrm{O}c\mathrm{C}$. As $s,\mathrm{OC}:\mathrm{R}::s,\mathrm{C}c:s,\angle\mathrm{CO}c=\frac{s,\mathrm{C}c}{s,\mathrm{OC}}\times\mathrm{R}$.

In $\triangle \mathrm{O}t\mathrm{N}$. As $s,\mathrm{ON}:\mathrm{R}::s,\mathrm{N}t:s,\angle\mathrm{NO}t=\frac{s,\mathrm{N}t}{s,\mathrm{ON}}\times\mathrm{R}$.

Then $\frac{s,\mathrm{C}c}{s,\mathrm{OC}}=\left(\frac{s,\mathrm{N}t}{s,\mathrm{ON}}=\right)\frac{s,\mathrm{N}t}{s,\mathrm{C}c}$. And $ss,\mathrm{C}c=s,\mathrm{OC}\times s,\mathrm{N}t$.

Or $s$, alt. W. × $s$, alt. at 6 = $ss$,decl. Or $\frac{\mathrm{L}s,\text{ alt. W.}+\mathrm{L}s,\text{ alt. at 6}}{2}=\mathrm{L}s,\text{decl.}$

And L`$s$,alt. W. + L$s$,decl. = L$s$,lat.

| | | |
|---|---|---|
| OC = $s$,27° 24′ its L,$s$ | | 9,66295 |
| N$t$ = 14 43½ its L,$s$ | | 9,40514 |
| | Sum | 19,06809 |
| C$c$ = 20° 00′ | | 9,53404 |
| O$c$ = 27 24 its L` | | 0,33705 |
| $\angle$ QOZ = 48 00 | | 9,87109 |

## 194. PROBLEM L.

*At a place in the weſtern ocean, the Sun at riſing was obſerved to be 59° 15′ 40″ from the true north point of the horizon; and at 6 h. A. M., the altitude was obſerved 14° 43½′: Required the latitude and declination.*

Let V, and N, be the places of the Sun at riſing and at 6 h. A. M.
N$t$, the ſine of the alt. at 6. ON, V$v$ are arcs of declination.
O$v$, the aſcenſional diff. $\angle \mathrm{NO}t$ = lat.; $\angle \mathrm{VO}v$ = co-latitude.
OV, = co-amplitude.

Now in △ $\text{OV}v$. As $\text{R} : s,\text{OV} :: s,\text{VO}v : s,\text{V}v = \frac{s,\text{OV} \times s,\text{VO}v}{\text{R}}$.

in △ $\text{NO}t$. As $s,\text{NO}t : \text{R} :: s,\text{N}t : s,\text{ON} = \frac{\text{R} \times s,\text{N}t}{s,\text{NO}t}$.

Then $\frac{s,\text{OV} \times s\text{VO}v}{\text{R}} = \frac{s,\text{N}t \times \text{R}}{s,\text{NO}t}$; and $s,\text{VO}v \times s,\text{NO}t = \frac{s,\text{N}t}{s,\text{OV}} \times \text{RR}$.

That is, $\frac{s,\text{N}t}{s,\text{OV}} = s,\text{NO}t \times s^{\backprime},\text{NO}t$; hence $\frac{2s,\text{N}t}{s,\text{OV}} = (2s,\text{NO}t \times s^{\backprime},\text{NO}t =) s,2\text{NO}t$.

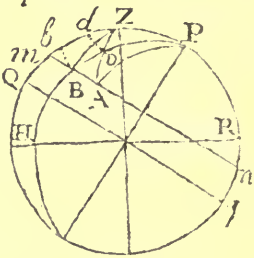
(IV. 189)

Therefore $\text{L}^{\backprime}s^{\backprime}$, ampl. + $\text{L}s$, alt. at 6 + $\text{L},2 = \text{L}s$, double the latitude. If the latitude is leſs than 45°; otherwiſe it is double the co-latitude.

| | | | | | |
|---|---|---|---|---|---|
| $\text{L}^{\backprime}s^{\backprime}$, ampl. | 59° 15′ 40″ | 0,29147 | Rad. | R | 10,00000 |
| $\text{L}s$, alt. 6 | 14 43 30 | 9,40514 | $s^{\backprime}$, ampl. | 59° 15′ 40″ | 9,70853 |
| L, 2 | | 0,30103 | $s^{\backprime}$, lat. | 47 59 | 9,82565 |
| $\text{L}s$, | 84° 02′ | 9,99764 | $s$, decl. | 20 00 | 9,53418 |
| Co-lat. is | 42 01 | | | | |

## 195. PROBLEM LI.

*Being at ſea in the weſtern ocean, ſome time in the night, the diſtance of two ſtars when both were on the meridian, was obſerved to be* 20°; *and* $1^h\ 49^m$ *after, the difference of their altitudes was* 14° 35½′, *and the difference of their azimuths* 30° 9½′: *Required the latitude of the place.*



As the ſtars were on the meridian when firſt obſerved; their diſtance, difference of declination, and difference of altitudes, at that time, are equal: If they are firſt at *d*, *b*, and in the difference of time revolve to D, B; then is known ∠P, ∠DZB, DB, and ZB—ZD. Let ZB—ZD = N; and find ZB + ZD = M.

Now (IV. 239) $s,\text{ZB} \times s,\text{ZD} \times s^{\backprime},\text{DZB} + s^{\backprime},\text{ZD} \times s^{\backprime}\text{ZB} = s^{\backprime},\text{DB}$.

Or (IV. 181, 174) $\overline{\tfrac{1}{2}s^{\backprime},\text{N} - \tfrac{1}{2}s^{\backprime},\text{M}} \times s^{\backprime},\text{DZB} + \tfrac{1}{2}s^{\backprime},\text{N} + \tfrac{1}{2}s^{\backprime},\text{M} = s^{\backprime},\text{DB}$.

Or $s^{\backprime},\text{DZB} \times \tfrac{1}{2}s^{\backprime},\text{N} - s^{\backprime},\text{DZB} \times \tfrac{1}{2}s^{\backprime}\text{M} + \tfrac{1}{2}s^{\backprime}\text{N} + \tfrac{1}{2}s^{\backprime}\text{M} = s^{\backprime},\text{DB}$.

Or $\overline{s^{\backprime},\text{DZB} + 1} \times \tfrac{1}{2}s^{\backprime},\text{N} + \overline{1 - s^{\backprime},\text{DZB}} \times \tfrac{1}{2}s^{\backprime},\text{M} = s^{\backprime},\text{DB}$.

Then $s^{\backprime},\text{M} = \left(\frac{2s^{\backprime},\text{DB} - \overline{1 + s^{\backprime}\text{DZB}} \times s^{\backprime},\text{N}}{1 - s^{\backprime}\ \text{DZB}} = \right) \frac{2s^{\backprime},\text{DB} - v^{\backprime},\text{DZB} \times s^{\backprime},\text{N}}{v,\text{DZB}}$.

Or $s^{\backprime},\text{M} = \left(\frac{2s^{\backprime},\text{DB}}{v,\text{DZB}} - \frac{v^{\backprime},\text{DZB} \times s^{\backprime}\text{N}}{v,\text{DZB}} = \right) \frac{2s^{\backprime},\text{DB}}{2ss,\tfrac{1}{2}\text{DZB}} - \frac{2s^{\backprime}s^{\backprime},\tfrac{1}{2}\text{DZB} \times s^{\backprime},\text{N}}{2ss,\tfrac{1}{2}\text{DZB}}$.

(IV. 193, 197)

Let $2\text{L}^{\backprime}s,\tfrac{1}{2}\text{DZB} + \text{L}s^{\backprime},\text{DB} = \text{L},\text{A}$; and $2\text{L}^{\backprime}s,\tfrac{1}{2}\text{DZB} + 2\text{L}^{\backprime}s,\tfrac{1}{2}\text{DZB} + \text{L}s^{\backprime},\text{N} = \text{L},\text{B}$.

| | | | | | |
|---|---|---|---|---|---|
| Then $s^{\backprime}\text{M} = \text{A} - \text{B}$. | | $2\text{L}^{\backprime}s.\tfrac{1}{2}\text{DZB}$ | 1,16952 | $2\text{L}^{\backprime}s,\tfrac{1}{2}\text{DZB}$ | 1,16952 |
| Here | DB = 20° 00′ | $\text{L}s^{\backprime},\text{DB}$ | 9,97200 | $2\text{L}s^{\backprime},\tfrac{1}{2}\text{ZB}$ | 9,96960 |
| | ∠DZB = 30 9½ | A = 13,884 | 1,14251 | $\text{L}s^{\backprime},\text{N}$ | 9,98576 |
| | ½∠DZB = 15 4¾ | B = 13,331 | | B = 13,331 | 1,12488 |
| ZB—ZD = N = 14 35½ | | $s^{\backprime},\text{M} = 0,553$; and ZB + ZD = 56° 25½′. | | | |

Then $\frac{1}{2}$ ſum + $\frac{1}{2}$ diff. $= ZB = 35^\circ\ 30\frac{1}{2}'$; and $\frac{1}{2}$ ſum — $\frac{1}{2}$ diff. $= ZD = 20^\circ\ 55'$. Now $s,DB : s,DZB :: s,ZB : s,PDZ$; and $s,ZPD : s,PDZ :: s,ZD : s,ZP\ 42^\circ\ 1'$. Hence the latitude is $47^\circ\ 59'$ N.

## 196. PROBLEM LII.

*By obſervations made at a place in the weſtern ocean, it was found that the Sun's altitude was 36° S., when his azimuth was N. 100° 5′ E. and his alt. was 46° S., when his azimuth was N. 114° 28′ E. What was the latitude of the place and the Sun's declination?*

Let A, B, be the Sun's places when obſerved.
In the triangles PAZ, PBZ, where $PA = PB$.

By IV. 239. $\begin{cases} \grave{s},ZP\times\grave{s}ZA-\grave{s},PZA\times s,ZP\times s,ZA=\grave{s},PA. \\ \grave{s},ZP\times\grave{s},ZB-\grave{s},PZB\times s,ZP\times s,ZB=\grave{s},PA. \end{cases}$

Then $\grave{s},PZB\times s,ZB\times s,ZP-\grave{s},PZA\times s,ZA\times s,ZP=\grave{s},ZB\times\grave{s},ZP-\grave{s},ZA\times\grave{s},ZP$.

Or $\overline{\grave{s},PZB\times s,ZB-\grave{s},PZA\times s,ZA}\times s,ZP=\overline{s',ZB-\grave{s},ZA\times\grave{s},ZA}\times\grave{s},ZP$.

Then $\frac{\grave{s},PZB\times\overline{s,ZB\backsim\grave{s},PZA}\times s,ZA}{\grave{s},ZB-\grave{s},ZA}=\left(\frac{\grave{s},ZP}{s,ZP}=(\text{III. }33)\ t,ZP=\right)\grave{s},RP$ the latitude.

Let $L\grave{s},PZB+Ls,ZB=L,A$: And $L\grave{s},PZA+Ls,ZA=L,B$. Then $\frac{A\backsim B}{\grave{s}ZB-\grave{s},ZA}=\grave{t},RP$.

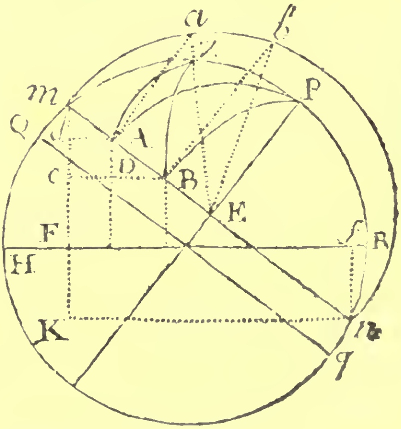
Here $A=0{,}14164$; $B=0{,}28770$; $\grave{s},ZB=0{,}71933$; $\grave{s},ZA=0{,}58779$.

Then $\frac{0{,}13154}{0{,}14606}={,}9005=t,42^\circ$. Hence lat. is 48° N. decl. 20° N.

## 197. PROBLEM LIII.

*Given two altitudes of the Sun and the time from noon when thoſe altitudes were taken; thence to find the latitude and declination.*

EXAM. *At* 8 *h.* 8 *m.* 11 *s. A. M., the alt. was* 36°; *and at* 9 *h.* 10 *m.* 51 *s. the alt. was* 46°; *at a place in the weſtern ocean, ſome time in May*, 1763.



Let B, A, be the Sun's places obſerved;
$m$, $n$, thoſe of noon and midnight; $Fe$, $Fd$, $Fm$, FK, repreſent the ſines of the diſtances of thoſe places from the horizon HR, to the diameter $Qq$; and $d = Fd - Fe$.

On $mn$ deſcribe the ſemicircle $mbn$, repreſenting half the parallel of declination, and let $Aa$, $Bb$, be at right angles to $mn$: Then will the angles $mEb$, $mEa$, repreſent the times from noon, at the obſervations B, A; and $mB$, $mA$, are as the verſed ſines of thoſe times.

And $mB - mA = AB$.

Then $AB : de :: mB : me$; and $me + Fe = Fm$, the ſine of $Hm$.

Alſo $AB : de :: mn : mK$; and $Km - Fm = FK$, the ſine of $Rn$. Hence the latitude and declination are found. (185)

$$\text{Here } AB = (\grave{s},ma - \grave{s},mb = s,\frac{mb+ma}{2} \times 2s,\frac{mb-ma}{2} = )\, s,M \times 2s,N.$$

(IV. 181)

$$\text{And } de = (s,\text{2d alt.} - s,\text{1ſt alt.} = \grave{s},\frac{\text{2d}+\text{1ſt}}{2} \times 2s,\frac{\text{2d}-\text{1ſt}}{2} = \grave{s},W \times 2s,V.$$

(IV. 182)

$$\text{Now } L,\frac{de}{AB} \times mn = L,mK = L\grave{s},M + L\grave{s},N + Ls,W + L,sV,L,2.$$

$$\text{And } L,\frac{de}{AB} \times mB = L,me = L,mK + 2Ls,\tfrac{1}{2}\angle mEb.$$

In this example, the $\angle mEb = 3$ h. 51 49 = 57° 57¼′; its ½ = 28° 58⅝′.
$\angle mEa = 2$ h. 49 9 = 42 17¼.

| | | | |
|---|---|---|---|
| 57° 57¼′ | | 46 | |
| 42 17¼ | | 36 | |
| 100 14½ | 50° 7¼′ = M | 82 | 41° 0′ = W. |
| 15 40 | 7 50 = N | 10 | 5 0 = V. |

$s$,36° = $Fe$ = 0,58778
$me$ = 0,29521

$Fm$ = 0,88299 = 62° 0′ 19″
$Km$ = 1,25810

$FK$ = 0,37511 = 22 1 51
84 2 10
39 58 28

| | | |
|---|---|---|
| $L\grave{s},M$ | 50° 7¼′ | 0,11498 |
| $L\grave{s},N$ | 7 50 | 0,86553 |
| $Ls\grave{},W$ | 41 0 | 9,87778 |
| $Ls,V$ | 5 0 | 8,94030 |
| $L,2$ | | 0,30103 |
| $L,Km$ | 1,25783 | 10,09962 |
| $2Ls,\frac{1}{2}mb$ | 28° 58⅝′ | 9.37050 |
| $L,me$ | 0,29521 | 9,47012 |

42 0′ 55″ the co-latitude.

19 59 14 the decl. on May 20th.

198. The following is another ſolution, on different principles.
In the triangles $BPZ$, $APZ$, are given, (ſee the foregoing figure.)
$BZ$, $AZ$; $BPZ$, $APZ$, hour angles; and $PB = PA$;
To find $PB$, $PZ$; for whoſe ſum and diff. put $M$ and $N$.

$$\text{Now (IV. 239)} \begin{cases} s,PB \times s,PZ \times \grave{s},BPZ + \grave{s},PB \times \grave{s},PZ = \grave{s},BZ. \\ s,PA \times s,PZ \times \grave{s},APZ + \grave{s},PA \times \grave{s},PZ = \grave{s},AZ. \end{cases}$$

$$\text{Or (IV. 181. 174)} \begin{cases} \overline{\tfrac{1}{2}\grave{s},N - \tfrac{1}{2}\grave{s},M} \times \grave{s},BPZ + \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M} = \grave{s},BZ. \\ \overline{\tfrac{1}{2}\grave{s},N - \tfrac{1}{2}\grave{s},M} \times \grave{s},APZ + \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M} = \grave{s},AZ. \end{cases}$$

$$\text{Therefore } (\tfrac{1}{2}\grave{s},N - \tfrac{1}{2}\grave{s},M = )\frac{\grave{s},BZ - \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M}}{\grave{s},CPZ} = \frac{\grave{s},AZ - \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M}}{\grave{s},APZ}.$$

$$\text{And } (\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M = )\,\grave{s},BZ - \overline{\tfrac{1}{2}\grave{s},N - \tfrac{1}{2}\grave{s},M} \times \grave{s},BPZ = \grave{s},AZ - \overline{\tfrac{1}{2}\grave{s},N - \tfrac{1}{2}\grave{s},M} \times \grave{s},APZ.$$

$$\text{Hence } \grave{s},BZ \times \grave{s},APZ - \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M} \times \grave{s},APZ = \grave{s},AZ \times \grave{s},BPZ - \overline{\tfrac{1}{2}\grave{s},N + \tfrac{1}{2}\grave{s},M} \times \grave{s},BPZ.$$

Therefore $\grave{s},\mathrm{BZ}\times\grave{s},\mathrm{APZ}-\grave{s},\mathrm{AZ}\times\grave{s},\mathrm{BPZ}=\overline{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}\times\overline{\tfrac{1}{2}\grave{s},\mathrm{N}+\tfrac{1}{2}\grave{s},\mathrm{M}}.$

Again $\grave{s},\mathrm{AZ}-\grave{s},\mathrm{BZ}=\overline{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}\times\overline{\tfrac{1}{2}\grave{s},\mathrm{N}-\tfrac{1}{2}\grave{s},\mathrm{M}}.$

$$\left.\begin{array}{ll}\text{Therefore} & \tfrac{1}{2}\grave{s},\mathrm{N}+\tfrac{1}{2}\grave{s},\mathrm{M}=\dfrac{\grave{s},\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-\grave{s},\mathrm{APZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s}\,\mathrm{BPZ}.}\\ \text{And} & \tfrac{1}{2}\grave{s},\mathrm{N}-\tfrac{1}{2}\grave{s},\mathrm{M}=\dfrac{\grave{s},\mathrm{AZ}-\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}\end{array}\right\}\text{Hence IV. 216.}$$

$$\grave{s},\mathrm{M}=\left(\frac{\grave{s},\mathrm{AZ}+\grave{s},\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-\grave{s},\mathrm{BZ}-\grave{s},\mathrm{APZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}=\right)\frac{\overline{1+\grave{s},\mathrm{BPZ}}\times\grave{s},\mathrm{AZ}-\overline{1+\grave{s},\mathrm{APZ}}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s}\text{‘}\mathrm{BPZ}}.$$

$$\grave{s},\mathrm{N}=\left(\frac{\grave{s},\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-\grave{s},\mathrm{AZ}-\grave{s},\mathrm{APZ}\times\grave{s},\mathrm{BZ}+\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}=\right)\frac{\overline{1-\grave{s},\mathrm{BPZ}}\times\grave{s},\mathrm{AZ}-\overline{1-\grave{s},\mathrm{APZ}}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}.$$

$$\text{Or, } s\mathrm{M}=\left(\frac{\grave{v},\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-\grave{v},\mathrm{APZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}=\right)\frac{2\grave{s}\grave{s}\tfrac{1}{2}\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-2\grave{s}\grave{s},\tfrac{1}{2}\mathrm{APZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}.$$

$$\text{And } \grave{s},\mathrm{N}=\left(\frac{v,\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-v,\mathrm{ABZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}=\right)\frac{2ss,\tfrac{1}{2}\mathrm{BPZ}\times\grave{s},\mathrm{AZ}-2ss,\tfrac{1}{2}\mathrm{APZ}\times\grave{s},\mathrm{BZ}}{\grave{s},\mathrm{APZ}-\grave{s},\mathrm{BPZ}}.$$

Here BPZ = 57° $57\frac{1}{4}'$, its $\frac{1}{2}$ = 28° $58\frac{5}{8}'$; APZ = 42° $17\frac{1}{4}'$, its $\frac{1}{2}$ = 21° $8\frac{5}{8}'$. BZ = 54°; AZ = 44°; $\grave{s}$,BPZ = 0,53060; $\grave{s}$,APZ = 0,73978, their difference = 0,20918.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2L$\grave{s}\grave{s}$, | 28° $58\frac{5}{8}'$ | | 0,30103<br>9,88384 | 2E$ss$, | 28° $58\frac{5}{8}'$ | | 0,30103<br>9,37050 |
| L$\grave{s}$, | 44 | | 9,85693 | L$\grave{s}$, | 44 | | 9,85693 |
| L,A | 1,10104 | | 10,04180 | L,$a$ | 0,33764 | | 9,52846 |
| 2L$\grave{s}\grave{s}$, | 21° $8\frac{5}{8}'$ | | 0,30103<br>9,93946 | 2L$ss$, | 21° $8\frac{5}{8}'$ | | 0,30103<br>9,11438 |
| L$\grave{s}$, | 54 | | 9,76922 | L$\grave{s}$, | 54 | | 9,76922 |
| l,B | 1,02260 | | 10,00971 | L,$b$ | 0,15298 | | 9,18463 |
| L,$\overline{\mathrm{A}-\mathrm{B}}$ | 0.07844 | | 8,89454 | L,$\overline{a-b}$ | 0,18466 | | 9,26637 |
| L$\grave{s}$,APZ—$\grave{s}$,BPZ | | 0,20918 | 9,32052 | L$\grave{s}$,APZ—$s$,BPZ | | 0,20918 | 9,32052 |
| l$\grave{s}$,M | 112° $1\frac{1}{2}'$ | | 9,57402 | L$\grave{s}$,N | ·28° 0′ | | 9,94585 |

Hence the latitude is 48° 00′ N.; declination 19° 59′, which anſwers to the 20th of May.

199. And hence is readily derived the inveſtigation of that method, publiſhed in the year 1759, and then uſed by ſome for finding the true latitude at ſea, by knowing the latitude by account (or dead reckoning), the Sun's declination, two altitudes of the Sun, and the time between the obſervations. Thus. See the laſt figure.

Let $M$ and $N$ repreſent the half ſum and half diff. of the times from noon; $W$ and $V$, the half ſum and half diff. of the two altitudes.
$AB$ the diff. of the co-ſines of the times from noon, to the radius $Em$.
$AD$ the diff. of the ſines of the altitudes.
$\angle BAD$ repreſents the latitude; $Em$ the co-ſ. of the declination.

Now (197) $s,M \times 2s,N = AB$ reduced to the rad. $\frac{1}{2}Qq$; and $\grave{s},W \times 2s,V = AD$. But in the triangle $ABD$.

$$\grave{s},\text{lat.} : R :: AD : AB = \frac{1}{\grave{s},\text{lat.}} \times AD, \text{ to rad. } Em.$$

And $\grave{s},\text{decl.} : R :: AB : AB \times \frac{1}{\grave{s}\,\text{decl.}} = \frac{1}{\grave{s},\text{decl.}} \times \frac{1}{\grave{s},\text{lat.}} \times AD = AB$ reduced to the radius $\frac{1}{2}Qq$.

$$\text{Then } s,M \times 2s,N = \frac{1}{\grave{s},\text{decl.}} \times \frac{1}{\grave{s},\text{lat.}} \times \grave{s},W \times 2s,V.$$

$$\text{And } s,M = \frac{1}{\grave{s},\text{decl.}} \times \frac{1}{\grave{s},\text{lat.}} \times \frac{1}{s,N} \times \grave{s},W \times s,V.$$

Then $M + N = \angle mEb$, the time from noon at the leaſt altitude.
And $M - N = \angle mEa$, the time from noon at the greateſt altitude.
Hence the verſed ſines of the arcs $mb$ or $ma$, are known to rad. $\frac{1}{2}Qq$.
Now $R : Em :: v,ma : mA = Em \times v,ma$; or $mB = Em \times v,mb$.
And $R : s,mAd :: mA : md$ $(:: mB : me.)$
Then $Fm = Fd + dm$, or to $Fe + em$, is the ſine of the mer. alt.

Hence the two operations.

1ſt. $L\grave{}\grave{s},\text{decl.} + L\grave{}\grave{s},\text{lat.} + L\grave{}\grave{s},N + L\grave{s},W + Ls,V = Ls,M.$

Hence the times are known; viz. arcs $ma$, $mb$.

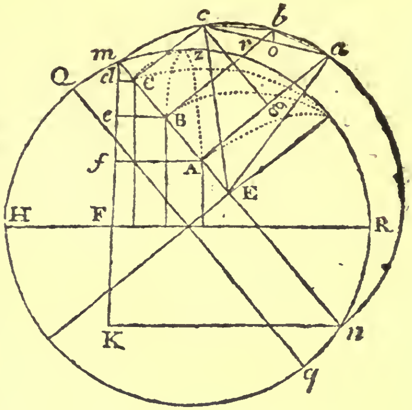
2d. $L\grave{s},\text{decl.} + L\grave{s},\text{lat.} + L,2 + 2Ls,\frac{1}{2}ma = L,md.$

Then by the merid. alt. and declination the latitude may be found. If this latitude and that aſſumed are the ſame, then the latitude by account, or dead reckoning, may be taken as the true latitude.

But if they differ, it is plain that the $\angle mAd$, the co-latitude, is leſs or greater than was aſſumed.

## 200. PROBLEM LIV.

*Given three deſcending (or aſcending) altitudes of the Sun, taken on the ſame day at unequal known intervals of time; thence to find thoſe times, the latitude of the place of obſervation, and the Sun's declination. Thus, ſuppoſe in July,* 1763, *the altitudes were* 54° 30′, 46°, *and* 36°; *and the intervals of time* 60 *m.* 55 *s. and* 62 *m.* 40 *s.*; *required the reſt.*



Let $man$ be the parallel of declination deſcribed on $m\,n$, and $a$, $b$, $c$, the places of the Sun when obſerved; $a\,\mathrm{A}$, $b\,\mathrm{B}$, and $c\,\mathrm{C}$ the ſines of the times from noon to the radius $\mathrm{E}m$; $m$, $n$, the places at noon and midnight; and let $f\mathrm{F}$, $e\mathrm{F}$, $d\mathrm{F}$, $m\mathrm{F}$, $\mathrm{FK}$, repreſent the ſines of the diſtances of thoſe places from the horizon HR. Draw $c\,g$ parallel to $\mathrm{AC}$; then if the $\angle gc\mathrm{E}$, which is equal to the $\angle m\mathrm{E}c$, the time from noon when the greateſt altitude was taken, could be found, the times from noon when the other two were taken would be known alſo. Now $c\,b$ being one interval, and $b\,a$ the other, $b\,c$ and $b\,a$, the chords of theſe arcs, will be known, as alſo $c\,a$, which is the chord of their ſum. Draw $b\,o$ perpendicular to $a\,c$. Now in the right angled $\triangle\, c\,b\,o$, the $\angle b\,c\,o = \frac{1}{2}$ the arc $b\,a$ (II. 120), and the ſide $c\,b =$ twice the ſine of $\frac{1}{2}$ the arc $b\,c$, conſequently $\mathrm{R} : s,b\,c\,o\ (= s\,\tfrac{1}{2}\,\text{arc}\ b\,a) :: b\,c\ (= 2\,s,\tfrac{1}{2}\,\text{arc}\ b\,c) : b\,o$; and $\mathrm{R} : s,c\,b\,o\ (\grave{s},\tfrac{1}{2}\,\text{arc}\ b\,a) :: b\,c\ (2\,s,\tfrac{1}{2}\,\text{arc}\ b\,c) : c\,o$. Now $d\,f$, $d\,e$, the diff. of the ſines of the altitudes, are known; and the lines $f\,d$, AC, $a\,c$, are cut proportionally in $e$, B and $r$; conſequently, $d\,f : d\,e :: (\mathrm{CA} : \mathrm{CB} ::)\ c\,a : c\,r$, and $c\,o - c\,r = r\,o$: In the triangle $b\,o\,r$, (III. 46) $r\,o : b\,o :: \mathrm{R} : \grave{t}, \angle\, r\,b\,o = \angle\, r\,cg$. Now $\angle\, a\,c\,\mathrm{E}$ ($=$compt. of $\frac{1}{2}$ whole interval $a\,c$) $- \angle\, r\,c\,g = \angle\, g\,c\,\mathrm{E} = \angle\, c\,\mathrm{E}\,m =$ time from noon when greateſt altitude was obſerved; therefore the time is known, as well as $cm$ the verſed ſine of that time from noon.

Again $\mathrm{R} : \grave{s},a\,c\,g :: a\,c : c\,g = \mathrm{AC}$.

And $\mathrm{AC} : mc :: f\,d : dm$. Then $\mathrm{F}d + dm = \mathrm{F}m$, the ſine of $\mathrm{H}m$.

And $\mathrm{AC} : mn :: f\,d : m\mathrm{K}$. Then $\mathrm{K}m - \mathrm{F}m = \mathrm{FK}$, the ſine of $\mathrm{R}n$.

This Problem has, at times, for near a century paſt, exerciſed the talents of many ingenious perſons, as well in Ruſſia, Germany, Holland, and France, as in England; perhaps, on account of its apparent uſe at ſea: And among the different ſolutions there ſeems none ſhorter or more intelligible, particularly to beginners, than that above; however, for the ſake of the more inquiſitive, another ſolution, which has been commonly given, is here ſubjoined.

201. In the laſt figure, the three triangles CPZ, BPZ, APZ, are thoſe concerned; in which the ſame two ſides are common in each.

$$\left.\begin{array}{l}\grave{s},\mathrm{CPZ} \times s,\mathrm{PC} \times s,\mathrm{PZ} + \grave{s},\mathrm{PC} \times \grave{s},\mathrm{PZ} = \grave{s},\mathrm{ZC}\\ \grave{s},\mathrm{BPZ} \times s,\mathrm{PC} \times s,\mathrm{PZ} + \grave{s},\mathrm{PC} \times \grave{s},\mathrm{PZ} = \grave{s},\mathrm{ZB}\\ \underline{\grave{s},\mathrm{APZ} \times s,\mathrm{PZ} \times s,\mathrm{PC} + \grave{s},\mathrm{PC} \times \grave{s},\mathrm{PZ} = \grave{s},\mathrm{ZA}}\end{array}\right\}\text{by IV. 239.}$$

Then $\grave{s},\mathrm{CPZ} - \grave{s},\mathrm{BPZ} \times s,\mathrm{PC} \times s,\mathrm{PZ} = (\grave{s},\mathrm{ZC} - \grave{s},\mathrm{ZB} =)\ d$. (II. 48.)

And $\grave{s},\mathrm{CPZ} - \grave{s},\mathrm{APZ} \times s,\mathrm{PC} \times s,\mathrm{PZ} = (\grave{s},\mathrm{ZC} - \grave{s},\mathrm{ZA} =)\ \mathrm{D}$.

Hence $\mathrm{D} \times \grave{s},\mathrm{CPZ} - \grave{s},\mathrm{BPZ} = d \times \grave{s},\mathrm{CPZ} - \grave{s},\mathrm{APZ}$. (II. 147)

Then $\mathrm{D} \times \grave{s},\mathrm{CPZ} - d \times \grave{s},\mathrm{CPZ} = \mathrm{D} \times \grave{s},\mathrm{BPZ} - d \times \grave{s},\mathrm{APZ}$.

Or, $\mathrm{D} \times s^{\backprime},\mathrm{CPZ} - d \times s^{\backprime},\mathrm{CPZ}\ (= \overline{\mathrm{D}-d} \times s^{\backprime},\mathrm{CPZ}) = \mathrm{D} \times s^{\backprime},\mathrm{BPZ} - d \times s^{\backprime},\mathrm{APZ}$.
But, $\mathrm{BPZ} = \mathrm{CPZ} + \mathrm{BPC}$, and $\mathrm{APZ} = \mathrm{CPZ} + \mathrm{APC}$.

Conſequently, $\left\{\begin{array}{l} s^{\backprime},\mathrm{BPZ} = s^{\backprime},\mathrm{CPZ} \times s^{\backprime},\mathrm{BPC} - s,\mathrm{CPZ} \times s,\mathrm{BPC}. \\ s^{\backprime},\mathrm{APZ} = s^{\backprime},\mathrm{CPZ} \times s^{\backprime},\mathrm{APC} - s^{\backprime},\mathrm{CPZ} \times s,\mathrm{APC}. \end{array}\right\}$ (IV. 216.)

Hence $\overline{\mathrm{D}-d} \times s^{\backprime}$. CPZ =. $\left\{\begin{array}{l} \mathrm{D} \times s^{\backprime},\mathrm{CPZ} \times s^{\backprime},\mathrm{BPC} - \mathrm{D} \times s,\mathrm{CPZ} \times s,\mathrm{BPC} - \\ d \times s^{\backprime},\mathrm{CPZ} \times s^{\backprime},\mathrm{APC} + d \times s,\mathrm{CPZ} \times s,\mathrm{APC}. \end{array}\right\}$ by ſubſtitu.

Or, $\overline{\mathrm{D} - \mathrm{D} \times s^{\backprime},\mathrm{BPC} - d + d \times s^{\backprime},\mathrm{APC}} \times s^{\backprime},\mathrm{CPZ} = \overline{d \times s,\mathrm{APC} - \mathrm{D} \times \mathrm{S},\mathrm{BPC}} \times s,$ CPZ. Wherefore,

$$\frac{\mathrm{D} - \mathrm{D} \times s^{\backprime},\mathrm{BPC} \backsim d - d \times s^{\backprime},\mathrm{APC}}{d \times s,\mathrm{APC} \backsim \mathrm{D} \times s,\mathrm{BPC}.} \left( = \frac{\overline{1 - s^{\backprime},\mathrm{BPC}} \times \mathrm{D} \backsim \overline{1 - s^{\backprime},\mathrm{APC}} \times d}{d \times s,\mathrm{APC} \backsim \mathrm{D} \times s,\mathrm{BPC}} \right)$$

$$= \frac{v,\mathrm{BPC} \times \mathrm{D} \backsim v.\ \mathrm{APC} \times d}{s,\mathrm{APC} \times d \backsim s,\mathrm{BPC} \times \mathrm{D}} = \frac{s,\mathrm{CPZ}}{s,\mathrm{CPZ}} = t,\ \mathrm{CPZ},$$

the meaſure of the time from noon when the greateſt altitude was obſerved.

Here $\left.\begin{array}{l} \mathrm{F}d = s,54^{\circ}\ 30' = 0,81412 \\ \mathrm{F}e = s,46\ \ 00 = 0,71934 \\ \mathrm{F}f = s,36\ \ 00 = 0,58779 \end{array}\right\}$ Then $df = 0,22633$; $de = 0,09478$.

The arc $c\,b$, or ∠CPB, = 1 h. 0 m. 55 ſ. = 15° $13\frac{3}{4}'$; its $\frac{1}{2}$ = 7° $36\frac{7}{8}'$.
arc $a\,b$, or ∠BPA, = 1 h. 2 m. 40 ſ. = 15 40 ; its $\frac{1}{2}$ = 7 50.
arc $c\,a$, or ∠CPA, = 2 h. 3 m. 35 ſ. = 30 $53\frac{3}{4}$ ; its $\frac{1}{2}$ = 15 $26\frac{7}{8}$.

*To find the hour by the 1ſt method.*

| | | |
|---|---|---|
| Radius | 90° 00′ | 10,00000 |
| Is to $s$, $\frac{1}{2}$ $ba$ | 7 50 | 9,13447 |
| As $s$, $\frac{1}{2}bc$ | 7 $36\frac{7}{8}$ | 9.12224 |
| To $\frac{1}{2}$ $ob$ | ,01806 | 8,25671 |
| Radius | 90° 00′ | 10,00000 |
| Is to $s$, $\frac{1}{2}$ $bc$ | 7 $36\frac{4}{8}$ | 9,12224 |
| As $s^{\backprime}$ $\frac{1}{2}$ $ba$ | 7 50 | 9.99593 |
| To $\frac{1}{2}$ $co$ | ,13127 | 9.11817 |
| As $fd$ | 22633 A.C. | 0,64526 |
| Is to $de$ | ,09478 | 8,97672 |
| So is $\frac{1}{2}$ $ac$ ($s$, $\frac{1}{2}c\mathrm{E}a$) | 15° $26\frac{7}{8}'$ | 9.42547 |
| To $\frac{1}{2}$ $cr$ | ,11154 | 9,04745 |
| $\frac{1}{2}$ $co$ | ,13127 | |
| As $\frac{1}{2}$ $ro$ | ,01973 | 1,70480 |
| Is to $\frac{1}{2}$ $bo$ | ,01806 | 8,25671 |
| So is Rad. | 90 00 | 10,00000 |
| To $t^{\backprime}$, ∠$rbo$ | 47 $32\frac{1}{8}$ | 0,96151 |
| ∠$ac$E | 74 $33\frac{1}{8}$ | h m s |
| ∠$gc$E = ∠$m$E$c$ = 27 | 1 = | 1 48 4 |
| Time of 2d obſervation | | 2 48 59 |
| Time of 3d obſervation | | 3 51 39 |

*To find the hour by the 2d method.*

| | | |
|---|---|---|
| D (= $fd$) | ,22633 | 9,35474 |
| $s$, ∠BPC | 15° $13\frac{3}{4}'$ | 9,41943 |
| D × $s$,BPC | ,05945 | 8,77417 |
| $d$ (= $de$) | ,09478 | 8,97672 |
| $s$, ∠APC | 30° $53\frac{3}{4}'$ | 9,71052 |
| $d$ × $s$,APC | ,04867 | 8,68724 |
| D (= $fd$) | ,22633 | 9,35474 |
| $v$, ∠BPC | 15° $13\frac{3}{4}'$ | 8,54552 |
| D × $v$,BPC | ,00795 | 7,90026 |
| $d$ (= $de$) | ,09478 | 8,97672 |
| $v$,APC | 30° $53\frac{3}{4}'$ | 9,15198 |
| $d$ × $v$,APC | ,01345 | 8,12870 |
| D × $v$,BPC | ,00795 | |
| $d$ × $v$,APC − D × $v$,BPC | ,00550 | 17,74036 |
| D × $s$,BPC − $d$ × $s$,APC | ,01078 | 8,03262 |
| tang. | 27° $1\frac{3}{4}'$ | 9,70774 |
| Hence the h.fr.n. of 1ſt ob. is | | 1h 48′ 7″ |
| Of the ſecond obſervation | | 2 49 2 |
| Of the third obſervation | | 3 51 42 |

Conſequently from any two of theſe, with their correſponding altitudes the latitude and declination may be found by Problem 53.

Here the altitudes are ſuppoſed to be deſcending ones, or in the afternoon.

202. If the altitudes were taken at equal intervals of time; the two firſt proportions are uſeleſs: For $ab$ and $bc$ being equal, the point $o$ falls in the middle of the chord $ca$; and $bo$, the verſed ſine, is known.
Then $df : de :: (\mathrm{CA} : \mathrm{CB} ::)\ ca : cr$; and $co - cr = ro$.
And $bo : ro ::$ Rad. : $t,rbo, = acg$; and $at\mathrm{E} - acg = gc\mathrm{E} = c\mathrm{E}m$.
Hence the times from noon are known; and alſo $m\mathrm{C}$, the verſed ſine of $c\mathrm{E}m$.
Again, in $\triangle\ acg$. $\mathrm{R} : \grave{s},acg :: ac : cg = \mathrm{AC}$.
And $\mathrm{AC} : m\mathrm{C} :: fd : dm$. Then $\mathrm{F}d + dm = \mathrm{F}m$.
Alſo $\mathrm{AC} : mn :: fd : m\mathrm{K}$. Then $\mathrm{K}m - \mathrm{F}m = \mathrm{FK}$.

Here $d\mathrm{F} = 0,81412$, $e\mathrm{F} = 0,71934$; $f\mathrm{F} = 0,58779$; $df = 022633$; $de = 0,09478$.
Alſo, arc $cb = 1$ h. 1 m. $47\frac{1}{2}$ ſ. $= 15^\circ\ 26\frac{7}{8}'$; $co = 0,26636$; $bo = 0,03613$.
Then $df : de :: ca : cr = 0,22309$; and $co - cr = 0,04327$.
And $bo : ro ::$ Rad. : $t,rbo = 50^\circ\ 8\frac{3}{8}'$ — ; then $74^\circ\ 33'\frac{1}{8} - 50^\circ\ 8'\frac{3}{8} = 24^\circ\ 24\frac{3}{4}'$.
h m s
Then $24^\circ\ 24\frac{3}{4}' =$ 1 37 39 from noon at firſt alt.
And $24^\circ\ 24\frac{3}{4}' + 15^\circ\ 26\frac{7}{8} = 39^\circ\ 51\frac{5}{8} = 2$ 39 $26\frac{1}{2}$ from noon at 2d alt.
Alſo $39\ 51\frac{5}{8} + 15\ 26\frac{7}{8} = 55\ 18\frac{1}{2} = 3$ 41 14 from noon at 3d alt.
Here the altitudes are ſuppoſed to be aſcending ones, or in the forenoon.
Again, rad. : $s,cag :: ac : \mathrm{AC} = 0,34142$.
And $\mathrm{AC} : m\mathrm{C} :: fd : dm = 0,05927$; then $m\mathrm{F} = 0,87338 = s$, $60^\circ\ 51'$.
Alſo $\mathrm{AC} : mn :: fd : m\mathrm{K} = 1,32581$; then $\mathrm{FK} = 0,45243 = s$, $26^\circ\ 54'$.
Hence the latitude is $46^\circ\ 7\frac{1}{2}$ N. and the declin. $16^\circ\ 58\frac{1}{2}'$; or on May 7th.

203. But the operation in this caſe may be much contracted. Thus, ſince $(df : de :: ca :)\ cr = \frac{de \times ca}{df}$; thence $ro = \left(co - \frac{de \times 2co}{df} =\right) \frac{\overline{df - 2de} \times co}{df}$.

And $(bo : ro :: \mathrm{R} :)\ t,rbo = \left(\frac{ro}{bo} = \frac{\mathrm{D} \times co}{df \times 2ss,\frac{1}{2}cb} =\right) \frac{\mathrm{D} \times \frac{1}{2}co}{df \times ss,\frac{1}{2}cb}$;
where $\mathrm{D} = df - 2de$.
But $\mathrm{R} \times v,cb = ss,\frac{1}{2}cb = \frac{1}{2}s,cb \times t,\frac{1}{2}cb$. (IV. 193, 195)

Then $t,rbo = \frac{\mathrm{D}}{df \times t,\frac{1}{2}cb}$; or $\mathrm{L}t,rbo = \grave{\mathrm{L}}t,\frac{1}{2}cb + \grave{\mathrm{L}},df + \mathrm{L},\mathrm{D}$.
Alſo $(\mathrm{R} : \grave{s},acg :: ac :)\ \mathrm{AC} = (\grave{s},acg \times ac =)\ \grave{s},acg \times 2s,cb$.
And $(\mathrm{AC} : mn :: fd :)\ m\mathrm{K} = \left(\frac{mn \times fd}{\mathrm{AC}} = \frac{2 \times fd}{s,acg \times 2s,cb} =\right) \frac{fd}{s,acg \times s,cb}$.
And $(\mathrm{AC} : m\mathrm{C} :: fd :)\ md = \left(\frac{m\mathrm{C} \times fd}{\mathrm{AC}} = \frac{fd \times 2ss,\frac{1}{2}m\mathrm{E}c}{\grave{s},acg \times 2s,cb} =\right) \frac{fd}{\grave{s},acg \times s,cb}$ $\times ss\frac{1}{2}m\mathrm{E}c$.
Hence $\grave{\mathrm{L}}\grave{s},acg + \grave{\mathrm{L}}s,cb + \mathrm{L},fd = \mathrm{L},m\mathrm{K}$.
And $\grave{\mathrm{L}}\grave{s},acg + \grave{\mathrm{L}}s,cb + \mathrm{L},fd + 2\mathrm{L}s,\frac{1}{2}m\mathrm{E}c = \mathrm{L},md$.

Here $df = 0,22633$; $2de = 0,18955$; and $\mathrm{D} = (df - 2de =)\ 0,03678$.
Alſo $\frac{1}{2}cb = 7^\circ\ 43'\ 26''$; and $ac\mathrm{E} = 74^\circ\ 33\frac{1}{8}'$.

$\mathrm{L},\grave{t},$

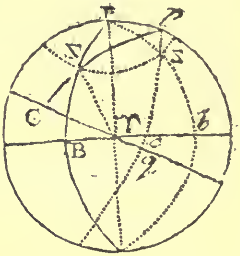
| | | | |
|---|---|---|---|
| L,t‘, 7° 43′ 26″ | 0,86764 | L‘s‘,acg = 50° 09′ | 0,19329 |
| L‘,df = 0,22633 | 0,64526 | L‘s,bc = 15 26$\frac{7}{8}$ | 0,57452 |
| L,D = 0,03678 | 8,56561 | L,fd = 22633 | 9,3547 |
| L,t,rbc = 50° 09′ | 10,07851 | L,mK, = 1,32602 | 10,12255 |
| acE = 74 33$\frac{1}{8}$ | | 2Ls,$\frac{1}{2}$mEc = 12° 12$\frac{1}{16}$′ | 8,64998 |
| mEc 24 24$\frac{1}{8}$; half is 12° 12$\frac{1}{16}$′ | | L,m d = 0,05923 | 8,77253 |

## PROBLEM LV.

204. Given the obliquity of the ecliptic, the latitude of a place, and the apparent time at that place. To find the longitude of the nonageſimal degree.

### CONSTRUCTION.

The given time applied to the ſun's right aſcenſion, gives the right aſcenſion of the mid-heaven.



Then let the primitive circle repreſent the ſolſtitial colure, where P, *p*, repreſent the poles of the equator ♈ B, and ecliptic ♈ C, the center ♈ being their interſection. In the equator apply the right aſcenſion of the mid-heaven from ♈ to B, and the circle PZB, being deſcribed, is that meridian, the interſection of which by a parallel of latitude deſcribed about P, gives Z, the place of the zenith; through Z deſcribe a circle of longitude *p*ZC, and the point C is the nonageſimal degree, and ♈C its longitude.

### COMPUTATION.

In the triangle P *p* Z. Given P *p* the obliquity of the ecliptic,
PZ the co-latitude.
∠ Z P *p* the right aſ. of the mid-heaven.
Required Z *p* —— {IV. 237, 238, and 144.}
∠ P *p* Z

### DETERMINATION.

When the right aſcenſion of the mid-heaven falls in the firſt quadrant, its quantity in degrees, increaſed by 90, gives the angle Z P *p*, and the acute angle P *p* Z is the complement of the longitude of the nonageſimal degree.

When in the ſecond quadrant, the ſaid right aſcenſion in degrees taken from 270, leaves the angle Z P *p*; and the acute angle P *p* Z, increaſed by 90 degrees, is the longitude of the nonageſimal degree.

When the ſaid right aſcenſion falls in the third quadrant, its degrees, taken from 270, leaves the angle Z P *p*, and the angle P *p* Z, increaſed by 90 degrees, is the longitude ſought.

When in the fourth quadrant, the ſaid right aſcenſion in degrees, leſſened by 270, leaves the angle z P *p*; and the ſupplement of the angle, P *p* z, ſo long as it continues to be obtuſe, being added to three right angles, or 270 degrees, gives the longitude of the nonageſimal degree; but after it becomes acute, its complement is the longitude required.

Note. In theſe determinations the latitude of the place is ſuppoſed to be north, and leſs than the diſtance of the tropic from the neareſt pole.

EXAMPLE. *At Greenwich, in latitude* 51° $28\frac{1}{3}'$ *N. the obliquity of the ecliptic being* 23° 28′: *What is the longitude of the nonageſimal degree on the* 14*th of May* 1780, *at* 1 *h.* 24*m.* 24*s. P. M. at* 3 *h.* 40 *m.* 2 *s. P. M. at* 13 *h.* 22 *m.* 26 *s. P. M. and at* 15 *h.* 38 *m.* 4 *s. P. M.?*

The ſeveral right aſcenſions of the mid-heaven are thus found:

| | h m s | h m s | h m s | h m s |
|---|---|---|---|---|
| 1780. May 14th, at | 1 24 24 | 3 40 2 | 13 22 26 | 15 38 4 |
| The Sun's right aſcen. | 3 27 36 | 3 27 58 | 3 29 34 | 3 29 56 |
| Right aſc. mid. hea. in time | 4 52 0 | 7 8 0 | 16 52 0 | 19 8 0 |
| Right aſc. mid hea. in degrees | 73° 00′<br>90 00 | 107° 00′<br>270 00 | 253° 00′<br>270 00 | 287° 00′<br>270 00 |
| The angle z P *p*, or z P *p* | 163 00 | 163 00 | 17 00 | 17 00 |

The two following operations are wrought by IV. 237. and 238.

| | | | | |
|---|---|---|---|---|
| P Z | 38° $31\frac{1}{3}'$ | | | |
| P *p* | 23 28 | | | |
| Sum | 61 $59\frac{1}{3}$, half is | 30° $59\frac{2}{3}'$ | Ar. co. *s.* 0,28823 | Ar. co. *s̀.* 0,06691 |
| Diff. | 15 $3\frac{1}{3}$, half is | 7 $31\frac{2}{3}$ | *s.* 9,11729 | *s̀.* 9,99624 |
| ∠ z P *p* | 163 00, half is | 81 30 | *t̀.* 9,17450 | *t̀.* 9,17450 |
| Half diff. ∠s z and *p* | | 2 $10\frac{2}{3}$ | *t.* 8,58002 | |
| Half ſum | | 9 $49\frac{1}{3}$ | —— —— | —— *t.* 9,23765 |
| The angle *p* | | 11 59<br>90 00 | | |
| The difference | | 78 01 | is the long. nonag. in 1ſt quad. | |
| The ſum | | 101 59 | is the long. nonag. in 2d quad. | |

| | | | | |
|---|---|---|---|---|
| Sum | 61° $59\frac{1}{3}'$, half | 30° $59\frac{2}{3}'$ | Ar. co. *s.* 0,28823 | Ar. co. *s̀.* 0,06691 |
| Diff. | 15 $3\frac{1}{3}$, half | 7 $31\frac{2}{3}$ | *s.* 9,11729 | *s̀.* 9,99624 |
| ∠ z P *p* | 17 00, half | 8 30 | *t̀.* 10,82550 | *t̀.* 10,82550 |
| Half diff. ∠'s z and *p* | | 59 34 | *t.* 10,23102 | |
| Half ſum | | 82 38 | —— —— | —— *t.* 10,88865 |
| The angle at *p* | | 142 12, | its ſupplement is | 37° 48′ |
| | | 90 00 | | 270 00 |
| Long. nonag. in 3d qd. | | 232 12 | Long. nonag. in 4th qd. | 307 48 |

⁂ The altitude of the nonageſimal being equal to z *p*, the diſtance of the zenith from the pole of the ecliptic, it is found by the ſines of oppoſite ſides and angles in the ſpheric triangle z P *p*: that is, ſin. long. nonag. : co-ſ. lat. : : ſin. ∠ z P *p* : ſin. alt. nonageſimal.

## SECTION VII.

## *Of Practical Aſtronomy.*

### 205. DESCRIPTION and USE of ASTRONOMICAL INSTRUMENTS.

By PRACTICAL ASTRONOMY is meant the knowledge of obſerving the celeſtial bodies with reſpect to their poſition, and time of the year; and of deducing from thoſe obſervations, certain concluſions uſeful in calculating the time, when any propoſed poſition of thoſe bodies ſhall happen.

For this purpoſe the Aſtronomer, or Obſerver, ſhould have an obſervatory properly furniſhed.

An OBSERVATORY is a room, or place, conveniently ſituated, contrived, and furniſhed with proper aſtronomical inſtruments for obſerving the motions of the heavenly bodies: it ſhould have an uninterrupted view, from the zenith, down to (or even below) the horizon, at leaſt towards its cardinal points; and for this purpoſe that part of the roof which lies in the direction of the meridian, in particular, ſhould have moveable covers, which may be eaſily removed and put on again: by which means an inſtrument may be directed to any point of the heavens between the horizon and zenith, as well to the northward as ſouthward.

The furniture ſhould conſiſt of ſome, if not all, of the following inſtruments.

1ſt. A PENDULUM CLOCK for ſhewing equal time.

2d. An Achromatic REFRACTING TELESCOPE, or a REFLECTING ONE, of two feet at leaſt in length, for obſerving particular phænomena.

3d. A MICROMETER for meaſuring ſmall angular diſtances.

4th. An ASTRONOMICAL QUADRANT for obſerving meridian altitudes of the celeſtial bodies.

5th. A TRANSIT INSTRUMENT for obſerving objects as they paſs over the meridian.

6th. An EQUATORIAL SECTOR to obſerve angular diſtances of ſeveral degrees, and the differences of right aſcenſion and declination.

7th. An EQUAL ALTITUDE INSTRUMENT for finding when an object has the ſame altitude on both ſides of the meridian.

It is not intended to give in this work any other than a general account of theſe inſtruments, moſt of which have met with conſiderable improvements (if they were not contrived) by the late Mr. George Graham, F. R. S. one of the moſt eminent artiſts in mechanical contrivances that this, or any other nation has produced: thoſe readers who are curious to ſee a minute deſcription of ſuch, and other, inſtruments, together with their uſe fully exemplified, may conſult the ſecond volume of Dr. Smith's complete Treatiſe of Optics, Stone's Treatiſe of Mathematical Inſtruments, the Philoſophical Tranſactions, and the works of many writers who have treated on ſuch ſubjects,

206. *Of*

## 206. *Of the Pendulum Clock.*

A clock which ſhews time in hours, minutes, and ſeconds, ſhould be choſen; with which the obſerver, by hearing the beats of the pendulum, may count them by his ear, while his eye is employed on the motion of the celeſtial object he is obſerving.

Juſt before the object arrives at the poſition deſired, the Obſerver ſhould look on the clock and remark the time; ſuppoſe it $9^h\ 15^m\ 25^s$; then ſaying 25, 26, 27, 28, &c. reſponſive to the beats of the pendulum, till he ſees through the inſtrument the object arrived at the poſition expected, which ſuppoſe to happen when he ſays 38; he then writes down $9^h\ 15^m\ 38^s$ for the time of obſervation, annexing the year and day of the month.

If two perſons are concerned in making the obſervation, one may read the time audibly, while the other obſerves through the inſtrument, the Obſerver repeating the laſt ſecond read, when the deſired poſition happens.

## 207. *Of the Teleſcope.*

The REFRACTING TELESCOPE is an inſtrument with which almoſt every perſon is acquainted, eſpecially the marine gentlemen; it will therefore be ſufficient to remark here, that an *aſtronomical teleſcope* has only two convex glaſſes; viz. the eye-glaſs, or that which is uſed next to the eye; and one at the other end, uſually called the object-glaſs, which has much the longer focal diſtance: ſuch an inſtrument, although it inverts all objects, is yet as uſeful for viewing thoſe in the heavens, as if it ſhewed them erect; the Obſerver knowing that the motions are in an oppoſite direction to thoſe he ſees through this teleſcope: But the Achromatic Refracting Teleſcope, which has been lately invented by Mr. Dolland, has its object-glaſs compounded of three glaſſes, and combined with two eye-glaſſes placed near each other. This inſtrument, which ſhews objects in their true poſition, need not exceed three feet and a half in length.

The REFLECTING TELESCOPE, as is generally well known, ſhews objects in their true poſitions; and as it is much ſhorter than the old refracter, it is therefore in much greater eſteem by ſome.

A *teleſcope*, uſed in aſtronomical obſervations, ſhould have a metal frame fixed in the focus of its object-glaſs, carrying fine ſilver wires ſtretched at right angles to one another; one of them is to be vertical, and the other horizontal; the interſection of thoſe wires ought to be exactly in the middle of the focus of the object-glaſs; a line paſſing through this interſection and the center of the object-glaſs, is called the line of ſight, or line of *collimation.*

## 208. *Of the Micrometer.*

A MICROMETER is an inſtrument uſed to meaſure ſmall angular diſtances by being placed in the focus of a teleſcope. This is effected by turning a ſcrew, which moves a fine wire in a poſition parallel to itſelf, and alſo parallel to a fixed wire; both being in a plane at right angles to the line of collimation: the diſtance of theſe parallel wires is meaſured by the number of turns the ſcrew has taken to cauſe their receſs; which number of turns is ſhewn on a graduated circular plate (like that of a clock) by an index, or hand, which revolves by the turning of the ſcrew: now the diviſions on the plate, anſwering to a known angle or arc intercepted between the parallel wires, being known by experiment, any other diſtance, to which the wires can recede, may be known by proportion; and ſo a table of angles anſwering to every diviſion on the circular plate may be formed, by which the obſerved angles will be readily known.

Thus in obſerving the diameter of a planet; when the wires are removed ſo far aſunder, as to become parallel tangents at the ſame time to oppoſite points of the planet, the meaſure of the receſs of the wires will ſhew the diameter of the planet in minutes and ſeconds.

There is another micrometer publiſhed by the late very ingenious Mr. Dollond *, an account of which was given to the Royal Society by Mr. James Short, F. R. S. and publiſhed in the Philoſophical Tranſactions for the year 1753, which is thus.

Let a good circular object-glaſs be neatly cut into two ſemicircles; and each ſemicircle fitted in a metal frame, ſo that their diameters ſliding on one another (by the means of a ſcrew) may have their centers ſo brought together as to appear like one glaſs, and ſo form one image; or by their centers receding may form two images of the ſame object: it being a property of ſuch glaſſes, for any ſegment, to exhibit a perfect image of an object, although not ſo bright as the whole glaſs would give it.

Now proper ſcales being fitted to this inſtrument, to ſhew how far the centers recede, relative to the focal length of the glaſs, will alſo ſhew how far the two parts of the ſame object are aſunder relative to its diſtance from the object-glaſs; and conſequently give the angle under which the diſtance of the parts of that object are ſeen.

## 209. *Of the Aſtronomical Qudrant.*

An Astronomical Quadrant is an inſtrument in the form of a quarter of a circle, contained under two radii at right angles to one another, and an arch equal to one fourth part of the circumference of the circle, and conſiſts of the following parts.

1ſt. *Its frame.* This is uſually compoſed of iron or braſs bars, ſet at right angles to one another in as ſtrong and neat a manner as a workman can contrive, to preſerve the face of the inſtrument in the ſame plane, and be as little affected by heat and cold as is poſſible.

2d. *Its center.* This center, which is a very fine point, ſhould be contained in a ſeparate piece of work ſcrewed to the bars; and ſo contrived, that if the index, or teleſcope, by frequent motion in a length of time, ſhould become irregular in its rotation, by the parts wearing, a new collar and ſocket may be fitted to the firſt center, and the inſtrument reſtored to its original accuracy.

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* The firſt notion of ſuch a micrometer was given by *Roemer*, a Dane, in the year 1675; Mr. *Savery*, an Engliſhman, alſo thought of ſuch a contrivance, which he communicated to the Royal Society in the year 1743; Mr. *Bouguer*, a Frenchman, alſo propoſed it in the year 1748; and Mr. *Dollond*, an Engliſhman, publiſhed it in the year 1753: but the public are obliged to Mr. *Short* for putting the theory into execution, otherwiſe it might ſtill have continued only as an ingenious thought.

3d. *Its limb.* This is a braſs arch of about two inches broad, well fixed to the ſaid frame-work, and generally continued a little farther at each end than the extent of 90 degrees; the two perpendicular radii may be alſo covered with plates of braſs ſcrewed to them; and the whole face of the inſtrument is to be worked ſmooth, and brought into the ſame plane with the greateſt care.

4th. *Its diviſions.* The arcs of 60, 30, and 90 degrees, and alſo the intermediate degrees, together with ſuch ſubdiviſions as the ſize of the degrees will conveniently contain, are laid down by accurate methods well known to good workmen.

5th. *Its index or teleſcope.* This, which is uſually a braſs tube containing the proper glaſſes and croſs wires, is fixed near the object end to a braſs plate, a little above a circular hole, or ſocket, in the plate: this ſocket goes round a collar concentric to the center, and fixed to the center-piece: ſo that, although the axis of the teleſcope does not, as a radius, paſs through the center, yet it always keeps at the ſame diſtance from it in every poſition: to the eye-end of the tube is ſcrewed a flat plate, which ſlides along the limb with the teleſcope; this plate, called the VERNIER, contains certain diviſions, which, uſed with thoſe on the limb, give the angle to minutes or ſeconds of a degree, according to the ſize of the inſtrument: the beginning of the diviſions called the index, on the *Vernier*, is as far diſtant from the axis, or line of collimation, as the center is; and therefore the poſition of an object is given as truly, as if the line of collimation coincided with a radius.

6th. *Its pedeſtal.* This part, which ſhould, by its conſtruction, be very ſteady, may be either moveable or fixed: the moveable pedeſtal is commonly a ſtrong pillar ſtanding on a tripod, or three-footed ſtand; with holes through each foot, either to ſcrew them to a floor, or to pin them to the ground: the fixed pedeſtal may be either a ſtrong timber frame, or the wall of the obſervatory, or a ſtone ſhaft built from the ground through the middle of the floor of the obſervatory. On the top of the pillar, of either ſort of pedeſtal, may be fixed a piece of machinery called the *arm*, which is attached by ſcrews to the middle of the plane of the quadrant, on the under ſide. The arm is contrived to give to the inſtrument, either an horizontal, vertical, or oblique motion; which motions ſhould be ſteady, and free from jerks, or ſhakes: but when a wall, or ſtone ſhaft, is uſed as the ſupporter, the quadrant is then fixed to the wall, or ſhaft (without its arm attached) and is called a MURAL ARCH; its plane is adjuſted to that of the meridian, and this is the beſt method of fixing the quadrant for taking the meridian altitude of the ſtars, or planets.

7th. *Its plummet.* This is a ſufficient ball, or weight, hanging to one end of a very fine ſilver wire, the upper end being fixed in the radius continued above the center. Now when the face of the quadrant is ſet in the plane of an azimuth circle, one of its radii is brought into a vertical poſition by the help of the plummet, the wire being made to biſect the center-point and the diviſion of 90° on the arch; and to diſtinguiſh theſe biſections with accuracy, they are to be examined with a ſmall proſpect, or magnifying-glaſs: the ball ſhould hang freely in a veſſel of water to check its vibrations.

## 210. *Of the tranſit Inſtrument.*

This inſtrument conſiſts of a teleſcope fixed at right angles to an horizontal axis, which axis muſt be ſo ſupported, that the line of collimation of the teleſcope may move in the plane of the meridian.

The axis, to the middle of which the teleſcope is fixed, ſhould gradually taper towards its ends, and terminate in cylinders well turned and ſmoothed: and a proper balance is to be put on the tube, ſo that it may ſtand at any elevation when its axis reſts on the ſupporters.

Two upright poſts of wood or ſtone, firmly fixed at a proper diſtance, are to ſuſtain the ſupporters of this inſtrument: theſe ſupporters are two thick braſs plates, having well ſmoothed angular notches in their upper ends to receive the cylindrical arms of the axis: each of the notched plates are contrived to be moveable by a ſcrew, which ſlides them upon the ſurfaces of two other plates immoveably fixed to the two upright poſts; one plate moving in a vertical, and the other in an horizontal, direction, to adjuſt the teleſcope to the planes of the horizon and meridian: to the plane of the horizon, by a ſpirit level hung in a parallel poſition to the axis, and to the plane of the meridian in the following manner.

Obſerve the times by the clock when a circumpolar ſtar, ſeen through this inſtrument, tranſits both above and below the pole: and if the times of deſcribing the eaſtern and weſtern parts of its circuit are equal, the teleſcope is then in the plane of the meridian; otherwiſe, the notched plates muſt be gently moved till the time of the ſtar's revolution is biſected by both the upper and lower tranſits, taking care at the ſame time that the axis remains perfectly horizontal.

When the teleſcope is thus adjuſted, a mark muſt be ſet at a conſiderable diſtance (the greater the better) in the horizontal direction of the interſection of the croſs-wires, and in a place where it can be illuminated in the night-time by a lanthorn hanging near it; which mark being on a fixed object, will ſerve at all times afterwards to examine the poſition of the teleſcope by, the axis of the inſtrument being firſt adjuſted by means of the level.

## 211. *To adjuſt the Clock by the Sun's Tranſit over the Meridian.*

Note the times by the clock, when the preceding and following edges of the ſun's limb touch the croſs wires: the difference between the middle time and 12 hours, ſhews how much the mean, or time by the clock, is faſter or ſlower than the apparent, or ſolar time for that day; to which the equation of time being applied, will ſhew the time of mean noon for that day, by which the clock may be adjuſted.

## 212. *Of the Equatorial Sector.*

This is an inſtrument contrived for finding the difference in right aſcenſion and declination between two objects, the diſtance of which is too great to be obſerved by means of a micrometer. It conſiſts of the following particulars.

1ſt. A braſs plate called a ſector, formed like a T, having the ſhank (as a radius) of about $2\frac{1}{2}$ feet long, and 2 inches broad, and the croſs piece (as an arch) of about 6 inches long, and $1\frac{1}{2}$ inch broad; upon which, with a radius of 30 inches, is deſcribed an arch of 10 degrees, each being ſubdivided into as ſmall parts as are convenient.

2d. Round a ſmall cylinder, containing the center of this arch, and fixed in the ſhank, moves a plate of braſs, to which is fixed a teleſcope, having its line of collimation parallel to the plane of the ſector, and paſſing through the center of the arch and the index of a *Vernier*'s dividing plate, which ſlides on the arch, and is fixed to the eye end of the teleſcope. This plate, with the teleſcope and Vernier, are moved on the cylinder, by means of a long ſcrew which is at the back of the arch, and communicates with the Vernier through a ſlit cut in the braſs work, parallel to the divided arch.

3d. A circular braſs plate, of 5 inches diameter, round the center of which there moves a braſs croſs, which has the oppoſite ends of one bar turned up perpendicularly about 3 inches. Theſe ſerve as ſupporters to the ſector, and are ſcrewed to the back of its radius, ſo that the plane of the ſector is parallel to the plane of the circular plate, and revolves round the center of that plate in this parallel poſition.

4th. A flat axis of 18 inches long is ſcrewed to the back of the circular plate, along one of its diameters; ſo that the axis is parallel to the plane of the ſector: the whole inſtrument is ſupported on a proper pedeſtal, in ſuch a manner that the ſaid axis is parallel to the axis of the earth; and proper contrivances are annexed for fixing it in that poſition.

Now the inſtrument, thus ſupported, can revolve round its axis, parallel to the earth's axis, with a motion like that of the ſtars; the plane of the ſector being always parallel to the plane of ſome hour circle, and conſequently every point of the teleſcope deſcribes a parallel of declination: and if the ſector be turned round the joint of the circular plate, its graduated arch may be brought parallel to an hour circle; and conſequently any two ſtars, between which the difference of declination is not greater than the number of degrees in that arch, may be obſerved by the inſtrument.

213. *To obſerve their paſſage.* Direct the teleſcope to the preceding ſtar, and fix the plane of the ſector a little to the weſtward of it; move the teleſcope by the ſcrew, and obſerve the time ſhewn by the clock at the tranſit of each ſtar over the croſs wires, and alſo the diviſion ſhewn by the index; then is the difference of the arches the difference of declination; and that of the times ſhews the difference of right aſcenſion of thoſe ſtars.

## 214. *Of the Equal-Altitude Inſtrument.*

An Equal-Altitude Instrument is that uſed to obſerve a celeſtial object, when it has the ſame altitude on both the eaſt and weſt ſides of the meridian, or in the morning and afternoon; and conſiſts of a teleſcope of about 30 inches long (with 2 vertical, and 3 or 5 horizontal, wires in its focus) ſupported on the end of an iron bar, or axis, of 30 inches long, and about an inch in diameter. The axis is ſuſtained in a vertical poſition by paſſing through a hole in the upper end of a braſs box, whilſt its lower end ſupports the lower point of the axis. The box, which is about 21 inches long, with ends about 4 inches ſquare, has only two ſides, which are fixed at right angles to each other. To one of theſe ſides are fixed four flat arms, with a hole in each, by which the box is fixed in a vertical poſition to an upright poſt with ſcrews. On the lower end of the box lies a braſs plate, which ſlides in grooves, and can be moved gently backwards or forwards by means of a ſcrew. In this plate a fine hole is punched to receive the ſmooth conical point, which the lower end of the axis is formed into. On the upper end of the box are two plates, which ſlide alſo in grooves; and, by the means of ſcrews, can be moved gently ſideways, till their angular notches embrace the axis; which, in this part, is made perfectly cylindrical, and very ſmooth.

To the upper part of the axis is fixed, by its radius, a braſs ſextant (or arch of 60°, to a radius of ſeven or eight inches) with the arch downwards, ſo that the center is juſt above the top of the axis: alſo a ſpirit level is fixed at right angles acroſs the axis, juſt under the arch, ſo as to be clear of the upper end of the box.

To the under part of the teleſcope is fixed a braſs ſemicircle, of the ſame radius with the ſextant, both arches having a common center-pin. In the ſemicircle is a groove cut through the plate parallel to its limb, to receive two ſcrew-pins, which go into the ſextantal arch near its ends; by theſe ſcrew-pins the two arches may be preſſed cloſe, and the teleſcope fixed in any deſired elevation; which might be nearly aſcertained, by graduating the ſemicircle, and putting a Vernier's ſcale on the ſextant.

*To uſe the Inſtrument.* Fix the box to the poſt, put the axis into the box, letting the conical point drop into the punched hole, ſcrew on the level, and annex the teleſcope, obſerving to inſert the center and arch pins; then, by the help of the ſcrew-plates at the bottom and top ends of the box, correct the vertical poſition of the axis, ſo that the ſame end of the air-bubble in the level may ſtand at the ſame point throughout the whole revolution of the axis, which will thereby be known to be then truely vertical, ſo that the teleſcope will deſcribe a parallel of altitude: direct the tube to the ſun, or ſtar, and fix it at the deſired elevation by preſſing the two arches together with the two ſcrew pins.

Some inſtruments have been contrived to anſwer both kinds of obſervation; viz. either a tranſit, or equal altitudes.

## 215. *To adjuſt the Clock by equal altitudes of the Sun.*

Having rectified the inſtrument by the level, and being provided with a piece of tranſparently coloured or ſmoked glaſs to preſerve the eye; then at any convenient time from about 6 to 3 hours before noon, direct the teleſcope to the ſun, and fix it by the arch, ſo that the whole body of the ſun ſhall be above the upper wire (the aſcent of the ſun appearing through the teleſcope as a deſcent): mark the times ſhewn by the clock, when the preceding edge of the ſun touches each of the wires; and alſo when the following edge touches thoſe wires, writing down thoſe times; the inſtrument being turned horizontally on its axis to follow the Sun, and keep his center in the middle of the teleſcope between the vertical wires. About the ſame time after noon (taking care to be early enough) turn the inſtrument on its vertical axis, the teleſcope remaining fixed at the ſame elevation as in the morning, and rectifying its horizontal ſituation by the level, obſerve the Sun in its deſcent, which through the teleſcope apparently aſcends, and write down the times, when the preceding edge touches each wire, and alſo when they are touched by the following edge, keeping the Sun in the middle of the teleſcope; and the ſets of obſervations are made.

There are as many ſets of obſervations, as there are horizontal wires: for the fore and afternoon contacts of the ſame edge of the Sun with the ſame wire, make one ſet; and the ſame edge which precedes in the forenoon, follows in the afternoon; and that which follows in the forenoon, precedes in the afternoon; therefore the

$$\left.\begin{array}{l}\text{1ſt,}\\ \text{2d,}\\ \text{3d, \&c.}\end{array}\right\}\text{A. M. preceding, and the}\left\{\begin{array}{l}\text{laſt,}\\ \text{laſt but one,}\\ \text{laſt but 2, \&c.}\end{array}\right\}\text{P. M. following,}$$

make a ſet.

$$\left.\begin{array}{l}\text{1ſt,}\\ \text{2d,}\\ \text{3d, \&c.}\end{array}\right\}\text{A. M. following, and the}\left\{\begin{array}{l}\text{laſt,}\\ \text{laſt but one,}\\ \text{laſt but 2, \&c.}\end{array}\right\}\text{P. M. preceding,}$$

make a ſet.

Then to each ſet, or pair of obſervations, find the middle time, which added to the time of the morning obſervation, gives the time ſhewn by the clock when the Sun was on the meridian, if the obſervations were made within two or three days of the ſolſtice, when the Sun's declination would not ſenſibly alter between the fore and afternoon obſervations: but on other days, this time muſt be corrected, by applying an equation to it, ſhewing the alteration in time, ariſing from the alteration in declination between the fore and afternoon obſervations.

The time, by the clock, of the ſolar or apparent noon being thus obtained, the time of the mean noon may be had by applying the proper equation of time.

When the time of noon is ſought from two or more pairs of obſervations, if they give different times, it is beſt to take the medium between them, which is found by dividing the ſum of all the times by their number.

## 216. PROBLEM LV.

*Given the latitude, the declination of the Sun, and interval of time between the Sun's having equal altitudes before and after noon, to find the diſtance from noon of the middle point of time between the obſervations.*

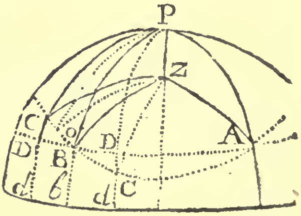
1ſt. Find the change made in the Sun's declination during the interval between the obſervations; which will nearly bear the ſame proportion to the change made between the noon of the day, on which the obſervations are made, and the noon of the day immediately preceeding or following, as the interval of time between the obſervations to 24 hours.

2dly. Add the co-tangent of the latitude to the co-ſine of half the interval of time reduced to degrees and minutes of the equator; the ſum, rejecting the radius, is the tangent of an arc to be taken leſs than a quadrant, when the interval of time is leſs than twelve hours, and greater than a quadrant, ſhould the interval of time exceed twelve hours.

3dly. Add together the arithmetical complements of the ſine of this arc, and of the ſine of the Sun's diſtance from the pole at noon, the logarithmic ſine of the difference of theſe arcs, the logarithmic co-tangent of half the interval of time in degrees, and the logarithm of half the change in the Sun's declination during the interval between the obſervations; the ſum, rejecting twice the radius, is the logarithm of an arc, which, divided by 15, gives the diſtance of the middle point between the obſervations from noon, in ſeconds of time.

4thly. When the Sun's diſtance from the elevated pole increaſes, this middle point of time precedes noon, otherwiſe it falls beyond.

### DEMONSTRATION.



Let P be the pole of the equator *bd*, Z the zenith, A the morning place, C the afternoon place, ABD a parallel of declination, ABC a parallel of altitude.

Then the afternoon hour angle, ZPC, differs from the morning hour angle, ZPA, by the hour angle BPC, the points A and B having the ſame declination and diſtance from the zenith; and the arc P*o* biſecting the ∠BPC, the ∠ZP*o* will be half the interval of time, which being increaſed or diminiſhed by half the ∠BPC, will give the poſition of the meridian to A or C; alſo P*o* will be the Sun's diſtance from the pole at noon.

Now here the difference between PB, P*o*, PC being but ſmall, the ∠BP*o* will be to the difference between P*o* and PB, or PC, that is, half PB ∽ PC, nearly as $t^{\backprime}, Poz$ to $s, Po$ *.

Again, in the triangle *o*PZ, an arc M being taken, that as rad. : $s^{\backprime}, ZPo$ : $t, PZ$ : $t, M$ the arc M to be leſs than a quadrant, when the ∠ZP*o* is acute, but greater, ſhould the ∠ZP*o* be obtuſe; then (IV. 123)
$\overline{s, Po \backsim M} : s, M :: t^{\backprime}, Poz : t^{\backprime}, ZPo$, and conſequently
$\overline{s, Po \backsim M} \times t^{\backprime}, ZPo = s, M \times t^{\backprime}, Poz$.

$$\text{But } \angle BPo : \frac{PB \backsim PC}{2} :: t^{\backprime}, Poz : s, Po :: s, M \times t^{\backprime}, Poz : s, M \times s, Po;$$

therefore $\angle BPo : \frac{PB \backsim PC}{2} :: \overline{s, Po \backsim M} \times t^{\backprime}, ZPo : s, M \times s, Po$, conformably to the rule above laid down.

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[*] See *Coteſ. Æſtimat. Error. in Mixt. Math. Theorem* 23.

217.

## EXAMPLE.

In the latitude 50° N. on the 27th of October, 1780, the Sun was obſerved to have equal altitudes at 9 h. 11 m. 50 s. A. M. and at 2 h. 22 m. 22 s. P. M. by a clock adjuſted nearly to the true meaſure of time, to find what correction may be wanted to ſet this clock to the true hour of the day, the Sun's diſtance from the pole on the 27th day at noon being 163° 6′ 34″, and on the 28th 103° 26′ 38″.

Here the interval of time is 5 h. 10 m. 32 s. its half is 2 h. 35 m. 16 s. in degrees and minutes of the equator 38° 49′, and the difference in declination in one day is 20′ 4″.

Then 24 h. : 5 h. 10 m. 32 s. : : 20′ 4″ : 4′ 20″=260″, the alteration in declination, the half of which is 2′ 10″=130″.

Then

| | | | | | |
|---|---|---|---|---|---|
| Latitude | 50° | log. t̀, 9,92381 | 33° 10′ 32″ | L̀, s, | 0,26185 |
| ½ time = | 38° 49′ | log. s̀, 9,89162 | 103 06 34 | L̀ s, | 0,01147 |
| | | | 69 56 02 | log. s, | 9,97280 |
| | 33° 10′ 32″ | log. t, 9,81543 | 38 49 0 | log. t̀, | 10,09447 |
| | | | 130″ | log. | 2,11394 |
| | | | corr. = 285″ | log. | 2,45453 |
| | | | or 19 ſec. of time. | | |

Hence, for ſetting the clock to the true hour of the day, add the half interval of time 2 h. 35 m. 16 s. to 9 h. 11 m. 50 s.; the ſum 11 h. 47 m. 6 s. is the middle time between the obſervations, as noted by the clock.

And 11 h. 47 m. 6 s.+19 s. = 11 h. 47 m. 25 s. will be the time pointed out by the clock, when the Sun paſſes the meridian, and ſhews the clock to be 12 m. 35 s. behind the Sun.

Though the clock ſhould not keep time with perfect exactneſs, yet if the deviation is but ſmall, the correction computed will not differ much from the truth; and the clock being examined again within a few days, will ſhew whether it keeps time truly, or moves too faſt or too ſlow, and its rate of going may be corrected accordingly.

218. The method here directed ſuppoſes the ſhip to be ſtationary: But the Abbé de la Caille propoſes a method of correcting a watch at ſea, even while the ſhip is in motion, by taking two equal altitudes of the Sun with a quadrant, one before and the other after noon.

His method is this. With the common altitude obſerved, together with the latitudes at the time of each obſervation, and the Sun's correct declination to thoſe times, the times from noon are to be computed at both obſervations, which times being applied to the two times of obſervation, give the reſpective times of noon: then the mean of the two noons being taken, will give the time ſhewn by the watch when it was the true midday.

Or. Half the difference of the computed noons being applied to either of them, will alſo give the true time of noon.

And although the latitudes uſed in the computations be ſomething erroneous, yet the altitudes being equal, the error in each of the computed diſtances from noon will be nearly the ſame, if the change in latitude and longitude between the obſervations be duly attended to.

## 219. *Of the Vernier's dividing plate.*

When the relative unit of any line is to be divided into many ſmall equal parts, thoſe parts may be too many to be conveniently introduced, or if introduced, they may be too cloſe to one another to be readily eſtimated; and on theſe accounts there has been a variety of methods contrived for eſtimating the aliquot parts of the ſmall diviſions, into which the relative unit of a line may be commodiouſly divided; among thoſe methods that is moſt juſtly preferred which was publiſhed by PETER VERNIER (a gentleman of Franche Comté) at Bruſſels, in the year 1631; and which, by ſome ſtrange fatality, is moſt unjuſtly, although commonly, called by the name of NONIUS: for *Nonius*'s method is not only very different from that of *Vernier*'s, but much leſs convenient.

*Vernier's method is derived from the following principle.*

If two equal right lines, or circular arcs A, B, are ſo divided, that the number of equal diviſions in B is one leſs than the number of equal diviſions of A; then will the exceſs of one diviſion of B above one diviſion of A be compounded of the ratios of one of A to A, and one of B to B.

For let A contain 11 parts; then one of A to A, is as 1 to 11; or $\frac{1}{11}$.

Let B contain 10 parts; then one of B to B, is as 1 to 10; or $\frac{1}{10}$.

Now $\frac{1}{10} - \frac{1}{11} = \left(\frac{1 \times 11}{10 \times 11} - \frac{1 \times 11}{11 \times 10}\right.$ (II. 148) $= \frac{11-10}{10 \times 11} = \frac{1}{10 \times 11} = \left.\right)$ $\frac{1}{10} \times \frac{1}{11}$.

Or. If B contains $n$ parts, and A is of $n+1$ parts;

Then $\frac{1}{n}$ is one part of B, and $\frac{1}{n+1}$ is one part of A.

And $\frac{1}{n} - \frac{1}{n+1} = \left(\frac{1 \times \overline{n+1}}{n \times \overline{n+1}} - \frac{1 \times n}{\overline{n+1} \times n}\right.$ (II. 148) $= \frac{n+1-n}{n \times \overline{n+1}} = \frac{1}{n \times \overline{n+1}} = \left.\right)$ $\frac{1}{n} \times \frac{1}{n+1}$.

Or thus. Let A and B be unequal right lines, or circular arcs; and let any part of A, conſidered as the relative unit, be divided into $n$ parts; and a part of B, equal to $m + 1$ parts of A, be divided into $m$ parts: then will $\frac{1}{m}$th of B $=$ $\frac{1}{n}$th of A $=$ $\frac{1}{m}$th of B $\times$ $\frac{1}{n}$th of A.

But $n$ parts of A : 1 unit of A : : $m + 1$ parts of A : $\frac{m+1}{n}$ units of A.

But $m$ parts of B $=$ ($m+1$ parts of A $=$) $\frac{m+1}{n}$ units of A.

Then $m$ parts of B : $\frac{m+1}{n}$ units of A : : 1 part of B : $\frac{m+1}{m \times n}$ units of A.

Therefore $\frac{m+1}{m \times n} - \frac{1}{n} = \left(\frac{m+1}{m \times n} - \frac{m \times 1}{n \times m} = \frac{m+1-m}{m \times n} = \frac{1}{n \times m} =\right)$ $\frac{1}{m} \times \frac{1}{n}$.

The moſt commodious diviſions, and their aliquot parts, into which the degrees on the circular limb of an inſtrument may be ſuppoſed to be divided, depend on the radius of that inſtrument.

Let R be the radius of a circle in inches; and a degree to be divided in $n$ parts, each degree being $\frac{1}{p}$th of an inch.

Now the circumference of a circle in parts of its diameter, 2R inches, is $3,1415926 \times 2\text{R}$ inches. (II. 197)

Then $360° : 3,1415926 \times 2\text{R} :: 1° : \frac{3,1415926}{360} \times 2\text{R}$ inches.

Or, $0,01745379 \times \text{R}$ is the length of one degree, in inches.
Or, $0,01745379 \times \text{R} \times p$ is the length of 1°, in $p$th parts of an inch.
But as every degree contains $n$ times ſuch parts,
Therefore $n = 0,01745379 \times \text{R} \times p$.

The moſt commodious perceptible diviſion is $\frac{1}{8}$ or $\frac{1}{10}$ of an inch.

EXAM. *Suppoſe an inſtrument of* 30 *inches radius: into how many convenient parts may each degree be divided? how many of thoſe parts are to go to the breadth of the Vernier, and to what parts of a degree may an obſervation be made by that inſtrument?*

Now $0,01745 \times \text{R} = 0,5236$ inches, the length of each degree.

And if $p$ be ſuppoſed about $\frac{1}{8}$ of an inch for one diviſion.

Then $0,5236 \times p = 4,188$, ſhews the number of ſuch parts in a degree.
But as this number muſt be an integer, let it be 4, each being 15′.
And let the breadth of the *Vernier* contain 31 of thoſe parts, or $7\frac{3}{4}$°, and be divided into 30 parts.

Here $n = \frac{1}{4}$; $m = \frac{1}{30}$; then $\frac{1}{4} \times \frac{1}{30} = \frac{1}{120}$ of a degree, or 30″,
Which is the leaſt part of a degree that inſtrument can ſhew.

If $n = \frac{1}{5}$, and $m = \frac{1}{36}$; then $\frac{1}{5} \times \frac{1}{36} = \frac{60}{5 \times 36}$ of a minute, or 20″.

220. The following table, taken as examples in the inſtruments commonly made from 3 inches to 8 feet radius, ſhews the diviſions of the limb to neareſt tenths of inches, ſo as to be an aliquot of 60's, and what parts of a degree may be eſtimated by the Vernier, it being divided into ſuch equal parts, and containing ſuch degrees, as their columns ſhew.

| Rad. inches | Parts in 2 deg. | Parts in Vernier. | Breadth of Ver. | Parts obſerved. | Rad, inches | Parts in a deg. | Parts in Vernier. | Breadth of Ver. | Parts obſerved. |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 15 | $15\frac{1}{4}°$ | 4′ 0″ | 30 | 5 | 30 | $7\frac{1}{2}°$ | 0′ 20″ |
| 6 | 1 | 20 | $20\frac{1}{4}$ | 3 0 | 36 | 6 | 30 | $5\frac{1}{4}$ | 0 20 |
| 9 | 2 | 20 | $10\frac{1}{4}$ | 1 30 | 42 | 8 | 30 | $3\frac{7}{8}$ | 0 15 |
| 12 | 2 | 24 | $12\frac{3}{4}$ | 1 15 | 48 | 9 | 40 | $4\frac{5}{9}$ | 0 10 |
| 15 | 3 | 20 | $6\frac{3}{4}$ | 1 0 | 60 | 10 | 36 | $3\frac{7}{10}$ | 0 10 |
| 18 | 3 | 30 | $10\frac{1}{4}$ | 0 40 | 72 | 12 | 30 | $2\frac{7}{12}$ | 0 10 |
| 21 | 4 | 30 | $7\frac{1}{4}$ | 0 30 | 84 | 15 | 40 | $2\frac{2}{3}$ | 0 6 |
| 24 | 4 | 36 | $9\frac{3}{4}$ | 0 25 | 96 | 15 | 60 | 4 | 0 4 |

By altering the number of diviſions, either in the degrees or in the Vernier, or in both, an angle can be obſerved to a different degree of accuracy. Thus to a radius of 30 inches, if a degree be divided into 12 parts, each being five minutes, and the breadth of the Vernier be 21 ſuch parts, or $1\frac{3}{4}°$, and divided into 20 parts, then $\frac{1}{12}\times\frac{1}{20}=\frac{1°}{240}=15''$: or taking the breadth of the Vernier of $2\frac{1}{12}°$, and divided into 30 parts; then $\frac{1}{12}\times\frac{1}{30}=\frac{1°}{360}$, or $10''$: Or $\frac{1}{12}\times\frac{1}{50}=\frac{1°}{600}=6''$; where the breadth of the Vernier is $4\frac{1}{4}°$.

## SECTION VIII.

## *Practical Aſtronomy.*

### The ELEMENTS of the EARTH'S MOTION.

221. By the theory of the Sun, or Earth, is meant the knowledge of all the requiſites, or elements, neceſſary for determining its place in the ecliptic at any propoſed time.

222. MEAN MOTION, or MEAN ANGULAR VELOCITY, is a motion made uniformly in the circumference of a circle, the center of motion being the center of that circle.

The mean motion of a planet is the degree and parts ſhewing its diſtance from the firſt point of Aries, reckoned in the order of the ſigns.

223. ANOMALY, or TRUE ANOMALY, is an angle made by two lines drawn from the center of motion, one to the Aphelion, or Apogee, and the other to the place of the revolving body, or planet: Or, ANOMALY is the angular diſtance of a planet from its Aphelion, the angular point being the center of motion.

224. MEAN ANOMALY is that made by an uniform circular motion about the center, and is the ſame as mean motion, beginning at the Aphelion.

225. Ex-

225. ECCENTRIC ANOMALY is an angular diſtance from the aphelion, determined in a circle on the tranſverſe axis by a normal to that axis, paſſing through the planet's place in its elliptical orbit.

226. The EQUATION OF THE CENTER, ſometimes called the *proſthaphæreſis*, is the diſtance between the mean and true anomalies.

227. The *motion of the equinoxes* is the ſame as the *preceſſion of the equinoxes*, which is backwards, or contrary to the order of the ſigns; by which the ſtars appear to have advanced forwards from the equinoctial point Aries: this motion is about 50 ſeconds of a degree in a year.

228. *The motion of the apſides* is a ſlow motion of the Earth's orbit around the Sun in the order of the ſigns; diſcovered by the apogeon changing its place among the fixed ſtars: this motion is found, by comparing diſtant obſervations together, to be about 16 ſeconds of a degree in a year, in reſpect to the fixed ſtars; and about 66 ſeconds $(=50''+16'')$ with reſpect to the equinoxes.

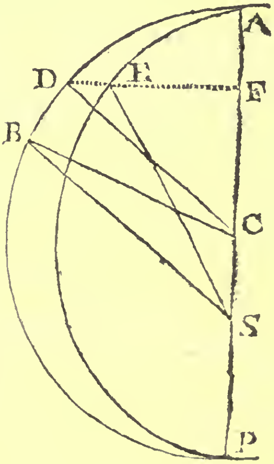
229. A TROPICAL or SOLAR YEAR is the time elapſed between two ſucceſſive paſſages of the Sun through the ſame Equinoctial or Solſtitial points of the ecliptic.

230. A SIDERIAL YEAR is the time the Sun takes between his departure from any fixed ſtar to his next return to that ſtar.

231. An ANOMALASTIC YEAR is the interval of time between two ſucceeding paſſages of the Sun through the ſame apſis.

232. By the annexed figure the foregoing articles may be eaſily comprehended.
On the line of the apſides AP deſcribe a circle ADP, called the excentric; and an ellipſis AEP for the Earth's orbit, having the excentricity CS.
Let S be the place of the Sun, C the center of the orbit, A the aphelion, P the perihelion; SA the aphelion, or apogeon diſtance; SP the perigeon diſtance.
Let E be a true place of the earth in its orbit; D a correſponding place in the excentric, in FE continued, normal to AP.
Let the $\angle$ ACB repreſent the mean anomaly; the $\angle$ ACD is the eccentric anomaly; and the $\angle$ ASE is the true anomaly; the difference between $\angle$ ACB and $\angle$ ASE is the equation of the center.

When the Earth is in the apſides, then B and E fall together in A and P, and here is no equation of the center, the mean and true anomalies being equal; but the greateſt equation of the center muſt be, when the Earth is at its mean diſtance from the Sun.



233. Obſervations ſhew, that in this age the Earth paſſes the apogee on the 30th of June, when its daily motion is $57' 12''$; and paſſes the perigee on the 30th of December, when the daily motion is $61' 12''$: and is at the mean diſtance about the 28th of March and 30th of September, when its daily motion is $59' 8''$.

234. PRO-

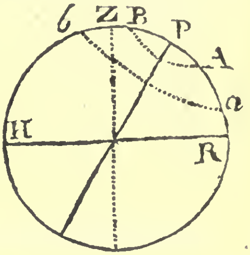
## 234. PROBLEM LVI.

### *To find the Latitude of a Place.*

Solution. Select a ſtar, the diſtance of which from the pole ſtar does not exceed 8 or 10 degrees; and obſervé with a quadrant the greateſt and leaſt meridional altitudes; then

If both obſervations are on the ſame ſide of the zenith;
Half the ſum of the *alts.* is the latitude, on the ſame ſide of the zenith.

If the obſervations are on different ſides of the zenith;
Half the difference of the altitudes is the co-latitude, on the ſame ſide of the zenith, with the leſſer altitude.



For let HZR be the meridian, HR the horizon, Z the zenith, RB, RA, two altitudes on the ſame ſide of the zenith; H*b*, R*a*, two altitudes on contrary ſides of the zenith.

Then, the arc AB, or *ab*, being biſected, will give P the poſition of the elevated pole.

For a ſtar is equally diſtant from P in its revolution.

Therefore PA = PB; or P*a* = P*b*; and RP equal to the latitude.

$$\text{Hence } RP = \left( RA + PA = \frac{2RA}{2} + \frac{2PA}{2} = \frac{RA + RA + AB}{2} = \right) \frac{RA + RB}{2}$$

$$\text{And } RP = \left( Ra + Pa = \frac{2Ra}{2} + \frac{2Pb}{2} = \frac{Ra + Ra + ab}{2} = \frac{RPb + Ra}{2} \right.$$

$$\left. = \frac{180^\circ - Hb \backsim RA}{2} = \right) 90^\circ - \frac{Hb \backsim Ra}{2}.$$

235. Remarks. 1. There will be about 12 hours between the two obſervations.

2. This method is ſubject to a ſmall error, on account of the leſſer altitude being more affected by refraction, than the greater.

## 236. PROBLEM LVII.

### *To find the Obliquity of the Ecliptic.*

Solution. Let the meridian altitude of the Sun's center be obſerved on the days of the ſummer and winter ſolſtice; the difference of thoſe altitudes will be the diſtance of the tropics; and half that diſtance will ſhew the obliquity of the ecliptic.

Or. The meridian altitude at the ſummer ſolſtice, leſſened by the co-latitude of the place, will give the obliquity of the ecliptic.

From good obſervations the obliquity of the ecliptic, about the time of the vernal equinox 1772, was found to be 23° 28′.

Diſtant obſervations compared together, ſhew that the obliquity is decreaſing at the rate of about one minute in 120 years.

237. Remark. By the ſecond method the declinations of the fixed ſtars, or of any other celeſtial phenomenon, may be found; obſerving that their declination is of the ſame name, viz. north or ſouth, with the latitude of the place, when its complement is leſs than the altitude; otherwiſe, of a contrary name with the latitude.

## 238. PROBLEM LVIII.

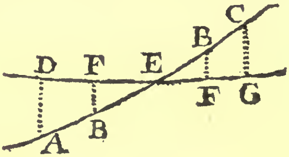
### *To find the Time of an Equinox.*

Solution. In a place the latitude of which is known, let the Sun's meridian altitude be taken on the day of the equinox, and on the day preceding, and that following it. Then the difference between thoſe altitudes and the co-latitude will be the Sun's declinations at the times of obſervations. (236)

If either of the altitudes is equal to the co-latitude, that obſervation was made at the time of the equinox.

But if the co-latitude is unequal to either of the altitudes, proceed thus. Let DG repreſent the equator; AC the ecliptic, E the equinoctial point; the points A, B, C, the places of the Sun at the times of obſervation; the arcs AD, BF, CG, the correſponding declinations.

Now uſing either the two firſt, or two laſt obſervations, ſuppoſe the latter, in the right-angled ſpherical triangles CEG, BEF, in which there are known the obliquity of the ecliptic, and the declinations; find EC, EB: then BC, the ſum or difference of EC, EB, is the ecliptic arc deſcribed in 24 hours. Then ſay,



As BC to BE, ſo 24 hours for BC, to the time correſponding to BE.

And this time ſhews the diſtance of the equinox from the time of the middle obſervation.

## 239. PROBLEM LIX.

### *To find the length of the tropical, periodical, and anomaliſtical revolutions of the earth.*

Solution. Let two obſervations be choſen, among the moſt authentic of thoſe on record, of the time when the Sun had like poſitions, viz.

1ſt. In regard to his longitude, or place in the ecliptic.

2d. In regard to the right aſcenſion of ſome noted ſtar.

3d. In reſpect to the line of the apſides.

The greater the interval (ſuppoſe 80 or 100 years) between each two obſervations, the more accurate will be the reſult: then that interval being divided by the number of revolutions made during that time, will give the time of one periodical revolution.

According to Mayer's tables the numbers are theſe,

| | | | | | |
|---|---|---|---|---|---|
| A tropical year | is made in | $365^d$ | $5^h$ | $48^m$ | 42". |
| A periodical, or ſiderial revolution | | 365 | 6 | 9 | 7. |
| An anomaliſtic revolution | | 365 | 6 | 15 | 29. |

240. REMARKS. 1. The tropical year being ſhorter than the ſiderial by 20 m. 25 s., ſhews that the Sun has returned to the ſame point of the ecliptic, before he has made one complete revolution with regard to the ſtars; and conſequently every point of the ecliptic muſt have moved in *antecedentia* during that tropical period, and ſo have produced what is called the preceſſion of the equinoxes.

Now 365 d. 6 h. 9 m. 7 s. : 360° : : 20 m. 25 s. : 50″,3, or nearly 50″, for the preceſſion in one year.

If there was no preceſſion, the tropical and ſiderial years would be equal.

241. 2. A ſiderial revolution being performed ſooner by 6 m. 22 s. than the anomaliſtic, ſhews that the line of the apſides has a motion in *conſequentia:* now 365 d. 15 h. 29 m. : 360° : : 6 m. 22 s. : 15″,7, the yearly quantity by which the Sun's apogee is advanced in reſpect to the ſtars: and as the equinoxes move in *antecedentia*, and the apſides in *conſequentia*, their ſum 66″ (=50,3+15,7) ſhews the motion of the apſides from the equinoxes.

242. 3d. From the compariſon of many obſervations it appears, that the length of the ſolar year, deduced from two very diſtant obſervations made at the time when the Sun was in the ſame point of the ecliptic near its apogee, differs by many ſeconds from the length of the year deduced by like obſervations, when the Sun was in another part of the ecliptic, near its perigee; thoſe made near the apogee giving the revolutions leſs, and thoſe made near the perigee making them greater, than the revolutions deduced from obſervations taken at the Sun's mean diſtance; this alſo ſhews, that the line of the apſides has a motion in *conſequentia*; and that the length of a tropical revolution ſhould be determined from very diſtant obſervations, made at the times when the Sun is at its mean diſtance from the Earth; or that the mean revolution ſhould be taken between thoſe deduced from obſervations made on the Sun's place, when he is in both the apogee and perigee.

## 243. PROBLEM LX.

*To find the right aſcenſion of ſome noted fixed ſtar.*

Having a good clock well regulated to mean or equal time, a large aſtronomical quadrant fixed in the plane of the meridian, and an equal altitude or tranſit inſtrument: then, on ſome day a little before or after the vernal equinox, when the daily alteration of the Sun's declination is about 18 or 20 minutes, obſerve the Sun's meridian altitude; and by equal altitudes find the times when both Sun and ſtar come to the meridian; the difference of theſe times is their difference of right aſcenſion.

Again. At ſome time a little after or before the autumnal equinox, before the Sun has paſſed the ſaid declination, obſerve his meridian altitude; and by equal altitudes find the times of the Sun and ſame ſtar's coming to the meridian, the difference of thoſe times is alſo the difference of their right aſcenſions.

If the vernal and autumnal meridian altitudes are the ſame, then thoſe obſervations were made, when the Sun was on the ſame parallel of declination: now the ſum, or diff. of the two obſerved differences of right aſcenſion, ſhews the equatorial arc deſcribed by the Sun between thoſe times; which arc, being biſected, ſhews the diſtance of the neareſt ſolſtice from the Sun, at the time when the obſervations have equal altitudes; and that diſtance corrected and taken from 90°, ſhews the Sun's right aſcenſion at the vernal obſervation, or its complement to 360 degrees.

From hence, and the firſt difference of right aſcenſion between the Sun and ſtar, the ſtar's right aſcenſion will be obtained.

244. If the two meridian altitudes of the Sun are not the ſame, their difference ſhews the difference of the mid-day declinations, when thoſe obſervations were taken: now from ſome tables of right aſcenſion and declination take the Sun's daily alteration in declination and right aſcenſion on the day the leſſer altitude was taken; then ſay, *As the daily change of decl. is to that of right aſcen.*; *ſo is the diff. of the altitudes, to the correction in right aſcenſion.*

This correction being added to the vernal, or ſubtracted from the autumnal difference of right aſcenſion, as either is leaſt, reduces that difference of right aſcenſion to what it would be when the declination is the ſame with the other; and then the difference between thoſe two differences of right aſcenſion, ſo reduced, gives the equatorial arc, as before recited.

245. At the Royal Obſervatory at Greenwich, in the year 1770, obſervations were made on the Sun and the ſtar $\alpha$ Aquilæ.
March 15, Sun's mer. zen. diſt. cleared of refraction and parallax, was 53° 28′ 29″; and their diff. of rt. aſc. was 60° 30′ 7,8″.
Sep. 28th. Sun's mer. zen. diſt. cleared of refraction and parallax, was 53° 36′ 26″; and their diff. of rt. aſc. was 109° 59′ 22,8″.
Then 7′ 57″ is the diff. of zen. diſts. or the alteration in declination.
Alſo 23′ 40″ and ($3^{m}\ 39^{s}$ or) 54′ 45″ are the diffs. of decl. and rt. aſc. between the 15th and 16th days of March 1770.

Now 23′ 40″ : 7′ 57″ : : 54′ 45″ : 18′ 23,5″ the increaſe of the difference of right aſcenſion after the noon of the 15th of March.
Then 60° 30′ 07,8″ − 18′ 23,5″ = 60° 11′ 44,3″ which is the firſt diff. rt. aſc. when the Sun had the ſame decl. as at the ſecond obſervation.
Here, the times of the two obſervations fall neareſt the winter ſolſtice.
Then 60° 11′ 44,3″ + 109° 59′ 22,8″ = 170° 11′ 7″; its half 85° 5′ 33,5″ is the diſtance of the winter ſolſtice from the Sun.
Hence 270° + 85° 5′ 33,5″ + 18′ 23,5″ = 355° 23′ 57″ is the Sun's rt. aſc. on March 15th.
Alſo 355° 23′ 57″ − 60° 30′ 7,8″ = 294° 53′ 49,2″ is the rt. aſc. of $\alpha$ Aquilæ.

246. The right aſcenſion of one ſtar being known, the right aſcenſions of all the reſt are found by noting the times ſhewn by the clock, when thoſe ſtars come to the meridian: for the differences of thoſe times, from the tranſit of the choſen ſtar, are the differences of right aſcenſion; by which the right aſcenſion of all the obſerved ſtars will be known; taking care to augment or diminiſh the right aſcenſion of the choſen ſtar by thoſe differences, according as the choſen ſtar is preceded, or followed by the other obſerved ſtars.

## 247. PROBLEM LXI.

*To find the Sun's Place.*

Solution. Let the time be obſerved both when the Sun, and a ſtar (the right aſcenſion of which is known) paſſed the meridian, and hence the Sun's right aſcenſion is known.

With that right aſcenſion, and the obliquity of the ecliptic, compute (142) the longitude, and thus his place in the ecliptic will be known.

## 248. PROBLEM LXII.

*To find the greateſt Equation of the Center.*

Solution. At the times when the Sun is near his mean diſtance, let his longitude be found; their difference will ſhew the true motion for that interval of time.

Find alſo the Sun's mean motion for that interval of time.

Then half the difference between the true and mean motions will ſhew the greateſt equation of the center.

Obſervation made at the Royal Obſervatory at Greenwich, ſhews that
1769 October 1ſt. at $23^h$ $49^m$ $12^s$ mean time, ⊙ long. was $6^s$ 9° 32′ 0,6″
1770 March 29th. at 0 4 50 mean time, ⊙ long. was 0 8 50 27,5
The diff. of time 178d. 0 15 38; True diff. long. 5 29 18 27
The tropical year = 365 d. 5 h. 48 m. 42 s. = 365,2421527
The obſerved interval = 178 0 15 38 = 178,01085648.
Then 365,2421527 : 178,01085648 :: 360° : 175,455948 mean motion.
So 175° 27′ 21″ of mean motion, anſwers to 179° 18′ 27″ true motion.
Their diff. = 3° 51′ 6″; its half 1° 55′ 33″ is the greateſt equation of the center according to theſe obſervations.

## 249. PROBLEM LXIII.

*To find the eccentricity of the Earth's orbit.*

Solution. Say, As the diameter of a circle in degrees,
To the diameter in equal parts;
So the greateſt equation of the center in degrees,
To the eccentricity in equal parts.

The greateſt equation of the center 1° 55′ 33″ = 1°,9258333, &c.
The diam. of a circle being 1, its circumf. is 3,1415926. (II. 197)
Then 3,1415926 : 1 :: 360° : 114°,5915609 equal to the diameter.
And 114,5915609 : 1,00&c. :: 1,9258333 : 0,0168061 the eccentricity.
Hence 1,016806 (= 1,000000 + 0,016806) = aphelion diſtance.
And 0,983194 (= 1,000000 — 0,016806) = perihelion diſtance.

## 250. PROBLEM LXIV.

*To find the time and place of the Sun's Apogee.*

Solution. On each day of two ſucceſſive apſides let the Sun's place and the time be obſerved.

Then if the interval of thoſe times and places is equal to the halves of 365 d. 6 h. 15 m. 29 s. and 360° 1′ 6″; thoſe obſervations were made when the Sun was in the apſides.

For ſuch intervals of time and place belong to no other points of the Earth's orbit.

But if thoſe obſerved intervals of time and place differ from the ſaid halves, take the difference between the interval of place and 180° 0′ 33″.

Then to the daily motion of the Sun's apogee (233), the ſaid diff. and 24 h. find the proportional time; which proportional time and difference, being applied to the time, and places, of the apogeon obſervation, gives a time and place when it is 180° 0′ 33″ diſtant from the obſerved perigeon place: now if the interval of theſe times is equal to 182 d. 15 h. 7 m. 44½ s. the times and places of the apſides are known.

But if the interval of time differs from 182 d. 15 h. 7 m. 44½ s., ſay, *As the diff. between the perigeon and the apogeon daily motions, is to the daily motion of the apogee; ſo is the diff. of the interval of time, to a ſecond correction of the time of the apogee.*

This correction applied to the apogeon time, corrected as above, will give the true time of the Sun's apogee.

Alſo, to the laſt correction of time find the proportional motion of the Sun's apogee; and apply it to the laſt corrected place of the apogee, and the true place of the apogee will be obtained.

By obſervations made at the Royal obſervatory at Greenwich in the year 1769.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| July 1ſt. at | $0^h$ | $3^m$ | $20^s$ | mean time ⊙ long. = | $3^s$ | $9^\circ$ | $46^m$ | $38{,}5''$ |
| December 29th. at | 0 | 2 | 49 | mean time ⊙ long. = | 9 | 8 | 10 | 58,1 |
| Interval 180 d. | 23 | 59 | 29. | Interval of place = | 5 | 28 | 24 | 19,6 |
| The Sun's motion in half an anomaliſtic year | | | | | 6 | 0 | 0 | 33 |
| The Sun's place at firſt obſervation is too forward by | | | | | | 1 | 36 | 13,4 |

Then 57′ 12″ : 1° 36′ 13,4″ : : 24 h. : 40 h. 22 m. 24 s. to be taken from the time of July 1ſt, to make the diſtance of the times anſwer to the half of 360° 1′ 6″; and it leaves June 29 d. 7 h. 40 m. 56 s.; at which time, the ſun was in 3ˢ 8° 10′, 25,1″, which is diſtant from the December obſervation by 180° 0′ 33″: But here the interval of time is 182 d. 16 h. 21 m. 53 s.; which is greater than 182 d. 15 h. 7 m. 44½ s. the half anomaliſtic revolution, by 1 h. 14 m. 8½ s.; therefore the Sun has ſome time to run before he comes to the apogee.

Now 4′ 0″ : 57′ 12″ : : 1 h. 14 m. 8½ s. : 17 h. 13 m. 14 s. correction of time.
And 24 h. : 17 h. 13 m. 14 s. : : 57′ 12″ : 42′ 6,8″. correction of place.
Then June 29 d. 7 h. 4 m. 56 s. + 17 h. 13 m. 14 s. gives
June 30 d. 0 h. 21 m. 10 s. for the time of the apogee.
And 3ˢ 8° 20′ : 25,1″ + 42′ 6,8″ gives 3ˢ 8° 52′ 33″ for the place of the apogee.

## 251. PROBLEM LXV.

*At any given time to find the Sun's mean anomaly.*

Solution. Let an epocha of the Sun's paſſage through its aphelion be accurately determined. Then ſay,
As the time of a tropical revolution, or ſolar year,
To the interval between the aphelion and given time;
So is 360 degrees,
To the degrees ſhewing the mean anomaly.
Or. From the tables of mean motions find the Sun's mean motion for the given time, and this will be the mean anomaly.

252. If the Sun's motion in the ecliptic was uniform, his true place for any time could be found by the tables of his mean motion; but the Sun's longitude found by thoſe tables, called his mean longitude, muſt be corrected on account of his irregular motion.

As the Earth revolves in an elliptical orbit about the Sun, placed in one of its foci, its angular motion round the Sun will differ from the angular motion it would have, were the Sun in the center of the ellipſis.

Now the table of mean motions gives the angular motions from the center of the ellipſis in a circle deſcribed on the line of the apſides, and reckoned from the firſt point of Aries; this motion, leſſened by that of the apogee, gives the Sun's mean longitude, or mean anomaly, from the aphelion point.

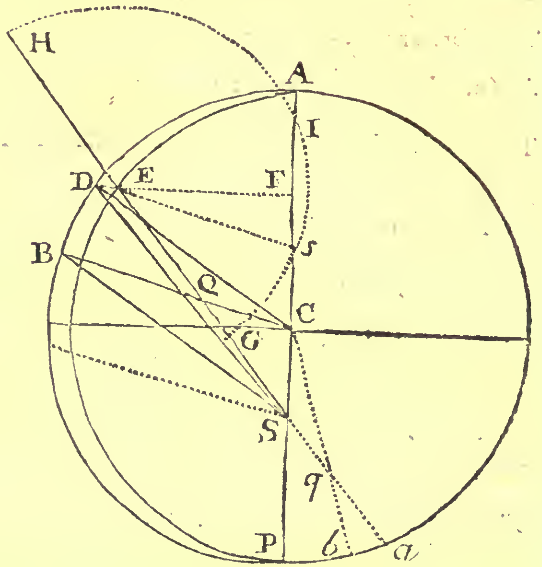
But the motion of the Earth being in an elliptic orbit, its true anomaly will differ from its mean; this difference, called the equation of the center, is the correction wanted to reduce the mean motions to the true ones.

253. To find the equation of the center, or to ſolve (what is called) the Keplerian problem, is the moſt difficult operation, particularly in orbits the eccentricity of which bears a conſiderable proportion to the mean diſtance: how to do this has been ſhewn by Newton, Gregory, Keil, La Caille, and many others, by methods little differing from one another: it conſiſts chiefly in finding an intermediate angle, called the eccentric anomaly, as ſhewn in the following problem.

## 254. PROBLEM LXVI.

*The Sun's mean anomaly being known, and the dimenſions of its orbit, to find the eccentric anomaly.*

Solution,



Solution. Say,
As the aphelion diſtance,
To the perihelion diſtance;
So is the tan. $\frac{1}{2}$ the mean anomaly,
To the tan. of an arc.
Which arc added to half the mean anomaly gives the excentric anomaly.

For let ADPB be the excentric. AEP the Earth's orbit, C the center, S the Sun A the aphelion, P the perihelion, E the true place, D the correſponding place in the excentric, and B the mean place.

Now it is evident, that the leſs the eccentricity is, the nearer will the elliptic orbit approach the excentric circle; the nearer will the true and mean places, E and B, approach one another; and the leſs will be the difference between the mean, the excentric, and the true anomalies; alſo the nearer will the lines CD, SB, approach to paralleliſm, or coincidence: ſo that in orbits of ſmall eccentricities CD and SB may be taken as parallel lines, particularly in the Earth's orbit, where CS is only about $\frac{1}{60}$ of CP.

Therefore $\angle$ ASB $=$ $\angle$ ACD the excentric anomaly.

Then in the triangle BCS, where the ſum of the ſides BC $+$ CS $=$ SA; the diff. of the ſides BC $-$ CS $=$ SP, and $\angle$ BCS ($=$ ſupplement of ACB) are known; the $\angle$ CSB may be found. (III. 48)

Thus SA : SP : : tan. $\frac{1}{2}$ (ſum $\angle s$, CSB $+$ B $=$) $\angle$ ACB : tan. of an arc.

Then $\frac{1}{2}$ $\angle$ ACB $+$ that arc $=$ $\angle$ CSB (III 47.) the excentric anomaly.

## 255. PROBLEM LXVII.

*The Sun's excentric anomaly, and the dimenſions of its orbit being known, to find the true anomaly.*

Solution. Say, As the ſquare root of the aphelion diſtance,
To the ſquare root of the perihelion diſtance,
So tangent of half the excentric anomaly,
To tangent of half the true anomaly.

For let a ſemicircle be deſcribed from E through the other focus $s$, cutting AP in $s$, I, and SE, produced, in G, H.

Then (II. 172) SH : SI : : S$s$ : SG $=$ $\left( \frac{\text{SI} \times \text{S}s}{\text{SH}} = \frac{\overline{\text{S}s + s\text{I}} \times \text{S}s}{\text{SE} + \text{E}s} = \right) \frac{2\text{CF} \times 2\text{CS}}{2\text{CA}}$.

Or $CF \times CS = (\tfrac{1}{2}SG =)\, \tfrac{1}{2}SE - \tfrac{1}{2}Es$; $CA\,(= \tfrac{1}{2}SE + \tfrac{1}{2}Es)$ being radius, and $= 1$.
Therefore $SE = (1 + CS \times CF$ (III. 47) $=)\, 1 + CS \times \grave{s},ACD$. (III. 9)

Again. In $\triangle\, SFE$. As $SE : R :: SF : \grave{s},ASE = \left(\frac{SF}{SE} =\right) \frac{SC + \grave{s},ACD}{1 + SC \times \grave{s},ACD}$.

$$\text{Then } 1 + \grave{s},ASE : 1 - \grave{s},ASE :: 1 + \frac{SC + \grave{s},ACD}{1 + SC \times \grave{s},ACD} : 1 - \frac{SC + \grave{s},ACD}{1 + SC \times \grave{s},ACD}.$$

$$\begin{aligned}
\text{Or } \frac{1 - \grave{s},ASE}{1 + \grave{s},ASE} &= \left(\frac{1 + SC \times \grave{s},ACD - SC - \grave{s},ACD}{1 + SC \times \grave{s},ACD + SC + \grave{s},ACD}\right. \\
&= \frac{1 - SC + SC \times \grave{s},ACD - \grave{s},ACD}{1 + SC + SC \times \grave{s},ACD + \grave{s},ACD} \\
&= \left.\frac{SP + \overline{CS-1} \times \grave{s},ACD}{SA + \overline{CS+1} \times \grave{s},ACD} = \frac{SP - \grave{s},ACD \times SP}{SA + \grave{s},ACD \times SA}\right) \\
&= \frac{1 - \grave{s},ACD}{1 + \grave{s},ACD} \times \frac{SP}{SA}.
\end{aligned}$$

But $\frac{1 - \grave{s},ASE}{1 + \grave{s},ASE} = tt,\tfrac{1}{2}ASE$; and $\frac{1 - \grave{s},ACD}{1 + \grave{s},ACD} \times \frac{SP}{SA} = tt,\tfrac{1}{2}ACD \times \frac{SP}{SA}$. (IV. 217)

Then $\frac{SP}{SA} \times tt,\tfrac{1}{2}ACD = tt,\tfrac{1}{2}ASE$. Or $\frac{SP}{SA} = \frac{tt,\tfrac{1}{2}ASE}{tt,\tfrac{1}{2}ACD}$.

Therefore $\left(\frac{\sqrt{SP}}{\sqrt{SA}} = \frac{t,\tfrac{1}{2}ASE}{t,\tfrac{1}{2}ACD}.\right.$ Or$\left.\right)$ $\sqrt{SA} : \sqrt{SP} :: t,\tfrac{1}{2}ACD : t,\tfrac{1}{2}ASE$.

256. REMARK. *The* $\angle CQS = \angle ACB \backsim \angle ASE$ (II. 95), *is the equation of the center, to be applied to the mean anomaly; and is ſubtractive from the aphelion to the perihelion, or in the firſt ſix ſigns of anomaly; and additive from the perihelion to the aphelion, or in the laſt ſix ſigns of anomaly.*

For the lines CB, SE; C*b*, SE, which coincide in SA, SP, will in every other poſition croſs one another; in Q while revolving from A to P, and in *q* while revolving from P to A: in the firſt half revolution, the mean anomaly, or the external ∠ ACB, exceeds the ∠ ASE, the true anomaly, by the ∠ CQS (II. 96): in the latter half, the true anomaly, or external ∠ P*sa*, exceeds the mean anomaly, or ∠ PC*b*, by the ∠ *sq*C.

SECTION

## SECTION IX.

## *Practical Aſtronomy.*

### Of the EQUATION of TIME.

257. Time, which of itſelf flows uniformly, has its parts meaſured by the motion of ſome viſible object; and the Sun being the moſt conſpicuous moving object in the heavens, its motion has been choſen as the moſt proper meaſure of the parts of time, as well for the day, as for the year.

258. The aſtronomical day, at any place, begins when the Sun's center is on the meridian of that place; and is divided into 24 hours, reckoned in a numeral ſucceſſion from 1 to 24: the firſt 12 are ſometimes diſtinguiſhed by the mark P. M., ſignifying *poſt meridiem*, or afternoon; and the latter 12 are marked A. M., ſignifying *ante meridiem*, or before noon: but aſtronomers generally reckon through the 24 hours, from noon to noon; and what is by the civil, or common way of reckoning, called morning hours, is by Aſtronomers reckoned in the ſucceſſion from 12, or midnight, to 24 hours.

Thus 5 o'clock in the morning of April the 10th, is by aſtronomers called April the 9th, at 17 h.

259. *The Sun's daily motion in longitude* is the arc of the ecliptic run through in that day; and his *daily motion in right aſcenſion* is the correſponding arc of the equator; and the mean daily motion in either circle is meaſured by 59′ 8″ nearly. For 365 d. : 1 d. : : 360° : 59′ 8″.

260. An ASTRONOMICAL or SOLAR DAY is the interval of time between two ſucceſſive tranſits of the Sun's center over the ſame meridian; and is meaſured by the ſum of the whole equator, and an arc of it equal to the daily motion in right aſcenſion.

For at the end of a diurnal rotation, which by obſervations is known to be uniform, the meridian has returned to the ſame ſtar, or point of the ecliptic, which it was againſt at the preceding noon; but the Sun, during this rotation, has removed from that ſtar to another, which has a greater right aſcenſion: therefore, before the meridian can be again oppoſite to the Sun, ſo much of another rotation muſt be deſcribed, as is equal to the daily motion in right aſcenſion.

261. A SIDERIAL DAY is the interval between two ſucceſſive returns of the ſame meridian to the ſame fixed ſtar, is leſs than the ſolar day, and is meaſured by 360°.

262. A MEAN or EQUATORIAL DAY is the time elapſed between two ſucceſſive tranſits of the Sun over the meridian, and is meaſured by 360° 59′ 8″ nearly.

263. MEAN or EQUAL TIME is that ſhewn by a *clock*, whoſe 24 hours meaſure the time which the Sun takes to deſcribe an equatorial arc equal to 360° 59′ 8″ nearly.

264. The difference between the meaſures of a mean ſolar day and a ſiderial day, viz. 59′ 8″, reduced to time (132), gives 3 m. 56 s.; which ſhews, that a ſtar which was on the meridian with the Sun on

one noon, will return to that meridian 3 m. 56 s. before the next noon; therefore a clock, which meaſures mean or equal days by 24 hours, will give 23 h. 56 m. 4 s. for the length of a ſiderial day.

265. Apparent, or True Time, is that ſhewn by a *ſun-dial*; where 24 hours, or a day, is meaſured by the ſum of 360°, and that day's motion in right aſcenſion.

266. The ſolar days are unequal to one another, for obſervations ſhew that the ſun's daily motion in right aſcenſion is continually varying.

The true and mean ſolar days are never equal, but when the Sun's daily motion in right aſcenſion is 59′ 8″; which happens about February 11th, May 14th, July 26th, and November 1ſt: at all other times the lengths of the *true* and *mean* days differ. The accumulation of theſe differences produces the equation of time; and ſometimes the apparent noon will precede the time of the mean noon, and ſometimes fall after it; their difference amounting to above 16 minutes at the beginning of November.

267. The Equation of Time is the difference between the times ſhewn by a *clock* and a *ſun-dial*; or between the *mean* and true noons; or between the Sun's right aſcenſion and his mean longitude when turned into time at the rate of 15° to an hour.

This difference ariſes on two accounts. Firſt, becauſe of the obliquity of the ecliptic the daily motions in longitude and right aſcenſion are unequal. Secondly, becauſe of the unequal motion of the Earth in an elliptic orbit.

In the firſt and third quadrants, or between the ſigns ♈♋, ♎♑, the right aſcenſion being leſs than the longitude (140), or the mean motion taken in the equator; the point of right aſcenſion is to the weſt, and therefore the apparent noon precedes, or comes in *conſequentia* to the meridian before the mean noon: but in the 2d and 4th quadrants, or between the ſigns ♋♎, ♑♈, the right aſcenſion being greater than the longitude or mean motion, taken in the equator, the mean noon is weſtward, and therefore precedes, or comes in *conſequentia* to the meridian before the apparent noon.

From the aphelion to the perihelion, or in the firſt ſix ſigns of anomaly, the mean noon precedes the apparent; and in the laſt ſix ſigns of anomaly the apparent noon precedes the true; their difference in either caſe is the equation of the center, which convert into time.

Now becauſe the points of Aries, and of the Sun's apogee, the places where the two parts of the equation of time commence, do conſtantly recede from one another; therefore the whole equation of time made up of thoſe two parts will ſerve only for a few years, and requires to be corrected from time to time.

268. *To calculate the equation, or difference between the mean and apparent noons, for any propoſed day.*

Find the mean and true anomalies for that time (255); their difference, or the equation of the center, is one part.

The true anomaly gives the Sun's longitude; with which, and the obliquity of the ecliptic, compute the right aſcenſion (139); the difference between the longitude and right aſcenſion gives the other part.

The ſum, or diff. of the two parts, turned into time, gives the equation ſought.

## SECTION X.

## *Practical Aſtronomy.*

### To make SOLAR TABLES.

269. I. *Tables of the mean motions of the Sun.* (301, 302, 303, 304.)

Divide 360 degrees by a ſolar revolution, the quotient ſhews the mean motion for one day 0° 59′ 08″ &c.

Take the multiples of one day's motion from 1 to 365 for every day in the year; and theſe properly diſpoſed, according to the month days, will give the mean motions for every day of each month. (304)

The 24th part of one day's motion will give that for one hour, and its multiples to 24 times will ſhew the mean motions anſwering to each hour: from hence, thoſe for the minutes of an hour, the ſeconds of a minute, &c. are eaſily obtained. (303)

The mean motion of a year of 365 days (viz. for the laſt of December) being doubled, tripled, and quadrupled, thoſe for 1, 2, 3, and 4 years will be obtained, adding one day's mean motion to the 4th year, it being leap-year, and containing 366 days: the motion for leap-year being increaſed by thoſe of 1, 2, 3, and leap-years, give thoſe for 5, 6, 7, and 8 years: the mean motion for 8 years being increaſed by thoſe for 1, 2, 3, and 4 years, give thoſe for 9, 10, 11, and 12 years: and thus increaſing the mean motion for the laſt leap year by thoſe of 1, 2, 3, and 4 years, the mean motions may be continued for any number of abſolute years. (301)

270. In the following tables the numbers uſed were,

| | | | | |
|---|---|---|---|---|
| Length of the year | 365d. | 5h. | 48m. | $54\frac{3}{4}$s. |
| Yearly motion of the apogee, | | 0° | 1′ | 5″. |
| Place of the apogee, beginning the year 1760, | $3^s$ | 8 | 47 | 25. |
| Greateſt equation of the Earth's orbit. | | 1 | 55 | 39. |

271. Now 365 d. 5 h. 48 m. $54\frac{3}{4}$ ſ. = 365,2423003472 days.

Then 365,2423003472 d. : 360° : : 1 d. : 0,9856470613 degrees.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Hence the mean motion | | for 1 day = | $0^s$ | 0° | 59′ | 8″ | 19‴ | $45^{iv}$ | $54^{v}$ | $50^{vi}$ |
| January | 5th | 5 days = | 0 | 4 | 55 | 41 | 38 | 49 | 34 | 10 |
| January | 30th | 30 days = | 0 | 29 | 34 | 9 | 52 | 57 | 25 | 0 |
| March | 31ſt | 90 days = | 2 | 28 | 42 | 29 | 38 | 52 | 15 | 0 |
| June | 29th | 180 days = | 5 | 27 | 24 | 59 | 17 | 44 | 30 | 0 |
| December | 26th | 360 days = | 11 | 24 | 49 | 58 | 35 | 29 | 0 | 0 |
| December | 31ſt | 365 days = | 11 | 29 | 45 | 40 | 14 | 18 | 34 | 10 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Now 1 year's mean motion | = | 11ſ. | 29° | 45′ | 40″ | 14‴ | $18^{iv}$ | $34^{v}$ | $10^{vi}$. | |
| 2 years | = | 11 | 29 | 31 | 20 | 28 | 37 | 8 | 20. | |
| 3 years | = | 11 | 29 | 17 | 0 | 42 | 55 | 42 | 30. | |
| 4, or leap year | = | 0 | 0 | 1 | 49 | 17 | 0 | 11 | 30 | = 3y. + 1y. + 1d. |
| 5 years | = | 11 | 29 | 47 | 29 | 31 | 18 | 45 | 40 | = 4y. + 1y. |
| 6 years | = | 11 | 29 | 33 | 9 | 45 | 37 | 19 | 50 | = 4y. + 2y. |
| 7 years | = | 11 | 29 | 18 | 49 | 59 | 55 | 54 | 0 | = 4y. + 3y. |
| 8 years B | = | 0 | 0 | 3 | 38 | 34 | 0 | 23 | 0 | = 4y. × 2. |
| 20 years B | = | 0 | 0 | 9 | 6 | 25 | 0 | 57 | 30 | = 4y. × 5. |
| 100 years B | = | 0 | 0 | 45 | 32 | 5 | 4 | 47 | 30 | = 20y. × 5. |
| 1000 years B | = | 0 | 7 | 35 | 20 | 50 | 47 | 55 | 0 | = 100y. × 10. |

Where B ſtands for biſſextile, or leap-year.

272. But to find the mean motions for the years related to any particular epocha, the mean motion for ſome particular time in that epocha muſt be known. Thus,

Let the mean motion of the Sun be determined by obſervation (or otherwiſe) when the Sun is in ſome noted point of the ecliptic, ſuppoſe near Aries: or let the time of its entrance into the ſign Aries be well aſcertained. Take the difference between the time of that ingreſs and the 31ſt of December at noon, in days, hours, minutes, and ſeconds (reckoning the end of the 31ſt of December to be the beginning of January at noon,) and find the mean motions for thoſe days, hours, minutes, and ſeconds, and it will ſhew the motion from Aries, for the 31ſt of December, or the mean motion at the beginning of the year propoſed; or the radix for that year with relation to the propoſed epocha.

The relative mean motions for one year being known, thoſe for any number of ſucceeding years belonging to that epocha, may be had, by adding ſuch of the before found abſolute years to the firſt relative year, as will make the number wanted: and the mean motions for any paſt year of that epocha will be found by leſſening the radical years by ſuch a number of the abſolute years, as will produce the relative years required. (302)

And in this manner are tables conſtructed, by which the mean motions of the Sun for any time, paſt, or to come, may be computed.

273. Suppoſe in the year 1760, the Sun entered Aries on the 20th of March, at 13 h. 42 m. $3\frac{1}{4}$s. P. M.: required the Sun's mean motion for the beginning of the year 1760.

Now 1760 being leap-year, February has 29 days, and from the equinox to the commencement of the year is 80 d. 13 h. 42 m. $3\frac{1}{4}$ s.

Then 1 d. : 0,9856470613 deg. : : 80 d. 13 h. 42 m. $3\frac{1}{4}$ s. : 79,41444 &c. degrees.

Therefore at the beginning of the year 1760, the Sun's mean longitude was $2^{s}$ 19° 24′ 52″ ſhort of Aries; or his mean longitude was $9^{s}$ 10° 35′ 8″, which is the radical mean place for the year 1760.

## 274. II. *Of the mean motions of the Sun's apogees.* (301, 302)

The yearly motion of the apogee being determined (241); the motion for any number of abſolute years will be that multiple of one year's motion, and ſo for any part of a year: the monthly motions will be 5 ſeconds for ſome months, and 6 for others, to make 65 in the 12 months.

Let the time of the Sun's paſſage through the aphelion be accurately determined by obſervation (250), and alſo its place in the ecliptic; then the diſtance of the place of the apogee from Aries will be known at that time: let this diſtance be leſſened by the apogee's motion from the laſt day of the year preceding the propoſed epocha to the time of the apogeon paſſage, and the mean motion of the apogee will be known for the beginning of that year, taken as a radix.

Then that radical mean motion, increaſed by the multiples of the yearly motion, will give thoſe for ſucceeding years: but being diminiſhed by thoſe multiples will give them for paſt years.

The Sun's mean motion for any time, leſſened by that of the apogee for that time, gives the Sun's mean anomaly.

## 275. III. *Of the equation of the Sun's center.* (305)

To every degree of the firſt ſix ſigns of mean anomaly aſſumed, find the true anomaly (253, 254): the difference between the mean and true anomalies will be the equations of the center to thoſe degrees of mean anomaly; which ſerve alſo for the degrees of the laſt ſix ſines; as equal anomalies are at equal diſtances on both ſides of either apſide.

Set the equations of the center orderly to their ſigns and degrees of anomaly, the firſt ſix being reckoned from the top of the table downwards, and ſigned at top with the title *ſubtract*; the laſt ſix, for which the ſame equations ſerve, but taken in a contrary order, viz. from the bottom of the table reckoned upwards, are ſigned at bottom with the title *add*; and let the difference between every adjacent two equations, called tabular differences, be ſet in another column.

From theſe equations of the center, augmented or diminiſhed by the proportional parts of their reſpective tabular differences for any given minutes and ſeconds, are deduced equations of the center to any given mean anomaly.

276. Aſtronomical tables are uſually computed to anſwer to two given denominations only; as to ſigns and degrees: degrees and minutes; months and days; &c.: for if made to more names, ſuch tables would ſwell into a bulk ſo great, as to be tedious to compute, expenſive to print, and of no great advantage in the uſe; but it generally happens in calculations, that numbers are wanted from tables to anſwer to given numbers of three, or more denominations as to ſigns, degrees, minutes, and ſeconds; months, days, hours, minutes, and ſeconds; &c.: and to obtain from the tables numbers anſwering to all the given names, the tabular numbers are to be increaſed or diminiſhed by a proportional part of their difference.

Thus. *To find the equation of the center to* $4^s\,21^\circ\,44'\,36''$?

Now the equation to this number will fall between thoſe belonging $4^s\,21^\circ$ and $4^s\,22^\circ$; which equations are $1^\circ\,13'\,59''$ and $1^\circ\,12'\,24''$ (205) Their diff. is $1'\,35''=95''$; the pro. pt. of which is to be taken for $44'\,36''$.

And as the diff. 1° or 60′ : diff. 95″ : : 44′,6 : 70″,6 = 1′ 11″ the proportional part.

Now to $4^s\ 21^\circ\ 0'\ 0''$, $1^\circ\ 13'\ 59''$ is the equation of the center.
And to $0\ \ 0\ \ 44\ \ 36$ , $1\ \ 11$ is the prop. part to be ſubtracte
Then to $4\ \ 21\ \ 44\ \ 36$ , $1\ \ 12\ \ 48$ is the equation of the center.

When the tabular numbers are increaſing, the proportional part is to be added; but when decreaſing, the proportional part is to be ſubtracted.

## 277. IV. *Tables of the Sun's true place.* (308)

The Sun's true place at any propoſed time is thus found.

Collect together the mean motions of the Sun, and alſo thoſe of the apogee, for the given year, month, day, (hour, minute, and ſecond, if given); and their ſum will be the mean motions of the Sun and its apogee.

The Sun's mean motion, leſſened by that of the apogee, gives the mean anomaly; to which find the proper equation of the center by proportioning for the minutes and ſeconds.

Then the Sun's mean motion, augmented or diminiſhed by the equation of the center, as the title of its table directs, gives the Sun's true longitude, or place, for that given time.

The Sun's place thus found to every day for four ſucceſſive years, viz. for leap-year, and 1, 2, 3 years after; and thoſe places ranged under their proper years, according to their reſpective months and days, conſtitute the tables of the Sun's place.

Theſe tables find the Sun's place at noon only; but the place for any intermediate time is found by applying to the noon-place the proportional part of the daily difference at that time.

## 278. *To find the Sun's longitude, ſuppoſe on May* 4, 1788, *at the time of apparent noon?*

In the table of the Sun's longitude (308) for 1788, againſt May 4th, ſtands $1^s\ 14^\circ\ 36'\ 02''$, which ſhews that the Sun's longitude, reckoned from Aries, is $44^\circ\ 36'\ 02''$; or that his place is in ♉ $14^\circ\ 36'\ 02''$.

But to find the Sun's place at any other hour, ſuppoſe on May 4th, at 7 h. 24 m. 36 s. apparent time, proceed thus,

The difference between the noon places of the 4th and 5th of May is $57'\ 59'' = 3479''$, anſwering to 24 hours in time. (308)

Then 24 h. : 7 h. 24 m. 36 s. : : $3479'' : 1074'' = 17'\ 54''$, the proportional part.

And $1^s\ 14^\circ\ 36'\ 02'' + 17'\ 54'' = 1^s\ 14^\circ\ 53'\ 56''$, the Sun's longitude at that time.

279. Or, the Sun's place may be found by the tables of mean motions.

From the apparent time $7^h\ 24'\ 36''$ take the equation of time (316) ual to $3'\ 35''$ and the remainder, $7^h\ 21'\ 1''$, is the mean time.

| | | | |
|---|---|---|---|
| 3. ⊙'s m. mot. | (302) | $9^s\ 10^\circ 47' 53''$ | |
| 4 | (305) | 4 2 13 13 | |
| hours | (303) | 17 15 | |
| minutes | (303) | 52 | |
| ſecond | (303) | 0 | |
| ⊙'s mean longitude | | 1 13 19 13 | |
| Equat. center | + | 1 34 44 | |
| Sun's true longitude | | 1 14 53 57 | |

| | | | |
|---|---|---|---|
| Mot. ap. | 3 9 17 45 | | (302) |
| | 22 | | (307) |
| | 3 9 18 07 | m. apogee. | |
| Diff. | 10 4 1 06 | m. anom. | |

Now to $10^s\ 4^\circ$ the equation of the center is $1^\circ\ 34'\ 45''$ (305)
And the diff. is $70''$ decreaſing. $60' : 70'' :: 1'\ 06'' : 1''$ the proportional part.
And $1^\circ\ 34'\ 45'' - 1'' = 1\ 34\ 44$ equation of the center.

### 280. *To find the Sun's longitude at any given time and place.*

Seek, in the table of the longitudes of places, at the end of Book VI. for the difference of longitude between London and the propoſed place; and convert the diff. of longitude into time.

If the propoſed place is to the eaſtward of London, take the diff. between the propoſed time and diff. of longitude, and this will ſhew the correſponding time at London; after noon, if the propoſed time is greater than the diff. of longitude; but before noon, if the propoſed time is leaſt.

If the propoſed place is to the weſtward of London, the ſum of the propoſed time and diff. of longitude will be the correſponding time at London.

The Sun's place found to the correſponding time at London, will be the Sun's longitude ſought for the propoſed time and place.

Thus. In a place 6 h. to the eaſt of London, when it is 8 h. P. M. at that place, it is 2 h. P. M. at London; and when it is 4 h. P. M. at that place, it is 2 h. before noon at London.

For when it is noon at London, it is 6 h. P. M. at the other place.

Alſo, in a place 6 h. to the weſt of London; when it is 8 h. P. M. at that place, it is 14 h. P. M. at London.

For when it is noon at the propoſed place, it is 6 h. P. M. at London.

### 281. V. *Tables of the Sun's declination.* (309)

With each of the Sun's longitudes, already found, and the obliquity of the ecliptic, find the declination to each day of the four years. (139)

Or thus. To each degree of the three firſt ſigns of the ecliptic, taken as longitudes, find the declination (139): and of theſe declinations, regularly ranged to their ſign and degree, take the difference of each adjacent two, which ſet againſt them in another column; and this auxiliary table is prepared, anſwering to each ſign and degree of longitude (306): for to equal longitudes, taken on both ſides of each equinox, belong equal declinations.

Now theſe auxiliary declinations augmented, or diminiſhed (according as they are increaſing or decreaſing, by the proportional part of their difference, for the minute and ſeconds in any given longitude, will give the declination for that longitude.

And this being done for every day in the four years, uſing the longitudes already computed, will give the declinations ſought: which are to be ranged according to their year, month, and day. (309)

282. To find the Sun's declination. Suppoſe on May 4, 1788, at noon. In the table of the Sun's declination (309) for 1788, againſt May 4, ſtands 16° 14′ 13″ for the Sun's declination, which is N. as being between the vernal and autumnal equinoxes.

283. But if the declination was wanted on May 4, 1788, at 7 h. 24 m. 36 ſ. P. M. proceed thus.

The difference between the noons of May 4 and 5, is 16′ 59″, which anſwers to 24 h. Then 24 h. : 17′ 0″ : : 7 h. 24 m. 36 ſ. : 5′ 15″, the proportional part.

And as the declination is increaſing; then 16° 14′ 13″ + 5′ 15″ gives 16° 19′ 28″ for the Sun's decl. at the propoſed time.

284. But art. 311 is a table for finding the proportional part at ſight, for fitting the noon declination to any other time. Thus.

Seek in the left-hand column for a daily difference, neareſt to the given one; againſt which, in a column marked at top with hours, neareſt to thoſe given, ſtands the proportional part ſought.

Thus againſt 17′ 0″ of daily diff. and to 7 h. 24 m. time, ſtand 5′ 15″, the proportional part ſought.

Although this table goes no farther than 8 h., yet it may be applied quite to 12 h. or 180 degrees.

285. Exam. *What will be the Sun's declination at London, on the 25th of Auguſt*, 1788, *at* 10 *h.* 35 *m. P. M.?*

In 1788, the daily diff. between the noons of the 25th and the 26th of Auguſt is 20′ 58″ decreaſing. (309)

Now 10 h. 35 m. is equal to 2 h. 35 m. + 8 h. 0 m.

To the diff. 20′ 58″, and to 2 h. 35 m., anſwers 2′ 16″. (311)

To the diff. 20′ 58″, and to 8 h. 0 m., anſwers 6′ 59″.

The ſum 9′ 15″ is the proportional part, by which the decl. 10° 28′ 29″ to Auguſt 25th, is to be diminiſhed; ſo 10° 19′ 14″ is the decl. ſought.

Here 20′ 58″, is taken as if it was 21′ 0″.

And 2 h. 35 m. is ¾, the interval between 2 h. 20 m. and 2 h. 40 m.

Now 21′ gives 2′ 2″ for $2^h$ 20′, and 2′ 20″ for $2^h$ 40′; the diff. is 18″, three fourths of which is 14″: and this being added to 2′ 2″ gives 2′ 16″ for 21′ with $2^h$ 35′. Moreover 21′ with 8 h. gives 7′ 00″; but I take one ſecond leſs becauſe the daily diff. in declination is 2″ leſs than 21′ 00″.

286. From the table of declination, fitted to the meridian of London, or Greenwich, the declination may be found at any time, under any other meridian, at a given difference of longitude from London. Thus.

*Required the Sun's declination at noon under a meridian* 110° *to the weſt of London, on the* 24*th of February*, 1788.

(311) Now at 110° to the weſt of London, it is noon 7 h. 20 m. after it is noon at London; that is, when it is 7 h. 20 m. P. M. at London, it will be noon at the propoſed place; ſo the declination found to that time at London (285) will be the declination ſought.

In 1788, the diff. between the declinations of the 24th and 25th of February, is 22′ 13″ decreaſing (309): and againſt 22′ 20″ of daily diff., and under 110°, or 7 h. 20 m., is 6′ 49″ in table, art. 311, which taken from 9° 27′ 33″, leaves 9° 20′ 44″, the declination ſought.

Exam. II. *What is the Sun's declination on September* 2*d*, 1788, *at* 20 *h.* 30 *m., under a meridian* 100° *to the eaſtward of London?*

Now under a meridian 100° to the eaſtward of London, it is noon 6 h. 40 m. before it is noon at London (311); or when it is noon at London, it is 6 h. 40 m. after noon at the propoſed place; and when 20 h. 30 m. after noon at that place, it is 13 h. 50 m. after noon at London; ſo the declination found at that time (285), will be the declination ſought.

In 1788, the daily diff. at September 2d, is 22′ 6″ (309), againſt which (in tab. art. 311), and under 8 h. and 5 h. 50 m., ſtand 7′ 21″ and 5′ 21″, their ſum 12′ 42″ taken from the decl. to September 2, viz. 7° 36′ 30″, leaves 7° 23′ 48″, the declination ſought.

Here 5 h. 50 m. fall in the middle between 5 h. 40 m. and 6 h. 0 m. ſo 5′ 21″, the middle between 5′ 12″ and 5′ 30″, is taken.

### 287. VI. *Tables of the Sun's right aſcenſion.* (310)

To the obliquity of the ecliptic, and each degree in the three firſt ſigns of longitude, find the right aſcenſions (139), and of each take the ſupplement.

Range the right aſcenſions according to their ſign and degree for the three firſt ſigns; and for the three next ſigns, range the ſupplements, ſo that the 4th ſign begins with the leaſt ſupplement, and the 6th ſign ends with the greateſt: becauſe the right aſcenſions in the 2d and 4th quadrants are the ſupplements of thoſe in the firſt and third.

Let the differences of theſe right aſcenſions, viz. each adjacent two, through the ſix ſigns be taken, and ſet in other columns. (307)

Then this auxiliary table, uſed like that of declination, will give the right aſcenſion to each day in the four years.

288. *To find the Sun's right aſcenſion, ſuppoſe on June* 12 *at noon, in the year* 1788, *at London.*

In the table of the Sun's right aſcenſion (310) for 1788, againſt June 12, ſtands 5 h. 25 m. 19 ſ., which is the right aſcenſion ſought, and ſhews how much later the Sun paſſed the meridian of London than the equinoctial point Aries, EXAM. II. *Required the Sun's right aſcenſion at London on the 2d of November*, 1788, *at* 9 *h.* 30 *m. P. M.?*

Between the 2d and 3d of November, 1788, the daily diff. is 3 m. 58 ſ. which anſwers to 24 h. Then 24 h.: 3 m. 58 ſ. : : 9 h. 30 m. : 1 m. 34 ſ., the proportional part.

Then the right aſcenſion on the 2d at noon, 14 h. 33 m. 20 ſ. + 1 m. 34 ſ. gives 14 h. 34 m. 54 ſ. for the right aſcenſion at the time required.

By this table the right aſcenſion may alſo be found at any time in places that are to the eaſtward, or weſtward, of London, the difference of longitude of thoſe places being known; by finding the time at London correſponding to the given time at the propoſed place, and ſeeking the right aſcenſion to that correſponding time at London.

The table at art. 311, pages 222, 223, may be applied to the tables of the Sun's longitude and right aſcenſion, as well as to thoſe of the declination, for finding the proportional parts of the difference between the noon of adjoining days, which ſhall anſwer to any intermediate hours.

Thus in the Ex. page 296. To find the pro. pts. of 58′ to 7 h. 24 m. 36 ſ.
Now $\frac{1}{4}$th of 58′ is 14′ 30″; which falls between 14′ 20″ and 14′ 40″.
And the time 7 h. 24 m. 36 ſ. falls between 7 h. 20 m. and 7 h. 40 m.
The mean of the equations under 7 h. 20 m. and 7 h. 40 m. and againſt 14′ 20″ and 14′ 40″, are 4′ 26″ and 4′ 38″, their diff. is 12″.
And 20 : 12 : : 4,6 : $2\frac{1}{2}$ : and 4′ 26″ + $2\frac{1}{2}$ = 4′ $28\frac{1}{2}$ the pro. pts. to $\frac{1}{4}$ of 58″.
Then the proportional parts to 58′, are 17′ 54″.
Again. In the Exam. above. To find the parts proportional to 3 m. 57 ſ. as 9 h. 30 m. is to 24 h.
Here 3 m. 57 ſ. being taken as 4 m.; and 4 h. 40 m. as the half of 9 h. 30 m. The equation is 47″; which doubled gives 1′ 34″ for the proportional parts required.

### 289. VII. *Of the right aſcenſions and declinations of the fixed Stars.* (312)

This table, which contains 120 of the principal fixed ſtars, viz. 60 having north declination, and 60 with ſouth declination, are fitted to the year 1780; and are ſelected partly from the catalogue which is given in the Nautical Almanac for 1773, as deduced from Dr. Bradley's Obſervations; and partly from that given by M. de La Caille, which, he ſays, “are all derived from his own obſervations made, during ten years atten- “tion to this buſineſs, either at Paris, or at the Cape of Good Hope; “that the poſitions are aſcertained with all the accuracy that could be de- “rived from the modern Aſtronomy; and that he had all proper helps, “with regard to inſtruments, aſſiſtants, and convenience, and neither “care or pains were wanting to perfect the work.

“The right aſcenſions were determined by a multitude of correſpond- “ing altitudes of each, taken with a quadrant of three feet radius, to have “their paſſage over the meridian with the greateſt exactneſs. Almoſt all “the ſtars in the northern hemiſphere have been compared with the “bright ſtar in the Harp; and thoſe in the ſouthern hemiſphere, with “Syrius; that is to ſay, on each day that the time of the ſtar's paſſing “the meridian had been found by equal altitudes, that of $\alpha$ Lyræ and “Syrius were found in like manner; the right aſcenſions of theſe two

" ſtars having been ſettled by a great many obſervations taken when " they were in the propereſt ſituation for this purpoſe.

" The declinations have been deduced from a ſufficient number of " obſervations of their zenith diſtances, taken with an inſtrument of ſix " feet radius, made with great care for this purpoſe."

The table conſiſts of nine columns; that on the left hand contains the name of the conſtellation; the next ſhews in what part of the conſtellation the ſtar is; in the 3d are the names by which certain ſtars are diſtinguiſhed; the 4th column ſhews the Greek characters by which the ſtar is marked in the cœleſtial charts, or maps of the conſtellations; the 5th ſhews the magnitude of the ſtars; the 6th and 7th contain the right aſcenſion in time, reckoned from Aries, and the yearly variation in right aſcenſion; the 8th and 9th contain the declinations and the yearly variation in declination; where thoſe which are marked + are augmented by the yearly variation; but thoſe which have the mark — annexed, are to be diminiſhed by the variation: by the help of theſe yearly variations the right aſcenſions and declinations of theſe ſtars may be fitted for any diſtant year.

Precepts for finding the culminating of the ſtars are at articles 133, 134.

## 290. VIII. *Tables of the Equation of Time.*

In page 318 are three tables, articles 313, 314, 315: Article 313 is a table of the Sun's right aſcenſion in degrees, to each degree of longitude in the firſt quadrant of the ecliptic; and alſo the differences between thoſe longitudes and right aſcenſions. The table, art. 314, contains the ſaid differences turned into time (132), of minutes, ſeconds, and the tenth part of ſeconds: the numbers in this table are the differences between the mean and true noons, ariſing from the obliquity of the ecliptic (267); and the table, art. 315, is nothing more than the equations of the center, table art. 305, converted into time; and are the differences between the times of the mean and true noons, ariſing from the eccentricity of the Earth's orbit: theſe two equations of time, properly put together, conſtitute another table, art. 316, of the abſolute equation of time with relation to the place of the Sun's apogee.

### 291. *To conſtruct the table* 316, *of the abſolute Equation of Time.*

1ſt. To the given time find the Sun's true place, or aſſume a place.

2d. The difference between that place, and the place of the apogee, gives the Sun's true anomaly.

3d. From the true anomaly find the mean. . (294)

4th. In table I. 314, ſeek the equation of time to the Sun's place.

5th. In table II. 315, ſeek the equation of time to the mean anomaly.

6th. The ſum, or difference of theſe equations, according as their titles, or ſigns direct, will be the abſolute equation of time to the Sun's place found at firſt, or to the correſponding time.

The tables, articles 314, 315, are made only to whole degrees of longitude and anomaly; the proportional parts of the differences are to be taken for minutes or ſeconds, above whole degrees of the Sun's longitude and anomaly.

292. Exam. I. *What is the Equation of Time, when the Sun's longitude is $7^{s}$ 12°?*

In table, art. 316, againſt 12° in the outſide column, and under ♏, or 7 ſ., ſtands — 16 m. 12 ſ.; which ſhews that 16 m. 12 ſ. is to be ſubtracted from the apparent time; to give the mean time of apparent noon, or the time which ſhould be ſhewn by a good clock, when the Sun's center is on the meridian.

293. Exam. II. *What is the Equation of Time when the Sun's longitude is $4^{s}$ 24° 30′ 42″?*

The difference between the equation in table, art. 316, to $4^{s}$ 24° and $4^{s}$ 25°, is $13^{s}$ decreaſing; and 30′ 42″ = 30,7′.
Then 60′ : 30,7′ : : $13^{s}$ : 6,65 or $7^{s}$, the proportional part decreaſing.
And + 3 m. 46 ſ. — 7 ſ. = + 3 m. 39 ſ., the equation ſought.
So 24 h. the apparent time of ſolar noon, increaſed by 3 m. 39 ſ. will give the mean time of noon.

*If the time was given, viz. the month, day, hour, &c., to find the Equation.*
To the given time find the Sun's longitude. (278)
Then to this longitude find the equation of time, as above.

294. *To find the mean anomaly from the true being given.*

Solution, Say, As the ſquare root of the perihelion diſtance,
To the ſquare root of the aphelion diſtance;
So the tangent of half the true anomaly,
To the tangent of half the excentric anomaly,
And As radius, to the ſign of the excentric anomaly,
So the degrees in an arc equal in length to the eccentricity,
To the degrees, &c. in the arc of correction.

The correction added to the excentric anomaly gives the mean anomaly.

295. Remarks. 1ſt. The greateſt equation of the center being taken at 1° 55′ 39″, the eccentricity (249) will be 0,01682; the aphelion diſtance will be 1,01682, and the perihelion 0,98318.

Hence the ratio of the ſquare root of the perihelion diſtance to the ſquare root of the aphelion diſtance will be expreſſed by the logarithm 0,00731; which conſtant logarithm, added to the logarithmic tangent of $\frac{1}{2}$ the true anomaly, will give the logarithmic tangent of $\frac{1}{2}$ the excentric anomaly.

296. 2d. In the 2d proportion, the arc equal to the length of the eccentricity 1682 is a conſtant quantity.
Now the radius, or mean diſtance, is equal to the length of an arc of 57°,29578 (249); then 100000 : 1682 : : 57°,29578 : 0°,96375, the length of the eccentricity in degrees; the conſtant logarithm of which is 9,98396, which added to the logarithmic ſine of the excentric anomaly, abating 10 in the index of the ſum, gives the logarithm of an arc, the degrees, minutes, and ſeconds of which being added to the excentric anomaly, give the mean anomaly.

## 297. IX. *Table of corrections for the middle time between equal altitudes of the Sun.* Art. 317.

This table, which is fitted to the latitudes of 30°, 40°, 50°, and 60°, will alſo ſerve, nearly, to all latitudes between 25° and 65°; by entering the table with the neareſt latitude to that given, and the given declination in degrees. It is conſtructed by art. 216.

Exam. I. *In latitude 50° N., when the Sun's declination is 16° N., and the interval between the morning and afternoon obſervation is 5 hours: what correction muſt be applied to the middle time, to give the time of apparent noon?*

In the table, art. 317, againſt 16° of declination taken in the outſide column, and under 50° latitude, and 5 hours, with N. declination, ſtand 12 ſeconds; which 12 ſ. applied to the middle time between the obſervations, give the time when the Sun was on the meridian.

The correction is applied to the middle time by the precepts at the bottom of the table.

298. Exam. II. *In latitude 50° N., on November 16th, 1761, obſervations at equal altitudes of the Sun were taken at the following times ſhewn by a clock, the equal altitude inſtrument having three horizontal wires.*

| Morning obſervations. | | Afternoon obſervations. | |
|---|---|---|---|
| ⊙ preceding limb | ⊙ following limb | ⊙ preceding limb | ⊙ following limb |
| $9^h\ 28^m\ 55''$ | $9^h\ 35^m\ 21\frac{1}{2}''$ | $1^h\ 46^m\ 43\frac{1}{2}''$ | $1^h\ 53^m\ 30\frac{1}{2}''$ |
| $9\ \ 32\ \ 46\frac{1}{2}$ | $9\ \ 39\ \ 23\frac{1}{2}$ | $1\ \ 50\ \ 53\frac{1}{2}$ | $1\ \ 57\ \ 28\frac{1}{2}$ |
| $9\ \ 36\ \ 44\frac{1}{2}$ | $9\ \ 43\ \ 30$ | $1\ \ 54\ \ 56$ | $2\ \ 1\ \ 20\frac{1}{2}$ |

$$\text{Now}\ \left.\begin{array}{l} \dfrac{9^h\ 28^m\ 55''+2^h\ \ 1^m\ 20\frac{1}{2}''+12^h}{2}=11^h\ 45^m\ 7\frac{3}{4}'' \\ \dfrac{9\ \ 32\ \ 46\frac{1}{2}+1\ \ 57\ \ 28\frac{1}{2}\ +12}{2}=11\ \ 45\ \ 7\frac{1}{2} \\ \dfrac{9\ \ 36\ \ 44\frac{1}{2}+1\ \ 53\ \ 30\frac{1}{2}\ +12}{2}=11\ \ 45\ \ 7\frac{1}{2} \end{array}\right\}\ \text{the mean} = 11^h\ 45^m\ 7{,}6''\ \text{by the preceding limb.}$$

$$\text{Again}\ \left.\begin{array}{l} \dfrac{9\ \ 35\ \ 21\frac{1}{2}+1\ \ 54\ \ 56\ \ +12}{2}=11\ \ 45\ \ 8\frac{3}{4} \\ \dfrac{9\ \ 39\ \ 23\frac{1}{2}+1\ \ 50\ \ 53\frac{1}{2}\ +12}{2}=11\ \ 45\ \ 8\frac{1}{2} \\ \dfrac{9\ \ 43\ \ 30\ \ +1\ \ 46\ \ 43\frac{1}{2}\ +12}{2}=11\ \ 45\ \ 6\frac{3}{4} \end{array}\right\}\ \text{the mean} = 11^h\ 45^m\ 8''\ \text{by the following limb.}$$

The mean time of obſervation from both limbs is 11 h. 45 m. 7,8 ſ.
The declination on the day of obſervation is 19° S. nearly; the interval between the obſervations is about 4 hours; and theſe give + 14 ſeconds for the correction of the middle time.
So the Sun was on the meridian when the clock ſhewed 11 h. 45 m. 21,8 ſ.
The Sun's place, at that time, was 7ˢ 24° 26′ 46″ nearly.
In table 316, to 7ˢ 24° the tabular difference is 12ˢ, decreaſing.

Then 60′ : 26,75′ : : 12 ſ. : 5,35 ſ.; and + 14 m. 56 ſ. — $5\frac{1}{3}$ ſ. = + 14 m. $50\frac{2}{3}$ ſ., which is the equation of time : hence 11 h. 45 m. 21,8 ſ. + 14 m. 50,6 ſ. = 12 h. 0 m. 12,4 ſ.
Which ſhews that the clock was 12 ſ., nearly, too faſt.

### 299. X. *Tables of Refraction and of the Sun's parallax.* (318, 319)

Theſe tables are the reſult of the experience of ſome of the moſt eminent Aſtronomers. By the refraction of the atmoſphere, objects appear more elevated than they really are, and therefore the apparent altitude is to be diminiſhed by the refraction, which is greateſt near the horizon, and gradually diminiſhes towards the zenith, where there is no refraction. The parallax in altitude is the difference between the altitude of an object, as ſeen from the cente rand ſurface of the Earth, that from the center being the true altitude, and the greateſt, except at the zenith, where parallax vaniſhes; therefore the apparent altitude is to be augmented by the parallax.

Exam. *The Sun's apparent altitude was obſerved to be* 18° 34′ 48″; *what was his true altitude?*

| | | | | | |
|---|---|---|---|---|---|
| Apparent altitude | 18° | 34′ | 48″ | Refraction 2′ 47″ | — |
| Correction is — | | 2 | 38 | Parallax 9 | + |
| Sun's true altitude | 18 | 32 | 10. | | |

* * *

### 300.

# ASTRONOMICAL TABLES,

## Fitted, in general, to the meridian of Greenwich.

Art.
I. Mean motions of the Sun and Apogee for years. (301)
II. Mean motions of the Sun and Apogee for radical years. (302)
III. Mean motions of the Sun for hours, minutes, and ſeconds. (303)
IV. Mean motions of the Sun and Apogee for months and days. (304)
V. Equations of the Sun's center. (305)
VI. Sun's declination to ſigns and degrees. (306)
VII. Sun's right aſcenſion in time to ſigns and degrees. (307)
VIII. Sun's longitude to each day for the years 1792, 1793, 1794, and 1795. (308)
IX. Sun's declination to each day for the ſame four years. (309)
X. Sun's right aſcenſion to each day for the ſame four years. (310)
XI. To fit the tables VIII. IX. X. to any meridian. (311)
XII. Right aſcenſions and declinations to 120 fixed ſtars. (312)
XIII. Sun's right aſcenſion in degrees, &c. to ſigns and degrees of longitude. (313)
XIV. Equation of time on the obliquity of the ecliptic. (314)
XV. Equation of time on the eccentricity of the Earth's orbit. (315)
XVI. Abſolute equation of time, to the Sun's longitude. (316)
XVII. Correction of the middle time between equal altitudes. (317)
XVIII. Correction of altitude for refractions. (318)
XIX. Correction of the Sun's altitude for his parallax. (319)

The tables of the Sun's place, declination, and right aſcenſion, are fitted to the years 1792, 1793, 1794, 1795; and will ſerve in moſt nautical operations as well for the four years preceding, viz. 1788, 1789, 1790, 1791, and alſo for the four years following, viz. 1796, 1797, 1798, 1799, as is mentioned at the heads of thoſe tables. But as a Nautical Almanack is publiſhed yearly under the direction of the commiſſioners of longitude, the tables contained therein ſhould be conſulted in caſes where the utmoſt preciſion is neceſſary.

## TABLES of the MEAN MOTIONS of the SUN and his APOGEE to MEAN SOLAR TIME.

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For years abſolute.

| Years. | M. lon. ⊙ |  |  |  | M. lon. ap. |  |  |
|---|---|---|---|---|---|---|---|
|  | s | ° | ′ | ″ | ° | ′ | ″ |
| 1 | 11 | 29 | 45 | 40 |  | 1 | 5 |
| 2 | 11 | 29 | 31 | 20 |  | 2 | 10 |
| 3 | 11 | 29 | 17 | 1 |  | 3 | 15 |
| 4 | 0 | 0 | 1 | 49 |  | 4 | 20 |
| 5 | 11 | 29 | 47 | 29 |  | 5 | 25 |
| 6 | 11 | 29 | 33 | 10 |  | 6 | 30 |
| 7 | 11 | 29 | 18 | 50 |  | 7 | 35 |
| 8 | 0 | 0 | 3 | 39 |  | 8 | 40 |
| 9 | 11 | 29 | 49 | 19 |  | 9 | 45 |
| 10 | 11 | 29 | 34 | 59 |  | 10 | 50 |
| 11 | 11 | 29 | 20 | 39 |  | 11 | 55 |
| 12 | 0 | 0 | 5 | 28 |  | 13 | 0 |
| 13 | 11 | 29 | 51 | 8 |  | 14 | 5 |
| 14 | 11 | 29 | 36 | 48 |  | 15 | 10 |
| 15 | 11 | 29 | 22 | 29 |  | 16 | 15 |
| 16 | 0 | 0 | 7 | 17 |  | 17 | 20 |
| 17 | 11 | 29 | 52 | 57 |  | 18 | 25 |
| 18 | 11 | 29 | 38 | 38 |  | 19 | 30 |
| 19 | 11 | 29 | 24 | 18 |  | 20 | 35 |
| 20 | 0 | 0 | 9 | 6 |  | 21 | 40 |
| 30 | 11 | 29 | 44 | 5 |  | 32 | 30 |
| 40 | 0 | 0 | 18 | 13 |  | 43 | 20 |
| 50 | 11 | 29 | 53 | 11 |  | 54 | 10 |
| 60 | 0 | 0 | 27 | 19 | 1 | 5 | 0 |
| 70 | 0 | 0 | 2 | 18 | 1 | 15 | 50 |
| 80 | 0 | 0 | 36 | 26 | 1 | 26 | 40 |
| 90 | 0 | 0 | 11 | 24 | 1 | 37 | 30 |
| 100 | 0 | 0 | 45 | 32 | 1 | 48 | 20 |
| 500 | 0 | 3 | 47 | 40 | 9 | 1 | 40 |
| 1000 | 0 | 7 | 35 | 20 | 18 | 3 | 20 |

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For the radical years, new ſtile.

| Years. | M. lon. ⊙ |  |  |  | M.lon.ap. |  |  |  | Years. | M. lon. ⊙ |  |  |  | M.lon.ap. |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | s | ° | ′ | ″ | s | ° | ′ | ″ |  | s | ° | ′ | ″ | s | ° | ′ | ″ |
| B 1752 | 9 | 10 | 31 | 29 | 3 | 8 | 38 | 45 | 1782 | 9 | 10 | 15 | 34 | 3 | 9 | 11 | 15 |
| 53 |  | 10 | 17 | 9 |  |  | 39 | 50 | 83 |  | 10 | 1 | 15 |  |  | 12 | 20 |
| 54 |  | 10 | 2 | 49 |  |  | 40 | 55 | B 1784 |  | 10 | 46 | 3 |  |  | 13 | 25 |
| 55 |  | 9 | 48 | 30 |  |  | 42 | 0 | 85 |  | 10 | 31 | 43 |  |  | 14 | 30 |
| B 1756 |  | 10 | 33 | 18 |  |  | 43 | 5 | 86 |  | 10 | 17 | 24 |  |  | 15 | 35 |
| 57 |  | 10 | 18 | 58 |  |  | 44 | 10 | 87 |  | 10 | 3 | 4 |  |  | 16 | 40 |
| 58 | 9 | 10 | 4 | 39 | 3 | 8 | 45 | 15 | B 1788 | 9 | 10 | 47 | 53 | 3 | 9 | 17 | 45 |
| 59 |  | 9 | 50 | 19 |  |  | 46 | 20 | 89 |  | 10 | 33 | 33 |  |  | 18 | 50 |
| B 1760 |  | 10 | 35 | 8 |  |  | 47 | 25 | 90 |  | 10 | 19 | 13 |  |  | 19 | 55 |
| 61 |  | 10 | 20 | 48 |  |  | 48 | 30 | 91 |  | 10 | 4 | 53 |  |  | 21 | 0 |
| 62 |  | 10 | 6 | 28 |  |  | 49 | 35 | B 1792 |  | 10 | 49 | 42 |  |  | 22 | 5 |
| 63 |  | 9 | 52 | 8 |  |  | 50 | 40 | 93 |  | 10 | 35 | 22 |  |  | 23 | 10 |
| B 1764 | 9 | 10 | 36 | 57 | 3 | 8 | 51 | 45 | 94 | 9 | 10 | 21 | 2 | 3 | 9 | 24 | 15 |
| 65 |  | 10 | 22 | 37 |  |  | 52 | 50 | 95 |  | 10 | 6 | 42 |  |  | 25 | 20 |
| 66 |  | 10 | 8 | 17 |  |  | 53 | 55 | B 1796 |  | 10 | 51 | 31 |  |  | 26 | 25 |
| 67 |  | 9 | 53 | 58 |  |  | 55 | 0 | 97 |  | 10 | 37 | 11 |  |  | 27 | 30 |
| B 1768 |  | 10 | 38 | 46 |  |  | 56 | 5 | 98 |  | 10 | 22 | 52 |  |  | 28 | 35 |
| 69 |  | 10 | 24 | 26 |  |  | 57 | 10 | 99 |  | 10 | 8 | 32 |  |  | 29 | 40 |
| 70 | 9 | 10 | 10 | 7 | 3 | 8 | 58 | 15 | C 1800 | 9 | 9 | 54 | 12 | 3 | 9 | 30 | 45 |
| 71 |  | 9 | 55 | 47 |  |  | 59 | 20 | B 1820 |  | 10 | 3 | 18 |  |  | 52 | 25 |
| B 1772 |  | 10 | 40 | 35 |  | 9 | 0 | 25 | B 1840 |  | 10 | 12 | 25 | 3 | 10 | 14 | 5 |
| 73 |  | 10 | 26 | 16 |  |  | 1 | 30 | B 1860 |  | 10 | 21 | 31 |  |  | 35 | 45 |
| 74 |  | 10 | 11 | 56 |  |  | 2 | 35 | B 1880 |  | 10 | 30 | 38 |  |  | 57 | 25 |
| 75 |  | 9 | 57 | 36 |  |  | 3 | 40 | C 1900 |  | 9 | 40 | 36 |  | 11 | 19 | 5 |
| B 1776 | 9 | 10 | 42 | 25 | 3 | 9 | 4 | 45 | B 1920 | 9 | 9 | 49 | 42 | 3 | 11 | 40 | 45 |
| 77 |  | 10 | 28 | 5 |  |  | 5 | 50 | B 1940 |  | 9 | 58 | 48 |  | 12 | 2 | 25 |
| 78 |  | 10 | 13 | 45 |  |  | 6 | 55 | B 1960 |  | 10 | 7 | 55 |  |  | 24 | 5 |
| 79 |  | 9 | 59 | 25 |  |  | 8 | 0 | B 1980 |  | 10 | 17 | 1 |  |  | 45 | 45 |
| B 1780 |  | 10 | 44 | 14 |  |  | 9 | 5 | B 2000 |  | 10 | 26 | 8 |  | 13 | 7 | 25 |
| 81 |  | 10 | 29 | 54 |  |  | 10 | 10 | C 2100 |  | 10 | 12 | 31 |  | 14 | 55 | 45 |

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For hours, minutes, and ſecs.

| H | Lon. ⊙ |  |  | M | Lon. ⊙ |  |  |
|---|---|---|---|---|---|---|---|
|  | ′ | ″ | ‴ |  | ′ | ″ | ‴ |
| 1 | 2 | 27 | 51 | 31 | 1 | 16 | 23 |
| 2 | 4 | 55 | 42 | 32 | 1 | 18 | 51 |
| 3 | 7 | 23 | 32 | 33 | 1 | 21 | 19 |
| 4 | 9 | 51 | 23 | 34 | 1 | 23 | 47 |
| 5 | 12 | 19 | 14 | 35 | 1 | 26 | 15 |
| 6 | 14 | 47 | 5 | 36 | 1 | 28 | 42 |
| 7 | 17 | 14 | 56 | 37 | 1 | 31 | 10 |
| 8 | 19 | 42 | 47 | 38 | 1 | 33 | 38 |
| 9 | 22 | 10 | 37 | 39 | 1 | 36 | 6 |
| 10 | 24 | 38 | 28 | 40 | 1 | 38 | 34 |
| 11 | 27 | 6 | 19 | 41 | 1 | 41 | 2 |
| 12 | 29 | 34 | 10 | 42 | 1 | 43 | 30 |
| 13 | 32 | 2 | 1 | 43 | 1 | 45 | 57 |
| 14 | 34 | 29 | 52 | 44 | 1 | 48 | 25 |
| 15 | 36 | 57 | 42 | 45 | 1 | 50 | 53 |
| 16 | 39 | 25 | 33 | 46 | 1 | 53 | 21 |
| 17 | 41 | 53 | 24 | 47 | 1 | 55 | 49 |
| 18 | 44 | 21 | 15 | 48 | 1 | 58 | 17 |
| 19 | 46 | 49 | 6 | 49 | 2 | 0 | 44 |
| 20 | 49 | 16 | 56 | 50 | 2 | 3 | 12 |
| 21 | 51 | 44 | 47 | 51 | 2 | 5 | 40 |
| 22 | 54 | 12 | 38 | 52 | 2 | 8 | 8 |
| 23 | 56 | 40 | 29 | 53 | 2 | 10 | 36 |
| 24 | 59 | 8 | 20 | 54 | 2 | 13 | 4 |
| 25 | 61 | 36 | 11 | 55 | 2 | 15 | 32 |
| 26 | 64 | 4 | 1 | 56 | 2 | 17 | 59 |
| 27 | 66 | 31 | 52 | 57 | 2 | 20 | 27 |
| 28 | 68 | 59 | 43 | 58 | 2 | 22 | 55 |
| 29 | 71 | 27 | 34 | 59 | 2 | 25 | 23 |
| 30 | 73 | 55 | 25 | 60 | 2 | 27 | 51 |

# TABLE of the MEAN MOTIONS of the SUN and APOGEE to MONTHS and DAYS.

| Days. | January. | February. | March. | April. | May. | June. | July. | August. | September. | October. | November. | December. | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ | ☉ M. lon.<br>s ° ′ ″ |  |
| 1 | 0 0 59 8 | 1 1 32 27 | 1 29 8 20 | 2 29 41 38 | 2 29 15 48 | 4 29 49 6 | 5 29 23 16 | 6 29 56 34 | 8 0 29 52 | 9 0 4 2 | 10 0 37 20 | 11 0 11 30 | 1 |
| 2 | 1 58 17 | 2 31 35 | 2 0 7 28 | 3 0 40 46 | 4 0 14 56 | 5 0 48 14 | 6 0 22 24 | 7 0 55 42 | 1 29 1 | 1 3 11 | 1 36 29 | 1 10 39 | 2 |
| 3 | 2 57 25 | 3 30 43 | 1 6 36 | 1 39 55 | 1 14 4 | 1 47 23 | 1 21 33 | 1 54 51 | 2 28 9 | 2 2 19 | 2 35 37 | 2 9 47 | 3 |
| 4 | 3 56 33 | 4 29 52 | 2 5 45 | 2 39 3 | 2 13 13 | 2 46 31 | 2 20 41 | 2 53 59 | 3 27 17 | 3 1 27 | 3 34 45 | 3 8 55 | 4 |
| 5 | 4 55 42 | 5 29 0 | 3 4 53 | 3 38 11 | 3 12 21 | 3 45 39 | 3 19 49 | 3 53 7 | 4 26 26 | 4 0 36 | 4 33 54 | 4 8 4 | 5 |
| 6 | 5 54 50 | 6 28 8 | 4 4 1 | 4 37 20 | 4 11 29 | 4 44 48 | 4 18 58 | 4 52 16 | 5 25 34 | 4 59 44 | 5 33 2 | 5 7 12 | 6 |
| 7 | 6 53 58 | 7 27 16 | 5 3 10 | 5 36 28 | 5 10 38 | 5 43 56 | 5 18 6 | 5 51 24 | 6 24 42 | 5 58 52 | 6 32 10 | 6 6 20 | 7 |
| 8 | 7 53 7 | 8 26 25 | 6 2 18 | 6 35 36 | 6 9 46 | 6 43 4 | 6 17 14 | 6 50 32 | 7 23 51 | 6 58 1 | 7 31 19 | 7 5 29 | 8 |
| 9 | 8 52 15 | 9 25 33 | 7 1 26 | 7 34 45 | 7 8 54 | 7 42 13 | 7 16 22 | 7 49 41 | 8 22 59 | 7 57 9 | 8 30 27 | 8 4 37 | 9 |
| 10 | 9 51 23 | 10 24 41 | 8 0 35 | 8 33 53 | 8 8 3 | 8 41 21 | 8 15 31 | 8 48 49 | 9 22 7 | 8 56 17 | 9 29 36 | 9 3 45 | 10 |
| 11 | 10 50 32 | 11 23 50 | 8 59 43 | 9 33 1 | 9 7 11 | 9 40 29 | 9 14 39 | 9 47 57 | 10 21 16 | 9 55 26 | 10 28 44 | 10 2 54 | 11 |
| 12 | 11 49 40 | 12 22 58 | 9 58 51 | 10 32 10 | 10 6 19 | 10 39 38 | 10 13 48 | 10 47 6 | 11 20 24 | 10 54 34 | 11 27 52 | 11 2 2 | 12 |
| 13 | 12 48 48 | 13 22 6 | 10 58 0 | 11 31 18 | 11 5 28 | 11 38 46 | 11 12 56 | 11 46 14 | 12 19 32 | 11 53 42 | 12 27 0 | 12 1 10 | 13 |
| 14 | 13 47 57 | 14 21 15 | 11 57 8 | 12 30 26 | 12 4 36 | 12 37 54 | 12 12 4 | 12 45 22 | 13 18 41 | 12 52 51 | 13 26 9 | 13 0 19 | 14 |
| 15 | 14 47 5 | 15 20 23 | 12 56 16 | 13 29 35 | 13 3 44 | 13 37 3 | 13 11 13 | 13 44 31 | 14 17 49 | 13 51 59 | 14 25 17 | 13 59 27 | 15 |
| 16 | 15 46 13 | 16 19 31 | 13 55 25 | 14 28 43 | 14 2 53 | 14 36 11 | 14 10 21 | 14 43 39 | 15 16 57 | 14 51 7 | 15 24 25 | 14 58 35 | 16 |
| 17 | 16 45 22 | 17 18 40 | 14 54 33 | 15 27 51 | 15 2 1 | 15 35 19 | 15 9 29 | 15 42 47 | 16 16 6 | 15 50 16 | 16 23 34 | 15 57 44 | 17 |
| 18 | 17 44 30 | 18 17 48 | 15 53 4 | 16 27 0 | 16 1 9 | 16 34 28 | 16 8 38 | 16 41 56 | 17 15 14 | 16 49 24 | 17 22 42 | 16 56 52 | 18 |
| 19 | 18 43 38 | 19 16 56 | 16 52 50 | 17 26 8 | 17 0 18 | 17 33 36 | 17 7 46 | 17 41 4 | 18 14 22 | 17 48 32 | 18 21 50 | 17 56 0 | 19 |
| 20 | 19 42 47 | 20 16 5 | 17 51 58 | 18 25 16 | 17 59 26 | 18 32 44 | 18 6 54 | 18 40 12 | 19 13 31 | 18 47 40 | 19 20 59 | 18 55 9 | 20 |
| 21 | 20 41 55 | 21 15 13 | 18 51 6 | 19 24 25 | 18 58 34 | 19 31 53 | 19 6 3 | 19 39 21 | 20 12 39 | 19 46 49 | 20 20 7 | 19 54 17 | 21 |
| 22 | 21 41 3 | 22 14 21 | 19 50 15 | 20 23 33 | 19 57 43 | 20 31 1 | 20 5 11 | 20 38 29 | 21 11 47 | 20 45 57 | 21 19 15 | 20 53 25 | 22 |
| 23 | 22 40 12 | 23 13 30 | 20 49 23 | 21 22 41 | 20 56 51 | 21 30 9 | 21 4 19 | 21 37 37 | 22 10 56 | 21 45 5 | 22 18 24 | 21 52 34 | 23 |
| 24 | 23 39 20 | 24 12 38 | 21 48 31 | 22 21 50 | 21 55 59 | 22 29 18 | 22 3 28 | 22 36 46 | 23 10 4 | 22 44 14 | 23 17 32 | 22 51 42 | 24 |
| 25 | 24 38 28 | 25 11 46 | 22 47 40 | 23 20 58 | 22 55 8 | 23 28 26 | 23 2 36 | 23 35 54 | 24 9 12 | 23 43 22 | 24 16 40 | 23 50 50 | 25 |
| 26 | 25 37 37 | 26 10 55 | 23 46 48 | 24 20 6 | 23 54 16 | 24 27 34 | 24 1 44 | 24 35 2 | 25 8 21 | 24 42 30 | 25 15 49 | 24 49 59 | 26 |
| 27 | 26 36 45 | 27 10 3 | 24 45 56 | 25 19 15 | 24 53 24 | 25 26 43 | 25 0 52 | 25 34 11 | 26 7 29 | 25 41 39 | 26 14 57 | 25 49 7 | 27 |
| 28 | 27 35 53 | 28 9 11 | 25 45 5 | 26 18 23 | 25 52 33 | 26 25 51 | 26 0 1 | 26 33 19 | 27 6 37 | 26 40 47 | 27 14 5 | 26 48 15 | 28 |
| 29 | 28 35 2 |  | 26 44 13 | 27 17 31 | 26 51 41 | 27 24 59 | 26 59 9 | 27 32 27 | 28 5 46 | 27 39 55 | 28 13 14 | 27 47 23 | 29 |
| 30 | 29 34 10 |  | 27 43 21 | 28 16 40 | 27 50 49 | 28 24 8 | 27 58 17 | 28 31 36 | 29 4 54 | 28 39 4 | 29 12 22 | 28 46 32 | 30 |
| 31 | 1 0 33 18 |  | 28 42 30 |  | 28 49 58 |  | 28 57 26 | 29 30 44 |  | 29 38 12 |  | 29 45 40 | 31 |
| M. M. ap. | 0 5 | 0 10 | 0 16 | 0 21 | 0 27 | 0 32 | 0 38 | 0 43 | 0 49 | 0 54 | 0 59 | 1 5 |  |

305. TABLE of the EQUATION of the SUN'S CENTER to each ſign and degree of mean Anomaly.

| Deg. | 0 Subt. | Diff. | I. Subt. | Diff. | II. Subt. | Diff. | III. Subt. | Diff. | IV. Subt. | Diff. | V. Subt. | Diff. | Deg. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | |
| 0 | 0 0 0 | 1 59 | 0 56 47 | 1 43 | 1 39 6 | 1 1 | 1 55 37 | 0 2 | 1 41 12 | 1 0 | 0 58 53 | 1 46 | 30 |
| 1 | 1 59 | 58 | 58 30 | 42 | 40 7 | 0 59 | 55 39 | 1 | 40 12 | 2 | 57 7 | 48 | 29 |
| 2 | 3 57 | 59 | 1 0 12 | 41 | 41 6 | 57 | 55 38 | 2 | 39 10 | 4 | 55 19 | 49 | 28 |
| 3 | 5 56 | 58 | 1 53 | 40 | 42 3 | 56 | 55 36 | 5 | 38 6 | 6 | 53 30 | 50 | 27 |
| 4 | 7 54 | 58 | 3 33 | 39 | 42 59 | 53 | 55 31 | 7 | 37 0 | 8 | 51 40 | 51 | 26 |
| 5 | 9 52 | 58 | 5 12 | 38 | 43 52 | 52 | 55 24 | 9 | 35 52 | 9 | 49 49 | 52 | 25 |
| 6 | 0 11 50 | 1 58 | 1 6 50 | 1 37 | 1 44 44 | 0 50 | 1 55 15 | 0 12 | 1 34 43 | 1 11 | 0 47 57 | 1 52 | 24 |
| 7 | 13 48 | 58 | 8 27 | 35 | 45 34 | 48 | 55 3 | 13 | 33 32 | 13 | 46 5 | 54 | 23 |
| 8 | 15 46 | 57 | 10 2 | 34 | 46 22 | 46 | 54 50 | 15 | 32 19 | 15 | 44 11 | 55 | 22 |
| 9 | 17 43 | 57 | 11 36 | 33 | 47 8 | 45 | 54 35 | 18 | 31 4 | 17 | 42 16 | 55 | 21 |
| 10 | 19 40 | 57 | 13 9 | 32 | 47 53 | 42 | 54 17 | 20 | 29 47 | 18 | 40 21 | 56 | 20 |
| 11 | 21 37 | 56 | 14 41 | 30 | 48 35 | 40 | 53 57 | 21 | 28 29 | 20 | 38 25 | 57 | 19 |
| 12 | 0 23 33 | 1 56 | 1 16 11 | 1 29 | 1 49 15 | 0 39 | 1 53 36 | 0 24 | 1 27 9 | 1 21 | 0 36 28 | 1 58 | 18 |
| 13 | 25 29 | 56 | 17 40 | 28 | 49 54 | 36 | 53 12 | 26 | 25 48 | 23 | 34 30 | 58 | 17 |
| 14 | 27 25 | 55 | 19 8 | 26 | 50 30 | 35 | 52 46 | 28 | 24 25 | 25 | 32 32 | 59 | 16 |
| 15 | 29 20 | 55 | 20 34 | 25 | 51 5 | 32 | 52 18 | 30 | 23 0 | 26 | 30 33 | 2 0 | 15 |
| 16 | 31 15 | 54 | 21 59 | 23 | 51 37 | 31 | 51 48 | 33 | 21 34 | 28 | 28 33 | 0 | 14 |
| 17 | 33 9 | 53 | 23 22 | 22 | 52 8 | 28 | 51 15 | 34 | 20 6 | 30 | 26 33 | 0 | 13 |
| 18 | 0 35 2 | 1 53 | 1 24 44 | 1 21 | 1 52 36 | 0 27 | 1 50 41 | 0 36 | 1 18 36 | 1 31 | 0 24 33 | 2 1 | 12 |
| 19 | 36 55 | 52 | 26 5 | 19 | 53 3 | 24 | 50 5 | 39 | 17 5 | 32 | 22 32 | 2 | 11 |
| 20 | 38 47 | 52 | 27 24 | 17 | 53 27 | 23 | 49 26 | 40 | 15 33 | 34 | 20 30 | 2 | 10 |
| 21 | 40 39 | 51 | 28 41 | 16 | 53 50 | 20 | 48 46 | 43 | 13 59 | 35 | 18 28 | 2 | 9 |
| 22 | 42 30 | 50 | 29 57 | 14 | 54 10 | 18 | 48 3 | 44 | 12 24 | 37 | 16 26 | 2 | 8 |
| 23 | 44 20 | 49 | 31 11 | 13 | 54 28 | 16 | 47 19 | 47 | 10 47 | 38 | 14 24 | 3 | 7 |
| 24 | 0 46 9 | 1 48 | 1 32 24 | 1 11 | 1 54 44 | 0 14 | 1 46 32 | 0 48 | 1 9 9 | 1 40 | 0 12 21 | 2 3 | 6 |
| 25 | 47 57 | 48 | 33 35 | 10 | 54 58 | 12 | 45 44 | 51 | 7 29 | 40 | 10 18 | 4 | 5 |
| 26 | 49 45 | 47 | 34 45 | 8 | 55 10 | 10 | 44 53 | 52 | 5 49 | 42 | 8 14 | 3 | 4 |
| 27 | 51 32 | 46 | 35 53 | 6 | 55 20 | 8 | 44 1 | 54 | 4 7 | 43 | 6 11 | 4 | 3 |
| 28 | 53 18 | 45 | 36 59 | 4 | 55 28 | 6 | 43 7 | 57 | 2 24 | 45 | 4 7 | 3 | 2 |
| 29 | 55 3 | 1 44 | 38 3 | 1 3 | 55 34 | 3 | 42 10 | 0 58 | 0 39 | 46 | 2 4 | 4 | 1 |
| 30 | 0 56 47 | | 1 39 6 | | 1 55 37 | | 1 41 12 | | 0 58 53 | | 0 0 0 | | 0 |
| | XI. Add. | Diff. | X. Add. | Diff. | IX. Add. | Diff. | VIII. Add. | Diff. | VII. Add. | Diff. | VI. Add. | Diff. | |

### 306. Table of the Sun's Declination to each ſign and degree of longitude.

| Deg. | ♈ 0 ♎ 6 ⊙ decl. | Diff. | ♉ 1 ♏ 7 ⊙ decl. | Diff. | ♊ 2 ♐ 8 ⊙ decl. | Diff. | Deg. |
|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | |
| 0 | 0 0 0 | 23 53 | 11 29 5 | 21 1 | 20 10 25 | 12 32 | 30 |
| 1 | 0 23 53 | 23 54 | 11 50 6 | 20 50 | 20 22 57 | 12 10 | 29 |
| 2 | 0 47 47 | 23 52 | 12 10 56 | 20 38 | 20 35 7 | 11 48 | 28 |
| 3 | 1 11 39 | 23 51 | 12 31 34 | 20 25 | 20 46 55 | 11 25 | 27 |
| 4 | 1 35 30 | 23 50 | 12 51 59 | 20 13 | 20 58 20 | 11 1 | 26 |
| 5 | 1 59 20 | 23 48 | 13 12 12 | 20 0 | 21 9 21 | 10 38 | 25 |
| 6 | 2 23 8 | 23 46 | 13 32 12 | 19 46 | 21 19 59 | 10 14 | 24 |
| 7 | 2 46 54 | 23 43 | 13 51 58 | 19 32 | 21 30 13 | 9 50 | 23 |
| 8 | 3 10 37 | 23 40 | 14 11 30 | 19 18 | 21 40 3 | 9 26 | 22 |
| 9 | 3 34 17 | 23 37 | 14 30 48 | 19 4 | 21 49 29 | 9 1 | 21 |
| 10 | 3 57 54 | 23 33 | 14 49 52 | 18 48 | 21 58 30 | 8 36 | 20 |
| 11 | 4 21 27 | 23 30 | 15 8 40 | 18 33 | 22 7 6 | 8 11 | 19 |
| 12 | 4 44 57 | 23 25 | 15 27 13 | 18 17 | 22 15 17 | 7 46 | 18 |
| 13 | 5 8 22 | 23 20 | 15 45 30 | 18 1 | 22 23 3 | 7 21 | 17 |
| 14 | 5 31 42 | 23 15 | 16 3 31 | 17 45 | 22 30 24 | 6 54 | 16 |
| 15 | 5 54 57 | 23 9 | 16 21 16 | 17 28 | 22 37 18 | 6 29 | 15 |
| 16 | 6 18 6 | 23 3 | 16 38 44 | 17 11 | 22 43 47 | 6 3 | 14 |
| 17 | 6 41 9 | 22 57 | 16 55 55 | 16 53 | 22 49 50 | 5 37 | 13 |
| 18 | 7 4 6 | 22 51 | 17 12 48 | 16 35 | 22 55 27 | 5 11 | 12 |
| 19 | 7 26 57 | 22 43 | 17 29 23 | 16 17 | 23 0 38 | 4 44 | 11 |
| 20 | 7 49 41 | 22 36 | 17 45 40 | 15 58 | 23 5 22 | 4 17 | 10 |
| 21 | 8 12 17 | 22 28 | 18 1 38 | 15 40 | 23 9 39 | 3 50 | 9 |
| 22 | 8 34 45 | 22 20 | 18 17 18 | 15 20 | 23 13 29 | 3 24 | 8 |
| 23 | 8 57 5 | 22 11 | 18 32 38 | 15 0 | 23 16 53 | 2 57 | 7 |
| 24 | 9 19 17 | 22 2 | 18 47 38 | 14 40 | 23 19 50 | 2 30 | 6 |
| 25 | 9 41 19 | 21 53 | 19 2 18 | 14 19 | 23 22 20 | 2 2 | 5 |
| 26 | 10 3 12 | 21 44 | 19 16 37 | 13 58 | 23 24 22 | 1 35 | 4 |
| 27 | 10 24 56 | 21 34 | 19 30 35 | 13 38 | 23 25 57 | 1 8 | 3 |
| 28 | 10 46 30 | 21 23 | 19 44 13 | 13 17 | 23 27 5 | 0 41 | 2 |
| 29 | 11 7 53 | 21 12 | 19 57 30 | 12 55 | 23 27 46 | 0 14 | 1 |
| 30 | 11 29 5 | | 20 10 25 | | 23 28 0 | | 0 |
| D | ♓ 11 ♍ | Diff. | ♒ 10 ♌ 4 | Diff. | ♑ 9 ♋ 3 | Diff. | D |

### 307. Table of the Sun's Right Ascension to each ſign and degree of longitude.

| Deg. | ♈ 0 ♎ 6 ⊙ rt. aſ. | Diff. | ♉ 1 ♏ 7 ⊙ rt. aſ. | Diff. | ♊ 2 ♐ 8 ⊙ rt. aſ. | Diff. | ♋ 3 ♑ 9 ⊙ rt. aſ. | Diff. | ♌ 4 ♒ 10 ⊙ rt. aſ. | Diff. | ♍ 5 ♓ 11 ⊙ rt. aſ. | Diff. | Deg. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. m. ſ. | m. ſ. | h. m. ſ. | m. ſ. | h. m. ſ. | m. ſ. | h. m. ſ. | m. ſ. | h. m. ſ. | m. ſ. | h. m. ſ. | m. ſ. | |
| 0 | 0 0 0 | 3 40 | 1 51 37 | 3 50 | 3 51 15 | 4 10 | 6 0 0 | 4 22 | 8 8 45 | 4 9 | 10 8 23 | 3 49 | 0 |
| 1 | 3 40 | 3 40 | 55 27 | 3 50 | 55 25 | 4 11 | 4 22 | 4 21 | 12 54 | 4 9 | 12 12 | 3 48 | 1 |
| 2 | 7 20 | 3 41 | 59 17 | 3 51 | 59 36 | 4 12 | 8 43 | 4 22 | 17 3 | 4 8 | 16 0 | 3 48 | 2 |
| 3 | 11 1 | 3 40 | 2 3 8 | 3 51 | 4 3 48 | 4 12 | 13 5 | 4 21 | 21 11 | 4 8 | 19 48 | 3 47 | 3 |
| 4 | 14 41 | 3 40 | 6 59 | 3 52 | 8 0 | 4 13 | 17 26 | 4 21 | 25 19 | 4 7 | 23 35 | 3 47 | 4 |
| 5 | 18 21 | 3 41 | 10 51 | 3 53 | 12 13 | 4 13 | 21 47 | 4 22 | 27 26 | 4 6 | 27 22 | 3 46 | 5 |
| 6 | 22 2 | 3 40 | 14 44 | 3 53 | 16 26 | 4 14 | 26 9 | 4 21 | 33 32 | 4 5 | 31 8 | 3 46 | 6 |
| 7 | 0 25 42 | 3 41 | 2 18 37 | 3 54 | 4 20 40 | 4 15 | 6 30 30 | 4 21 | 8 37 37 | 4 4 | 10 34 54 | 3 46 | 7 |
| 8 | 29 23 | 3 41 | 22 31 | 3 54 | 24 55 | 4 15 | 34 51 | 4 20 | 41 41 | 4 4 | 38 40 | 3 45 | 8 |
| 9 | 33 4 | 3 41 | 26 25 | 3 55 | 29 10 | 4 16 | 39 11 | 4 20 | 45 45 | 4 3 | 42 25 | 3 44 | 9 |
| 10 | 36 45 | 3 41 | 30 20 | 3 56 | 33 26 | 4 16 | 43 31 | 4 20 | 49 48 | 4 3 | 46 9 | 3 44 | 10 |
| 11 | 40 26 | 3 42 | 34 16 | 3 57 | 37 42 | 4 17 | 47 51 | 4 20 | 51 51 | 4 1 | 49 53 | 3 44 | 11 |
| 12 | 44 8 | 3 42 | 38 13 | 3 57 | 41 59 | 4 17 | 52 11 | 4 19 | 57 52 | 4 1 | 53 37 | 3 43 | 12 |
| 13 | 0 47 50 | 3 42 | 2 42 10 | 3 58 | 4 46 16 | 4 18 | 6 56 30 | 4 19 | 9 1 53 | 4 0 | 10 57 20 | 3 43 | 13 |
| 14 | 51 32 | 3 42 | 46 8 | 3 59 | 50 34 | 4 18 | 7 0 49 | 4 19 | 5 53 | 4 0 | 11 1 3 | 3 43 | 14 |
| 15 | 55 14 | 3 43 | 50 7 | 4 0 | 54 52 | 4 19 | 5 8 | 4 18 | 9 53 | 3 59 | 4 46 | 3 42 | 15 |
| 16 | 58 57 | 3 43 | 54 7 | 4 0 | 59 11 | 4 19 | 9 26 | 4 18 | 13 52 | 3 58 | 8 28 | 3 42 | 16 |
| 17 | 1 2 40 | 3 43 | 58 7 | 4 1 | 5 3 30 | 4 19 | 13 44 | 4 17 | 17 50 | 3 57 | 12 10 | 3 42 | 17 |
| 18 | 6 23 | 3 44 | 3 2 8 | 4 1 | 7 49 | 4 20 | 18 1 | 4 17 | 21 47 | 3 57 | 15 52 | 3 42 | 18 |
| 19 | 1 10 7 | 3 44 | 3 6 9 | 4 3 | 5 12 9 | 4 20 | 7 22 18 | 4 16 | 9 25 44 | 3 56 | 11 19 34 | 3 41 | 19 |
| 20 | 13 51 | 3 45 | 10 12 | 4 3 | 16 29 | 4 20 | 26 34 | 4 16 | 29 40 | 3 55 | 23 15 | 3 41 | 20 |
| 21 | 17 36 | 3 45 | 14 15 | 4 4 | 20 49 | 4 20 | 30 50 | 4 15 | 33 35 | 3 54 | 26 56 | 3 41 | 21 |
| 22 | 21 21 | 3 45 | 18 19 | 4 4 | 25 9 | 4 21 | 35 5 | 4 15 | 37 29 | 3 54 | 30 37 | 3 41 | 22 |
| 23 | 25 6 | 3 46 | 22 23 | 4 5 | 29 30 | 4 21 | 39 20 | 4 14 | 41 23 | 3 53 | 34 18 | 3 40 | 23 |
| 24 | 28 52 | 3 46 | 26 28 | 4 6 | 33 51 | 4 21 | 43 34 | 4 13 | 45 16 | 3 53 | 37 58 | 3 41 | 24 |
| 25 | 1 32 38 | 3 47 | 3 30 34 | 4 7 | 5 38 12 | 4 22 | 7 47 47 | 4 13 | 9 49 9 | 3 52 | 11 41 39 | 3 40 | 25 |
| 26 | 36 25 | 3 47 | 34 41 | 4 8 | 42 34 | 4 21 | 52 0 | 4 12 | 53 1 | 3 51 | 45 19 | 3 41 | 26 |
| 27 | 40 12 | 3 48 | 38 49 | 4 8 | 46 55 | 4 22 | 56 12 | 4 12 | 56 52 | 3 51 | 59 0 | 3 40 | 27 |
| 28 | 44 0 | 3 48 | 42 57 | 4 9 | 51 17 | 4 21 | 8 0 24 | 4 11 | 10 0 43 | 3 50 | 52 40 | 3 40 | 28 |
| 29 | 47 48 | 3 49 | 47 6 | 4 9 | 55 38 | 4 22 | 4 35 | 4 10 | 4 33 | 3 50 | 56 20 | 3 40 | 29 |
| 30 | 51 37 | | 51 15 | | 6 0 0 | | 8 45 | | 8 23 | | 12 0 0 | | 30 |
| D | | Diff. | | Diff. | | Diff. | | Diff. | | Diff. | | Diff. | D |

## A TABLE of the SUN'S LONGITUDE for the Years 1788, 1792, and 1796, being Leap Years.

| Days. | January. | | | | February. | | | | March. | | | | April. | | | | May. | | | | June | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ |
| 1 | 9 | 10 | 51 | 9 | 10 | 12 | 24 | 16 | 11 | 11 | 38 | 33 | 0 | 12 | 25 | 13 | 1 | 11 | 41 | 51 | 2 | 11 | 31 | 22 |
| 2 | | 11 | 52 | 20 | | 13 | 25 | 6 | | 12 | 38 | 38 | | 13 | 24 | 15 | | 12 | 39 | 56 | | 12 | 28 | 47 |
| 3 | | 12 | 53 | 31 | | 14 | 25 | 55 | | 13 | 38 | 39 | | 14 | 23 | 13 | | 13 | 38 | 0 | | 13 | 26 | 12 |
| 4 | | 13 | 54 | 42 | | 15 | 26 | 42 | | 14 | 38 | 39 | | 15 | 22 | 11 | | 14 | 36 | 2 | | 14 | 23 | 35 |
| 5 | | 14 | 55 | 53 | | 16 | 27 | 30 | | 15 | 38 | 38 | | 16 | 21 | 6 | | 15 | 34 | 1 | | 15 | 20 | 57 |
| 6 | | 15 | 57 | 4 | | 17 | 28 | 15 | | 16 | 38 | 34 | | 17 | 20 | 0 | | 16 | 31 | 59 | | 16 | 18 | 19 |
| 7 | | 16 | 58 | 15 | | 18 | 28 | 59 | | 17 | 38 | 28 | | 18 | 18 | 52 | | 17 | 29 | 57 | | 17 | 15 | 40 |
| 8 | | 17 | 59 | 25 | | 19 | 29 | 41 | | 18 | 38 | 21 | | 19 | 17 | 41 | | 18 | 27 | 53 | | 18 | 13 | 0 |
| 9 | | 19 | 0 | 34 | | 20 | 30 | 22 | | 19 | 38 | 12 | | 20 | 16 | 29 | | 19 | 25 | 46 | | 19 | 10 | 20 |
| 10 | | 20 | 1 | 44 | | 21 | 31 | 1 | | 20 | 38 | 1 | | 21 | 15 | 14 | | 20 | 23 | 39 | | 20 | 7 | 39 |
| 11 | | 21 | 2 | 53 | | 22 | 31 | 39 | | 21 | 37 | 47 | | 22 | 13 | 58 | | 21 | 21 | 30 | | 21 | 4 | 56 |
| 12 | | 22 | 4 | 1 | | 23 | 32 | 10 | | 22 | 37 | 31 | | 23 | 12 | 39 | | 22 | 19 | 19 | | 22 | 2 | 14 |
| 13 | | 23 | 5 | 8 | | 24 | 32 | 51 | | 23 | 37 | 13 | | 24 | 11 | 19 | | 23 | 17 | 7 | | 22 | 59 | 31 |
| 14 | | 24 | 6 | 15 | | 25 | 33 | 24 | | 24 | 36 | 54 | | 25 | 9 | 56 | | 24 | 14 | 54 | | 23 | 56 | 48 |
| 15 | | 25 | 7 | 22 | | 26 | 33 | 56 | | 25 | 36 | 33 | | 26 | 8 | 32 | | 25 | 12 | 39 | | 24 | 54 | 3 |
| 16 | | 26 | 8 | 28 | | 27 | 34 | 27 | | 26 | 36 | 9 | | 27 | 7 | 6 | | 26 | 10 | 24 | | 25 | 51 | 18 |
| 17 | | 27 | 9 | 35 | | 28 | 34 | 56 | | 27 | 35 | 43 | | 28 | 5 | 38 | | 27 | 8 | 6 | | 26 | 48 | 32 |
| 18 | | 28 | 10 | 41 | | 29 | 35 | 22 | | 28 | 35 | 16 | | 29 | 4 | 7 | | 28 | 5 | 48 | | 27 | 45 | 47 |
| 19 | | 29 | 11 | 45 | 11 | 0 | 35 | 47 | | 29 | 34 | 46 | 1 | 0 | 2 | 36 | | 29 | 3 | 28 | | 28 | 43 | 2 |
| 20 | 10 | 0 | 12 | 48 | | 1 | 36 | 12 | 0 | 0 | 34 | 14 | | 1 | 1 | 2 | 2 | 0 | 1 | 7 | | 29 | 40 | 16 |
| 21 | | 1 | 13 | 50 | | 2 | 36 | 34 | | 1 | 33 | 40 | | 1 | 59 | 27 | | 0 | 58 | 44 | 3 | 0 | 37 | 30 |
| 22 | | 2 | 14 | 53 | | 3 | 36 | 55 | | 2 | 33 | 4 | | 2 | 57 | 49 | | 1 | 56 | 20 | | 1 | 34 | 44 |
| 23 | | 3 | 15 | 54 | | 4 | 37 | 13 | | 3 | 32 | 26 | | 3 | 56 | 9 | | 2 | 53 | 55 | | 2 | 31 | 55 |
| 24 | | 4 | 16 | 53 | | 5 | 37 | 30 | | 4 | 31 | 47 | | 4 | 54 | 28 | | 3 | 51 | 30 | | 3 | 29 | 6 |
| 25 | | 5 | 17 | 52 | | 6 | 37 | 45 | | 5 | 31 | 5 | | 5 | 52 | 45 | | 4 | 49 | 3 | | 4 | 26 | 19 |
| 26 | | 6 | 18 | 51 | | 7 | 37 | 58 | | 6 | 30 | 21 | | 6 | 51 | 1 | | 5 | 46 | 34 | | 5 | 23 | 33 |
| 27 | | 7 | 19 | 47 | | 8 | 28 | 10 | | 7 | 29 | 35 | | 7 | 49 | 14 | | 6 | 44 | 6 | | 6 | 20 | 44 |
| 28 | | 8 | 20 | 43 | | 9 | 38 | 20 | | 8 | 28 | 46 | | 8 | 47 | 26 | | 7 | 41 | 35 | | 7 | 17 | 57 |
| 29 | | 9 | 21 | 38 | | 10 | 38 | 28 | | 9 | 27 | 54 | | 9 | 45 | 37 | | 8 | 39 | 3 | | 8 | 15 | 8 |
| 30 | | 10 | 22 | 32 | | | | | | 10 | 27 | 1 | | 10 | 43 | 44 | | 9 | 36 | 30 | | 9 | 12 | 20 |
| 31 | | 11 | 23 | 24 | | | | | | 11 | 26 | 8 | | | | | | 10 | 33 | 57 | | | | |

| July. | | | | Auguſt | | | | September. | | | | October. | | | | November. | | | | December. | | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | |
| 3 | 10 | 9 | 32 | 4 | 9 | 45 | 27 | 5 | 9 | 36 | 39 | 6 | 8 | 55 | 50 | 7 | 9 | 45 | 30 | 8 | 10 | 3 | 3 | 1 |
| | 11 | 6 | 45 | | 10 | 42 | 54 | | 10 | 34 | 50 | | 9 | 54 | 59 | | 10 | 45 | 41 | | 11 | 3 | 59 | 2 |
| | 12 | 3 | 56 | | 11 | 40 | 23 | | 11 | 33 | 1 | | 10 | 54 | 10 | | 11 | 45 | 55 | | 12 | 4 | 56 | 3 |
| | 13 | 1 | 9 | | 12 | 37 | 54 | | 12 | 31 | 17 | | 11 | 53 | 24 | | 12 | 46 | 10 | | 13 | 5 | 56 | 4 |
| | 13 | 58 | 21 | | 13 | 35 | 24 | | 13 | 29 | 35 | | 12 | 52 | 39 | | 13 | 46 | 26 | | 14 | 6 | 55 | 5 |
| | 14 | 55 | 33 | | 14 | 32 | 56 | | 14 | 27 | 51 | | 13 | 51 | 57 | | 14 | 46 | 45 | | 15 | 7 | 54 | 6 |
| | 15 | 52 | 44 | | 15 | 30 | 30 | | 15 | 26 | 12 | | 14 | 51 | 18 | | 15 | 47 | 6 | | 16 | 8 | 55 | 7 |
| | 16 | 49 | 59 | | 16 | 28 | 5 | | 16 | 24 | 33 | | 15 | 50 | 39 | | 16 | 47 | 28 | | 17 | 9 | 57 | 8 |
| | 17 | 47 | 12 | | 17 | 25 | 41 | | 17 | 22 | 57 | | 16 | 50 | 2 | | 17 | 47 | 53 | | 18 | 11 | 1 | 9 |
| | 18 | 44 | 25 | | 18 | 23 | 17 | | 18 | 21 | 23 | | 17 | 49 | 29 | | 18 | 48 | 18 | | 19 | 12 | 5 | 10 |
| | 19 | 41 | 37 | | 19 | 20 | 54 | | 19 | 19 | 52 | | 18 | 48 | 57 | | 19 | 48 | 45 | | 20 | 13 | 9 | 11 |
| | 20 | 38 | 52 | | 20 | 18 | 34 | | 20 | 18 | 20 | | 19 | 48 | 26 | | 20 | 49 | 14 | | 21 | 14 | 14 | 12 |
| | 21 | 36 | 6 | | 21 | 16 | 15 | | 21 | 16 | 51 | | 20 | 47 | 59 | | 21 | 49 | 44 | | 22 | 15 | 20 | 13 |
| | 22 | 33 | 20 | | 22 | 13 | 57 | | 22 | 15 | 25 | | 21 | 47 | 33 | | 22 | 50 | 17 | | 23 | 16 | 27 | 14 |
| | 23 | 30 | 34 | | 23 | 11 | 40 | | 23 | 13 | 59 | | 22 | 47 | 9 | | 23 | 50 | 50 | | 24 | 17 | 34 | 15 |
| | 24 | 27 | 50 | | 24 | 9 | 24 | | 24 | 12 | 36 | | 23 | 46 | 47 | | 24 | 51 | 25 | | 25 | 18 | 41 | 16 |
| | 25 | 25 | 7 | | 25 | 7 | 11 | | 25 | 11 | 15 | | 24 | 46 | 28 | | 25 | 52 | 3 | | 26 | 19 | 50 | 17 |
| | 26 | 22 | 25 | | 26 | 4 | 57 | | 26 | 9 | 56 | | 25 | 46 | 10 | | 26 | 52 | 41 | | 27 | 20 | 58 | 18 |
| | 27 | 19 | 40 | | 27 | 2 | 46 | | 27 | 8 | 39 | | 26 | 45 | 54 | | 27 | 53 | 20 | | 28 | 22 | 7 | 19 |
| | 28 | 16 | 58 | | 28 | 0 | 36 | | 28 | 7 | 24 | | 27 | 45 | 41 | | 28 | 54 | 2 | | 29 | 23 | 17 | 20 |
| | 29 | 14 | 16 | | 28 | 58 | 28 | | 29 | 6 | 10 | | 28 | 45 | 29 | | 29 | 54 | 44 | 9 | 0 | 24 | 26 | 21 |
| 4 | 0 | 11 | 35 | | 29 | 56 | 21 | 6 | 0 | 4 | 59 | | 29 | 45 | 19 | 8 | 0 | 55 | 29 | | 1 | 25 | 37 | 22 |
| | 1 | 8 | 55 | 5 | 0 | 54 | 15 | | 1 | 3 | 50 | 7 | 0 | 45 | 12 | | 1 | 56 | 14 | | 2 | 26 | 46 | 23 |
| | 2 | 6 | 15 | | 1 | 52 | 12 | | 2 | 2 | 43 | | 1 | 45 | 6 | | 2 | 57 | 1 | | 3 | 27 | 57 | 24 |
| | 3 | 3 | 36 | | 2 | 50 | 9 | | 3 | 1 | 38 | | 2 | 45 | 2 | | 3 | 57 | 50 | | 4 | 29 | 8 | 25 |
| | 4 | 0 | 58 | | 3 | 48 | 8 | | 4 | 0 | 36 | | 3 | 45 | 1 | | 4 | 58 | 38 | | 5 | 30 | 20 | 26 |
| | 4 | 58 | 20 | | 4 | 46 | 9 | | 4 | 59 | 34 | | 4 | 45 | 1 | | 5 | 59 | 28 | | 6 | 31 | 30 | 27 |
| | 5 | 55 | 44 | | 5 | 44 | 11 | | 5 | 58 | 35 | | 5 | 45 | 3 | | 7 | 0 | 21 | | 7 | 32 | 42 | 28 |
| | 6 | 53 | 8 | | 6 | 42 | 16 | | 6 | 57 | 38 | | 6 | 45 | 6 | | 8 | 1 | 14 | | 8 | 33 | 53 | 29 |
| | 7 | 50 | 34 | | 7 | 40 | 22 | | 7 | 56 | 44 | | 7 | 45 | 12 | | 9 | 2 | 8 | | 9 | 35 | 5 | 30 |
| | 8 | 48 | 0 | | 8 | 38 | 30 | | | | | | 8 | 45 | 19 | | | | | | 10 | 36 | 18 | 31 |

## A TABLE of the SUN's LONGITUDE for the Years 1789, 1793, and 1797, being the firſt after Leap Year.

| Days. | January. | | | | February. | | | | March. | | | | April. | | | | May. | | | | June. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ |
| 1 | 9 | 11 | 37 | 28 | 10 | 13 | 10 | 21 | 11 | 11 | 23 | 57 | 0 | 12 | 10 | 54 | 1 | 11 | 27 | 47 | 2 | 11 | 17 | 32 |
| 2 | | 12 | 38 | 39 | | 14 | 11 | 9 | | 12 | 24 | 2 | | 13 | 9 | 57 | | 12 | 25 | 53 | | 12 | 14 | 55 |
| 3 | | 13 | 39 | 50 | | 15 | 11 | 58 | | 13 | 24 | 6 | | 14 | 8 | 57 | | 13 | 23 | 57 | | 13 | 12 | 20 |
| 4 | | 14 | 41 | 0 | | 16 | 12 | 45 | | 14 | 24 | 6 | | 15 | 7 | 55 | | 14 | 21 | 59 | | 14 | 9 | 43 |
| 5 | | 15 | 42 | 12 | | 17 | 13 | 30 | | 15 | 24 | 5 | | 16 | 6 | 50 | | 15 | 20 | 0 | | 15 | 7 | 4 |
| 6 | | 16 | 43 | 23 | | 18 | 14 | 15 | | 16 | 24 | 2 | | 17 | 5 | 44 | | 16 | 17 | 59 | | 16 | 4 | 27 |
| 7 | | 17 | 44 | 32 | | 19 | 14 | 58 | | 17 | 23 | 57 | | 18 | 4 | 37 | | 17 | 15 | 56 | | 17 | 1 | 48 |
| 8 | | 18 | 45 | 41 | | 20 | 15 | 38 | | 18 | 23 | 50 | | 19 | 3 | 27 | | 18 | 13 | 51 | | 17 | 59 | 8 |
| 9 | | 19 | 46 | 52 | | 21 | 16 | 18 | | 19 | 23 | 41 | | 20 | 2 | 17 | | 19 | 11 | 46 | | 18 | 56 | 28 |
| 10 | | 20 | 48 | 0 | | 22 | 16 | 57 | | 20 | 23 | 20 | | 21 | 1 | 2 | | 20 | 9 | 39 | | 19 | 53 | 46 |
| 11 | | 21 | 49 | 10 | | 23 | 17 | 34 | | 21 | 23 | 16 | | 21 | 59 | 43 | | 21 | 7 | 31 | | 20 | 51 | 4 |
| 12 | | 22 | 50 | 17 | | 24 | 18 | 9 | | 22 | 23 | 1 | | 22 | 58 | 26 | | 22 | 5 | 20 | | 21 | 48 | 20 |
| 13 | | 23 | 51 | 26 | | 25 | 18 | 42 | | 23 | 22 | 44 | | 23 | 57 | 6 | | 23 | 3 | 8 | | 22 | 45 | 38 |
| 14 | | 24 | 52 | 33 | | 26 | 19 | 15 | | 24 | 22 | 26 | | 24 | 55 | 45 | | 24 | 0 | 56 | | 23 | 42 | 56 |
| 15 | | 25 | 53 | 39 | | 27 | 19 | 46 | | 25 | 22 | 5 | | 25 | 54 | 21 | | 24 | 58 | 42 | | 24 | 40 | 11 |
| 16 | | 26 | 54 | 45 | | 28 | 20 | 15 | | 26 | 21 | 42 | | 26 | 52 | 55 | | 25 | 56 | 26 | | 25 | 37 | 27 |
| 17 | | 27 | 55 | 51 | | 29 | 20 | 42 | | 27 | 21 | 17 | | 27 | 51 | 28 | | 26 | 54 | 9 | | 26 | 34 | 42 |
| 18 | | 28 | 56 | 54 | 11 | 0 | 21 | 8 | | 28 | 20 | 49 | | 28 | 49 | 57 | | 27 | 51 | 50 | | 27 | 31 | 55 |
| 19 | | 29 | 57 | 59 | | 1 | 21 | 32 | | 29 | 20 | 20 | | 29 | 48 | 26 | | 28 | 49 | 30 | | 28 | 29 | 10 |
| 20 | 10 | 0 | 59 | 2 | | 2 | 21 | 55 | 0 | 0 | 19 | 48 | 1 | 0 | 46 | 53 | | 29 | 47 | 10 | | 29 | 26 | 25 |
| 21 | | 2 | 0 | 3 | | 3 | 22 | 16 | | 1 | 19 | 16 | | 1 | 45 | 18 | 2 | 0 | 44 | 48 | 3 | 0 | 23 | 38 |
| 22 | | 3 | 1 | 4 | | 4 | 22 | 34 | | 2 | 18 | 41 | | 2 | 43 | 41 | | 1 | 42 | 24 | | 1 | 20 | 52 |
| 23 | | 4 | 2 | 3 | | 5 | 22 | 52 | | 3 | 18 | 3 | | 3 | 42 | 1 | | 2 | 40 | 0 | | 2 | 18 | 5 |
| 24 | | 5 | 3 | 2 | | 6 | 23 | 7 | | 4 | 17 | 24 | | 4 | 40 | 21 | | 3 | 37 | 34 | | 3 | 15 | 17 |
| 25 | | 6 | 4 | 1 | | 7 | 23 | 21 | | 5 | 16 | 43 | | 5 | 38 | 39 | | 4 | 35 | 7 | | 4 | 12 | 29 |
| 26 | | 7 | 4 | 58 | | 8 | 23 | 33 | | 6 | 15 | 59 | | 6 | 36 | 55 | | 5 | 32 | 39 | | 5 | 9 | 42 |
| 27 | | 8 | 5 | 44 | | 9 | 23 | 42 | | 7 | 15 | 14 | | 7 | 35 | 9 | | 6 | 30 | 10 | | 6 | 6 | 54 |
| 28 | | 9 | 6 | 50 | | 10 | 23 | 51 | | 8 | 14 | 26 | | 8 | 33 | 21 | | 7 | 27 | 40 | | 7 | 4 | 8 |
| 29 | | 10 | 7 | 43 | | | | | | 9 | 13 | 36 | | 9 | 31 | 32 | | 8 | 25 | 9 | | 8 | 1 | 20 |
| 30 | | 11 | 8 | 36 | | | | | | 10 | 12 | 44 | | 10 | 29 | 41 | | 9 | 22 | 36 | | 8 | 58 | 33 |
| 31 | | 12 | 9 | 29 | | | | | | 11 | 11 | 52 | | | | | | 10 | 20 | 3 | | | | |

| July. | | | | Auguſt. | | | | September. | | | | October. | | | | November. | | | | December. | | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | |
| 3 | 9 | 55 | 42 | 4 | 9 | 31 | 32 | 5 | 10 | 22 | 34 | 6 | 8 | 41 | 30 | 7 | 9 | 30 | 53 | 8 | 9 | 48 | 16 | 1 |
| | 10 | 52 | 55 | | 10 | 29 | 0 | | 10 | 20 | 44 | | 9 | 40 | 38 | | 10 | 31 | 4 | | 10 | 49 | 11 | 2 |
| | 11 | 50 | 7 | | 11 | 26 | 30 | | 11 | 18 | 57 | | 10 | 39 | 49 | | 11 | 31 | 17 | | 11 | 50 | 8 | 3 |
| | 12 | 47 | 19 | | 12 | 24 | 0 | | 12 | 17 | 11 | | 11 | 39 | 3 | | 12 | 31 | 32 | | 12 | 51 | 6 | 4 |
| | 13 | 44 | 31 | | 13 | 21 | 30 | | 13 | 15 | 27 | | 12 | 38 | 17 | | 13 | 31 | 48 | | 13 | 52 | 5 | 5 |
| | 14 | 41 | 43 | | 14 | 19 | 2 | | 14 | 13 | 44 | | 13 | 37 | 35 | | 14 | 32 | 6 | | 14 | 53 | 5 | 6 |
| | 15 | 38 | 56 | | 15 | 16 | 34 | | 15 | 12 | 4 | | 14 | 36 | 55 | | 15 | 32 | 27 | | 15 | 54 | 7 | 7 |
| | 16 | 36 | 8 | | 16 | 14 | 9 | | 16 | 10 | 26 | | 15 | 36 | 15 | | 16 | 32 | 48 | | 16 | 55 | 8 | 8 |
| | 17 | 33 | 21 | | 17 | 11 | 44 | | 17 | 8 | 48 | | 16 | 35 | 38 | | 17 | 33 | 12 | | 17 | 56 | 10 | 9 |
| | 18 | 30 | 35 | | 18 | 9 | 20 | | 18 | 7 | 13 | | 17 | 35 | 4 | | 18 | 33 | 38 | | 18 | 57 | 14 | 10 |
| | 19 | 27 | 47 | | 19 | 6 | 58 | | 19 | 5 | 41 | | 18 | 34 | 31 | | 19 | 34 | 4 | | 19 | 58 | 18 | 11 |
| | 20 | 25 | 2 | | 20 | 4 | 37 | | 20 | 4 | 10 | | 19 | 34 | 0 | | 20 | 34 | 32 | | 20 | 59 | 23 | 12 |
| | 21 | 22 | 17 | | 21 | 2 | 17 | | 21 | 2 | 40 | | 20 | 33 | 32 | | 21 | 35 | 2 | | 22 | 0 | 28 | 13 |
| | 22 | 19 | 30 | | 21 | 59 | 58 | | 22 | 1 | 13 | | 21 | 33 | 6 | | 22 | 35 | 34 | | 23 | 1 | 35 | 14 |
| | 23 | 16 | 45 | | 22 | 57 | 41 | | 22 | 59 | 48 | | 22 | 32 | 41 | | 23 | 36 | 8 | | 24 | 2 | 43 | 15 |
| | 24 | 14 | 1 | | 23 | 55 | 23 | | 23 | 58 | 24 | | 23 | 12 | 18 | | 24 | 36 | 42 | | 25 | 3 | 50 | 16 |
| | 25 | 11 | 16 | | 24 | 53 | 9 | | 24 | 57 | 3 | | 24 | 31 | 59 | | 25 | 37 | 19 | | 26 | 4 | 59 | 17 |
| | 26 | 8 | 33 | | 25 | 50 | 57 | | 25 | 55 | 43 | | 25 | 31 | 41 | | 26 | 37 | 58 | | 27 | 6 | 6 | 18 |
| | 27 | 5 | 50 | | 26 | 48 | 46 | | 26 | 54 | 25 | | 26 | 31 | 25 | | 27 | 38 | 36 | | 28 | 7 | 14 | 19 |
| | 28 | 3 | 6 | | 27 | 46 | 36 | | 27 | 53 | 10 | | 27 | 31 | 11 | | 28 | 39 | 17 | | 29 | 8 | 25 | 20 |
| | 29 | 0 | 25 | | 28 | 44 | 28 | | 28 | 51 | 55 | | 28 | 30 | 58 | | 29 | 39 | 59 | 9 | 0 | 9 | 35 | 21 |
| | 29 | 57 | 44 | | 29 | 42 | 20 | | 29 | 50 | 44 | | 29 | 30 | 49 | 8 | 0 | 40 | 42 | | 1 | 10 | 44 | 22 |
| 4 | 0 | 55 | 3 | 5 | 0 | 40 | 15 | 6 | 0 | 49 | 35 | 7 | 0 | 30 | 40 | | 1 | 41 | 27 | | 2 | 11 | 55 | 23 |
| | 1 | 52 | 24 | | 1 | 38 | 11 | | 1 | 48 | 28 | | 1 | 30 | 34 | | 2 | 42 | 14 | | 3 | 13 | 4 | 24 |
| | 2 | 49 | 44 | | 2 | 36 | 8 | | 2 | 47 | 21 | | 2 | 30 | 29 | | 3 | 43 | 2 | | 4 | 14 | 16 | 25 |
| | 3 | 47 | 6 | | 3 | 34 | 6 | | 3 | 46 | 18 | | 3 | 30 | 27 | | 4 | 43 | 51 | | 5 | 15 | 28 | 26 |
| | 4 | 44 | 29 | | 4 | 32 | 7 | | 4 | 45 | 16 | | 4 | 30 | 26 | | 5 | 44 | 40 | | 6 | 16 | 39 | 27 |
| | 5 | 41 | 52 | | 5 | 30 | 9 | | 5 | 44 | 16 | | 5 | 30 | 29 | | 6 | 45 | 31 | | 7 | 17 | 50 | 28 |
| | 6 | 39 | 16 | | 6 | 28 | 13 | | 6 | 43 | 19 | | 6 | 30 | 32 | | 7 | 46 | 25 | | 8 | 19 | 1 | 29 |
| | 7 | 36 | 41 | | 7 | 26 | 18 | | 7 | 42 | 24 | | 7 | 30 | 37 | | 8 | 47 | 20 | | 9 | 20 | 13 | 30 |
| | 8 | 34 | 7 | | 8 | 24 | 26 | | | | | | 8 | 30 | 45 | | | | | | 10 | 21 | 15 | 31 |

## A TABLE of the SUN'S LONGITUDE for the Years 1790, 1794, 1798, being the ſecond after Leap Year.

| Days. | January. | | | | February. | | | | March. | | | | April. | | | | May. | | | | June. | | | | July. | | | | Auguſt. | | | | September. | | | | October. | | | | November. | | | | December. | | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | |
| 1 | 9 | 11 | 22 | 35 | 10 | 12 | 55 | 32 | 11 | 11 | 9 | 23 | 0 | 11 | 56 | 36 | 1 | 11 | 13 | 44 | 2 | 11 | 3 | 30 | 3 | 9 | 41 | 55 | 4 | 9 | 17 | 39 | 5 | 9 | 8 | 29 | 6 | 8 | 27 | 10 | 7 | 9 | 16 | 17 | 8 | 9 | 33 | 28 | 1 |
| 2 | | 12 | 23 | 48 | | 13 | 56 | 22 | | 12 | 9 | 29 | | 12 | 55 | 38 | | 12 | 11 | 49 | | 12 | 1 | 2 | | 10 | 39 | 5 | | 10 | 15 | 6 | | 10 | 6 | 39 | | 9 | 26 | 19 | | 10 | 16 | 27 | | 10 | 34 | 23 | 2 |
| 3 | | 13 | 24 | 58 | | 14 | 57 | 12 | | 13 | 9 | 31 | | 13 | 54 | 39 | | 13 | 19 | 54 | | 12 | 58 | 27 | | 11 | 36 | 18 | | 11 | 12 | 34 | | 11 | 4 | 51 | | 10 | 25 | 28 | | 11 | 16 | 41 | | 11 | 35 | 20 | 3 |
| 4 | | 14 | 26 | 10 | | 15 | 57 | 59 | | 14 | 9 | 33 | | 14 | 53 | 37 | | 14 | 7 | 56 | | 13 | 55 | 51 | | 12 | 33 | 30 | | 12 | 10 | 5 | | 12 | 3 | 5 | | 11 | 24 | 41 | | 12 | 16 | 55 | | 12 | 36 | 18 | 4 |
| 5 | | 15 | 27 | 21 | | 16 | 58 | 45 | | 15 | 9 | 32 | | 15 | 52 | 34 | | 15 | 5 | 57 | | 14 | 53 | 12 | | 13 | 30 | 41 | | 13 | 7 | 35 | | 13 | 1 | 20 | | 12 | 23 | 55 | | 13 | 17 | 10 | | 13 | 37 | 17 | 5 |
| 6 | | 16 | 28 | 31 | | 17 | 59 | 30 | | 16 | 9 | 29 | | 16 | 51 | 29 | | 16 | 3 | 57 | | 15 | 50 | 35 | | 14 | 27 | 55 | | 14 | 5 | 7 | | 13 | 59 | 38 | | 13 | 23 | 12 | | 14 | 17 | 28 | | 14 | 38 | 16 | 6 |
| 7 | | 17 | 29 | 42 | | 19 | 0 | 13 | | 17 | 9 | 24 | | 17 | 50 | 21 | | 17 | 1 | 55 | | 16 | 47 | 56 | | 15 | 25 | 7 | | 15 | 2 | 39 | | 14 | 57 | 58 | | 14 | 22 | 31 | | 15 | 17 | 47 | | 15 | 39 | 16 | 7 |
| 8 | | 18 | 30 | 51 | | 20 | 0 | 53 | | 18 | 9 | 18 | | 18 | 49 | 12 | | 17 | 59 | 51 | | 17 | 45 | 17 | | 16 | 22 | 18 | | 16 | 0 | 13 | | 15 | 56 | 18 | | 15 | 21 | 51 | | 16 | 18 | 9 | | 16 | 40 | 17 | 8 |
| 9 | | 19 | 32 | 0 | | 21 | 1 | 34 | | 19 | 9 | 9 | | 19 | 48 | 1 | | 18 | 57 | 45 | | 18 | 42 | 37 | | 17 | 19 | 32 | | 16 | 57 | 48 | | 16 | 54 | 40 | | 16 | 21 | 14 | | 17 | 18 | 33 | | 17 | 41 | 21 | 9 |
| 10 | | 20 | 33 | 10 | | 22 | 2 | 13 | | 20 | 8 | 59 | | 20 | 46 | 46 | | 19 | 55 | 38 | | 19 | 39 | 55 | | 18 | 16 | 45 | | 17 | 55 | 24 | | 17 | 53 | 5 | | 17 | 20 | 39 | | 18 | 18 | 57 | | 18 | 42 | 24 | 10 |
| 11 | | 21 | 34 | 20 | | 23 | 2 | 50 | | 21 | 8 | 46 | | 21 | 45 | 31 | | 20 | 53 | 20 | | 20 | 37 | 14 | | 19 | 13 | 58 | | 18 | 53 | 1 | | 18 | 51 | 32 | | 18 | 20 | 7 | | 19 | 19 | 23 | | 19 | 43 | 29 | 11 |
| 12 | | 22 | 35 | 26 | | 24 | 3 | 26 | | 22 | 8 | 32 | | 22 | 44 | 14 | | 21 | 51 | 20 | | 21 | 34 | 32 | | 20 | 11 | 10 | | 19 | 50 | 40 | | 19 | 50 | 0 | | 19 | 19 | 36 | | 20 | 19 | 53 | | 20 | 44 | 34 | 12 |
| 13 | | 23 | 36 | 34 | | 25 | 4 | 1 | | 23 | 8 | 15 | | 23 | 42 | 54 | | 22 | 49 | 9 | | 22 | 31 | 49 | | 21 | 8 | 25 | | 20 | 48 | 20 | | 20 | 48 | 20 | | 20 | 19 | 6 | | 21 | 20 | 21 | | 21 | 45 | 40 | 13 |
| 14 | | 24 | 37 | 42 | | 26 | 4 | 34 | | 24 | 7 | 57 | | 24 | 41 | 33 | | 23 | 46 | 56 | | 23 | 29 | 5 | | 22 | 5 | 40 | | 21 | 46 | 1 | | 21 | 47 | 3 | | 21 | 18 | 40 | | 22 | 20 | 53 | | 22 | 46 | 45 | 14 |
| 15 | | 25 | 38 | 50 | | 27 | 5 | 5 | | 25 | 7 | 37 | | 25 | 40 | 9 | | 24 | 44 | 44 | | 24 | 26 | 22 | | 23 | 2 | 54 | | 22 | 43 | 44 | | 22 | 45 | 36 | | 22 | 18 | 14 | | 23 | 21 | 26 | | 23 | 47 | 51 | 15 |
| 16 | | 26 | 39 | 55 | | 28 | 5 | 34 | | 26 | 7 | 14 | | 26 | 38 | 44 | | 25 | 42 | 28 | | 25 | 23 | 37 | | 24 | 0 | 10 | | 23 | 41 | 28 | | 23 | 44 | 12 | | 23 | 17 | 51 | | 24 | 22 | 0 | | 24 | 49 | 0 | 16 |
| 17 | | 27 | 41 | 0 | | 29 | 6 | 1 | | 27 | 6 | 50 | | 27 | 37 | 17 | | 26 | 40 | 11 | | 26 | 20 | 50 | | 24 | 57 | 25 | | 24 | 39 | 13 | | 24 | 42 | 50 | | 24 | 17 | 31 | | 25 | 22 | 38 | | 25 | 50 | 8 | 17 |
| 18 | | 28 | 42 | 4 | 11 | 0 | 6 | 28 | | 28 | 6 | 23 | | 28 | 35 | 48 | | 27 | 37 | 53 | | 27 | 18 | 4 | | 25 | 54 | 41 | | 25 | 36 | 59 | | 25 | 41 | 30 | | 25 | 17 | 13 | | 26 | 23 | 16 | | 26 | 51 | 15 | 18 |
| 19 | | 29 | 43 | 8 | | 1 | 6 | 54 | | 29 | 5 | 55 | | 29 | 34 | 17 | | 28 | 35 | 33 | | 28 | 15 | 18 | | 26 | 51 | 58 | | 26 | 34 | 47 | | 26 | 40 | 12 | | 26 | 16 | 56 | | 27 | 23 | 55 | | 27 | 52 | 24 | 19 |
| 20 | 10 | 0 | 44 | 11 | | 2 | 7 | 17 | 0 | 0 | 5 | 24 | 1 | 0 | 32 | 44 | | 29 | 33 | 13 | | 29 | 12 | 30 | | 27 | 49 | 15 | | 27 | 32 | 36 | | 27 | 38 | 56 | | 27 | 16 | 42 | | 28 | 24 | 35 | | 28 | 53 | 35 | 20 |
| 21 | | 1 | 45 | 12 | | 3 | 7 | 38 | | 1 | 4 | 51 | | 1 | 31 | 10 | 2 | 0 | 30 | 52 | 3 | 0 | 9 | 43 | | 28 | 46 | 33 | | 28 | 30 | 27 | | 28 | 37 | 41 | | 28 | 16 | 28 | | 29 | 25 | 18 | | 29 | 54 | 43 | 21 |
| 22 | | 2 | 46 | 14 | | 4 | 7 | 58 | | 2 | 4 | 17 | | 2 | 29 | 33 | | 1 | 28 | 29 | | 1 | 6 | 55 | | 29 | 43 | 52 | | 29 | 28 | 20 | | 29 | 36 | 29 | | 29 | 16 | 18 | 8 | 0 | 26 | 0 | 9 | 0 | 55 | 53 | 22 |
| 23 | | 3 | 47 | 15 | | 5 | 8 | 14 | | 3 | 3 | 39 | | 3 | 27 | 54 | | 2 | 26 | 4 | | 2 | 4 | 7 | 4 | 0 | 41 | 11 | 5 | 0 | 26 | 15 | 6 | 0 | 35 | 20 | 7 | 0 | 16 | 9 | | 1 | 26 | 45 | | 1 | 57 | 2 | 23 |
| 24 | | 4 | 48 | 14 | | 6 | 8 | 30 | | 4 | 3 | 1 | | 4 | 26 | 14 | | 3 | 23 | 39 | | 3 | 1 | 9 | | 1 | 38 | 31 | | 1 | 24 | 9 | | 1 | 34 | 11 | | 1 | 16 | 12 | | 2 | 27 | 30 | | 2 | 58 | 13 | 24 |
| 25 | | 5 | 49 | 12 | | 7 | 8 | 45 | | 5 | 2 | 20 | | 5 | 24 | 32 | | 4 | 21 | 12 | | 3 | 58 | 31 | | 2 | 35 | 53 | | 2 | 22 | 6 | | 2 | 33 | 5 | | 2 | 15 | 58 | | 3 | 28 | 17 | | 3 | 59 | 25 | 25 |
| 26 | | 6 | 50 | 11 | | 8 | 8 | 57 | | 6 | 1 | 37 | | 6 | 22 | 48 | | 5 | 18 | 44 | | 4 | 55 | 41 | | 3 | 33 | 14 | | 3 | 20 | 4 | | 3 | 32 | 1 | | 3 | 15 | 55 | | 4 | 29 | 6 | | 5 | 0 | 35 | 26 |
| 27 | | 7 | 51 | 6 | | 9 | 9 | 8 | | 7 | 0 | 51 | | 7 | 21 | 3 | | 6 | 16 | 15 | | 5 | 52 | 52 | | 4 | 30 | 35 | | 4 | 18 | 4 | | 4 | 30 | 59 | | 4 | 15 | 53 | | 5 | 29 | 56 | | 6 | 1 | 47 | 27 |
| 28 | | 8 | 52 | 3 | | 10 | 9 | 17 | | 8 | 0 | 5 | | 8 | 19 | 16 | | 7 | 13 | 46 | | 6 | 50 | 4 | | 5 | 27 | 58 | | 5 | 16 | 6 | | 5 | 29 | 59 | | 5 | 15 | 55 | | 6 | 30 | 47 | | 7 | 2 | 58 | 28 |
| 29 | | 9 | 52 | 56 | | | | | | 8 | 59 | 15 | | 9 | 17 | 27 | | 8 | 11 | 16 | | 7 | 47 | 16 | | 6 | 25 | 22 | | 6 | 14 | 10 | | 6 | 29 | 1 | | 6 | 15 | 58 | | 7 | 31 | 40 | | 8 | 4 | 10 | 29 |
| 30 | | 10 | 53 | 50 | | | | | | 9 | 58 | 25 | | 10 | 15 | 38 | | 9 | 8 | 44 | | 8 | 44 | 27 | | 7 | 22 | 47 | | 7 | 12 | 14 | | 7 | 28 | 5 | | 7 | 16 | 3 | | 8 | 32 | 33 | | 9 | 5 | 21 | 30 |
| 31 | | 11 | 54 | 42 | | | | | | 10 | 57 | 32 | | | | | | 10 | 6 | 10 | | | | | | 8 | 20 | 12 | | 8 | 10 | 21 | | | | | | 8 | 16 | 10 | | | | | | 10 | 6 | 33 | 31 |

308. A TABLE of the SUN's LONGITUDE for the Years 1791, 1795, and 1799, being the third after Leap Year.

| | January. | | | | February. | | | | March. | | | | April. | | | | May. | | | | June. | | | | July. | | | | Auguſt. | | | | September. | | | | October. | | | | November. | | | | December. | | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | s | ° | ′ | ″ | |
| 1 | 9 | 11 | 7 | 43 | 10 | 12 | 40 | 47 | 11 | 10 | 54 | 48 | 0 | 11 | 42 | 17 | 1 | 10 | 59 | 40 | 2 | 10 | 49 | 44 | 3 | 9 | 28 | 3 | 4 | 9 | 3 | 46 | 5 | 8 | 54 | 24 | 6 | 8 | 12 | 51 | 7 | 9 | 1 | 41 | 8 | 9 | 18 | 40 | 1 |
| 2 | | 12 | 8 | 54 | | 13 | 41 | 37 | | 11 | 54 | 54 | | 12 | 41 | 20 | | 11 | 57 | 46 | | 11 | 47 | 9 | | 10 | 25 | 14 | | 10 | 1 | 12 | | 9 | 52 | 35 | | 9 | 11 | 58 | | 10 | 1 | 51 | | 10 | 19 | 34 | 2 |
| 3 | | 13 | 10 | 7 | | 14 | 42 | 26 | | 12 | 54 | 58 | | 13 | 40 | 21 | | 12 | 55 | 51 | | 12 | 44 | 34 | | 11 | 22 | 28 | | 10 | 58 | 41 | | 10 | 50 | 47 | | 10 | 11 | 7 | | 11 | 2 | 4 | | 11 | 20 | 31 | 3 |
| 4 | | 14 | 11 | 10 | | 15 | 43 | 13 | | 13 | 55 | 0 | | 14 | 39 | 20 | | 13 | 53 | 55 | | 13 | 41 | 58 | | 12 | 19 | 39 | | 11 | 56 | 11 | | 11 | 48 | 59 | | 11 | 10 | 20 | | 12 | 2 | 17 | | 12 | 31 | 29 | 4 |
| 5 | | 15 | 12 | 29 | | 16 | 44 | 0 | | 14 | 54 | 59 | | 15 | 38 | 17 | | 14 | 51 | 56 | | 14 | 39 | 20 | | 13 | 16 | 52 | | 12 | 53 | 41 | | 12 | 47 | 14 | | 12 | 9 | 34 | | 13 | 2 | 32 | | 13 | 22 | 27 | 5 |
| 6 | | 16 | 13 | 40 | | 17 | 44 | 44 | | 15 | 54 | 57 | | 16 | 37 | 12 | | 15 | 49 | 55 | | 15 | 36 | 43 | | 14 | 14 | 2 | | 13 | 51 | 12 | | 13 | 45 | 31 | | 13 | 8 | 49 | | 14 | 2 | 50 | | 14 | 23 | 28 | 6 |
| 7 | | 17 | 14 | 49 | | 18 | 45 | 29 | | 16 | 54 | 52 | | 17 | 36 | 5 | | 16 | 47 | 53 | | 16 | 34 | 5 | | 15 | 11 | 17 | | 14 | 48 | 44 | | 14 | 43 | 50 | | 14 | 8 | 8 | | 15 | 3 | 9 | | 15 | 24 | 28 | 7 |
| 8 | | 18 | 16 | 0 | | 19 | 46 | 10 | | 17 | 54 | 46 | | 18 | 34 | 56 | | 17 | 45 | 49 | | 17 | 31 | 25 | | 16 | 8 | 28 | | 15 | 46 | 17 | | 15 | 42 | 10 | | 15 | 7 | 28 | | 16 | 3 | 30 | | 16 | 25 | 29 | 8 |
| 9 | | 19 | 17 | 0 | | 20 | 46 | 51 | | 18 | 54 | 38 | | 19 | 33 | 46 | | 18 | 43 | 45 | | 18 | 28 | 45 | | 17 | 5 | 41 | | 16 | 43 | 52 | | 16 | 40 | 33 | | 16 | 6 | 51 | | 17 | 3 | 52 | | 17 | 26 | 32 | 9 |
| 10 | | 20 | 18 | 8 | | 21 | 47 | 31 | | 19 | 54 | 28 | | 20 | 32 | 32 | | 19 | 41 | 38 | | 19 | 26 | 4 | | 18 | 2 | 55 | | 17 | 41 | 28 | | 17 | 38 | 55 | | 17 | 6 | 14 | | 18 | 4 | 18 | | 18 | 27 | 34 | 10 |
| 11 | | 21 | 19 | 28 | | 22 | 48 | 7 | | 20 | 54 | 16 | | 21 | 31 | 16 | | 20 | 39 | 31 | | 20 | 23 | 22 | | 19 | 0 | 8 | | 18 | 39 | 5 | | 18 | 37 | 21 | | 18 | 5 | 41 | | 19 | 4 | 44 | | 19 | 28 | 38 | 11 |
| 12 | | 22 | 20 | 36 | | 23 | 48 | 46 | | 21 | 54 | 3 | | 22 | 30 | 0 | | 21 | 37 | 22 | | 21 | 20 | 41 | | 19 | 57 | 22 | | 19 | 36 | 43 | | 19 | 35 | 50 | | 19 | 5 | 10 | | 20 | 5 | 12 | | 20 | 29 | 43 | 12 |
| 13 | | 23 | 21 | 43 | | 24 | 49 | 19 | | 22 | 53 | 47 | | 23 | 28 | 41 | | 22 | 35 | 10 | | 22 | 17 | 59 | | 20 | 54 | 36 | | 20 | 34 | 23 | | 20 | 34 | 20 | | 20 | 4 | 39 | | 21 | 5 | 40 | | 21 | 30 | 48 | 13 |
| 14 | | 24 | 22 | 51 | | 25 | 49 | 52 | | 23 | 53 | 29 | | 24 | 27 | 21 | | 23 | 32 | 58 | | 23 | 15 | 15 | | 21 | 51 | 49 | | 21 | 32 | 4 | | 21 | 32 | 51 | | 21 | 4 | 12 | | 22 | 6 | 11 | | 22 | 31 | 55 | 14 |
| 15 | | 25 | 23 | 58 | | 26 | 50 | 25 | | 24 | 53 | 10 | | 25 | 25 | 58 | | 24 | 30 | 45 | | 24 | 12 | 32 | | 22 | 49 | 4 | | 22 | 29 | 46 | | 22 | 31 | 24 | | 22 | 3 | 48 | | 23 | 6 | 44 | | 23 | 33 | 0 | 15 |
| 16 | | 26 | 25 | 3 | | 27 | 50 | 54 | | 25 | 52 | 48 | | 26 | 24 | 33 | | 25 | 28 | 30 | | 25 | 9 | 47 | | 23 | 46 | 18 | | 23 | 27 | 30 | | 23 | 30 | 0 | | 23 | 3 | 23 | | 24 | 7 | 17 | | 24 | 34 | 7 | 16 |
| 17 | | 27 | 26 | 9 | | 28 | 51 | 21 | | 26 | 52 | 23 | | 27 | 23 | 6 | | 26 | 26 | 14 | | 26 | 7 | 2 | | 24 | 43 | 34 | | 24 | 25 | 15 | | 24 | 28 | 36 | | 24 | 3 | 3 | | 25 | 7 | 53 | | 25 | 35 | 17 | 17 |
| 18 | | 28 | 27 | 15 | | 29 | 51 | 48 | | 27 | 51 | 57 | | 28 | 21 | 38 | | 27 | 23 | 56 | | 27 | 4 | 16 | | 25 | 40 | 51 | | 25 | 23 | 2 | | 25 | 27 | 16 | | 25 | 2 | 44 | | 26 | 8 | 31 | | 26 | 36 | 25 | 18 |
| 19 | | 29 | 28 | 18 | 11 | 0 | 52 | 14 | | 28 | 51 | 30 | | 29 | 20 | 7 | | 28 | 21 | 37 | | 28 | 1 | 31 | | 26 | 38 | 8 | | 26 | 20 | 48 | | 26 | 25 | 57 | | 26 | 2 | 26 | | 27 | 9 | 9 | | 27 | 37 | 32 | 19 |
| 20 | 10 | 0 | 29 | 21 | | 1 | 52 | 37 | | 29 | 50 | 59 | 1 | 0 | 18 | 35 | | 29 | 19 | 17 | | 28 | 58 | 45 | | 27 | 35 | 26 | | 27 | 18 | 36 | | 27 | 24 | 41 | | 27 | 2 | 12 | | 28 | 9 | 49 | | 28 | 38 | 41 | 20 |
| 21 | | 1 | 30 | 23 | | 2 | 52 | 50 | 0 | 0 | 50 | 27 | | 1 | 17 | 2 | 2 | 0 | 16 | 57 | | 29 | 55 | 59 | | 28 | 32 | 43 | | 28 | 16 | 28 | | 28 | 23 | 26 | | 28 | 1 | 57 | | 29 | 10 | 31 | | 29 | 39 | 51 | 21 |
| 22 | | 2 | 31 | 25 | | 3 | 53 | 18 | | 1 | 49 | 53 | | 2 | 15 | 25 | | 1 | 14 | 34 | 3 | 0 | 53 | 13 | | 29 | 30 | 1 | | 29 | 14 | 20 | | 29 | 22 | 13 | | 29 | 1 | 46 | 8 | 0 | 11 | 13 | 9 | 0 | 41 | 1 | 22 |
| 23 | | 3 | 32 | 25 | | 4 | 53 | 36 | | 2 | 49 | 17 | | 3 | 13 | 47 | | 2 | 12 | 10 | | 1 | 50 | 20 | 4 | 0 | 27 | 19 | 5 | 0 | 12 | 13 | 6 | 0 | 21 | 3 | 7 | 0 | 1 | 37 | | 1 | 11 | 58 | | 1 | 42 | 11 | 23 |
| 24 | | 4 | 33 | 25 | | 5 | 53 | 53 | | 3 | 48 | 37 | | 4 | 12 | 8 | | 3 | 9 | 46 | | 2 | 47 | 38 | | 1 | 24 | 40 | | 1 | 10 | 9 | | 1 | 19 | 55 | | 1 | 1 | 29 | | 2 | 12 | 44 | | 2 | 43 | 20 | 24 |
| 25 | | 5 | 34 | 25 | | 6 | 54 | 8 | | 4 | 47 | 57 | | 5 | 10 | 26 | | 4 | 7 | 19 | | 3 | 44 | 50 | | 2 | 22 | 0 | | 2 | 8 | 6 | | 2 | 18 | 49 | | 2 | 1 | 24 | | 3 | 13 | 32 | | 3 | 44 | 32 | 25 |
| 26 | | 6 | 35 | 22 | | 7 | 54 | 21 | | 5 | 47 | 15 | | 6 | 8 | 42 | | 5 | 4 | 51 | | 4 | 42 | 3 | | 3 | 19 | 21 | | 3 | 6 | 3 | | 3 | 17 | 44 | | 3 | 1 | 21 | | 4 | 14 | 19 | | 4 | 45 | 44 | 26 |
| 27 | | 7 | 36 | 19 | | 8 | 54 | 32 | | 6 | 46 | 30 | | 7 | 6 | 57 | | 6 | 2 | 22 | | 5 | 39 | 16 | | 4 | 16 | 44 | | 4 | 4 | 3 | | 4 | 16 | 42 | | 4 | 1 | 20 | | 5 | 15 | 8 | | 5 | 46 | 55 | 27 |
| 28 | | 8 | 37 | 14 | | 9 | 54 | 41 | | 7 | 45 | 44 | | 8 | 5 | 10 | | 6 | 59 | 52 | | 6 | 36 | 29 | | 5 | 14 | 7 | | 5 | 2 | 3 | | 5 | 15 | 40 | | 5 | 1 | 21 | | 6 | 15 | 59 | | 6 | 48 | 6 | 28 |
| 29 | | 9 | 38 | 9 | | | | | | 8 | 44 | 55 | | 9 | 3 | 22 | | 7 | 57 | 22 | | 7 | 33 | 40 | | 6 | 11 | 30 | | 6 | 0 | 6 | | 6 | 14 | 41 | | 6 | 1 | 22 | | 7 | 16 | 52 | | 7 | 49 | 17 | 29 |
| 30 | | 10 | 39 | 3 | | | | | | 9 | 44 | 4 | | 10 | 1 | 33 | | 8 | 54 | 50 | | 8 | 30 | 52 | | 7 | 8 | 54 | | 6 | 58 | 10 | | 7 | 13 | 46 | | 7 | 1 | 27 | | 8 | 17 | 46 | | 8 | 50 | 29 | 30 |
| 31 | | 11 | 33 | 55 | | | | | | 10 | 4 | 11 | | | | | | 9 | 52 | 17 | | | | | | 8 | 6 | 20 | | 7 | 56 | 17 | | | | | | 8 | 1 | 33 | | | | | | 9 | 51 | 42 | 31 |

### 309. A Table of the Sun's Declination for the Years 1788, 1792, and 1796, being Leap Years.

| Days. | January. | February. | March. | April. | May. | June. | July. | August. | September. | October | Novemb. | December | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | South. | South. | South. | North. | North. | North. | North. | North. | North. | South. | South. | South | |
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | |
| 1 | 23 1 21 | 17 6 0 | 7 12 17 | 4 54 48 | 15 21 38 | 22 11 26 | 23 4 38 | 17 49 35 | 7 58 29 | 3 32 39 | 14 45 16 | 21 58 56 | 1 |
| 2 | 22 56 8 | 48 46 | 6 49 20 | 5 17 48 | 39 25 | 19 4 | 0 4 | 34 4 | 36 30 | 55 57 | 15 4 12 | 22 7 39 | 2 |
| 3 | 50 28 | 31 14 | 26 19 | 40 42 | 56 57 | 26 18 | 22 55 5 | 18 15 | 14 24 | 4 19 11 | 22 53 | 15 56 | 3 |
| 4 | 44 21 | 13 25 | 3 12 | 6 3 31 | 16 14 13 | 33 9 | 49 44 | 2 11 | 6 52 9 | 42 22 | 41 19 | 23 48 | 4 |
| 5 | 37 46 | 55 18 | 5 39 59 | 26 13 | 31 12 | 39 37 | 43 58 | 16 45 49 | 29 47 | 5 5 30 | 59 29 | 31 13 | 5 |
| 6 | 30 45 | 36 55 | 16 43 | 48 49 | 47 56 | 45 41 | 37 48 | 29 11 | 7 21 | 28 35 | 16 17 23 | 38 11 | 6 |
| 7 | 23 16 | 18 17 | 4 53 22 | 7 11 18 | 17 4 24 | 51 21 | 31 15 | 12 17 | 5 44 49 | 51 35 | 35 1 | 44 43 | 7 |
| 8 | 15 21 | 14 59 23 | 29 57 | 33 40 | 20 33 | 56 37 | 24 18 | 15 55 8 | 22 9 | 6 14 30 | 52 21 | 50 48 | 8 |
| 9 | 7 1 | 40 14 | 6 28 | 55 54 | 36 26 | 23 1 28 | 16 58 | 37 43 | 4 59 24 | 37 20 | 17 9 25 | 56 26 | 9 |
| 10 | 21 58 15 | 20 51 | 3 42 57 | 8 17 59 | 52 0 | 5 53 | 9 16 | 20 2 | 36 35 | 7 0 5 | 26 11 | 23 1 37 | 10 |
| 11 | 49 2 | 1 13 | 19 24 | 39 57 | 18 7 17 | 9 59 | 1 11 | 2 9 | 3 13 41 | 22 45 | 42 39 | 6 20 | 11 |
| 12 | 39 24 | 13 41 22 | 2 55 48 | 9 1 46 | 22 16 | 13 37 | 21 52 42 | 14 43 59 | 50 42 | 45 18 | 58 48 | 10 36 | 12 |
| 13 | 29 22 | 21 16 | 32 10 | 23 27 | 36 57 | 16 51 | 43 51 | 25 36 | 27 39 | 8 7 46 | 18 14 39 | 14 24 | 13 |
| 14 | 18 54 | 0 58 | 8 30 | 44 58 | 51 18 | 19 41 | 34 38 | 6 58 | 4 32 | 30 5 | 30 10 | 17 44 | 14 |
| 15 | 8 2 | 12 40 27 | 1 44 49 | 10 6 19 | 19 5 20 | 22 6 | 25 4 | 13 48 7 | 2 41 22 | 52 18 | 45 21 | 20 36 | 15 |
| 16 | 20 56 45 | 19 45 | 21 8 | 27 29 | 19 3 | 24 6 | 15 5 | 29 4 | 18 9 | 9 14 23 | 19 0 13 | 23 0 | 16 |
| 17 | 45 4 | 11 58 50 | 0 57 27 | 43 31 | 32 27 | 25 41 | 4 46 | 9 47 | 1 54 52 | 36 21 | 14 45 | 24 56 | 17 |
| 18 | 32 59 | 37 45 | 33 45 | 11 9 21 | 45 31 | 26 52 | 20 54 6 | 12 50 18 | 31 33 | 58 11 | 28 55 | 26 24 | 18 |
| 19 | 20 32 | 16 28 | S. 10 3 | 30 0 | 58 15 | 27 38 | 43 5 | 30 37 | 8 14 | 10 19 51 | 42 44 | 27 24 | 19 |
| 20 | 7 41 | 10 55 0 | N. 13 38 | 50 28 | 20 10 39 | 27 59 | 31 43 | 10 44 | 0 44 50 | 41 22 | 56 12 | 27 55 | 20 |
| 21 | 19 54 28 | 33 23 | 37 18 | 12 10 44 | 22 42 | 27 54 | 20 1 | 11 50 38 | N. 21 26 | 11 2 43 | 20 9 18 | 27 58 | 21 |
| 22 | 40 53 | 11 35 | 1 0 56 | 30 49 | 34 23 | 27 26 | 7 56 | 30 23 | S. 1 59 | 23 55 | 22 1 | 27 32 | 22 |
| 23 | 26 56 | 9 49 38 | 24 33 | 50 41 | 45 44 | 26 32 | 19 55 33 | 9 55 | 25 25 | 44 56 | 34 22 | 26 38 | 23 |
| 24 | 12 38 | 27 33 | 48 8 | 13 10 21 | 56 44 | 25 15 | 42 50 | 10 49 17 | 48 51 | 12 5 46 | 46 20 | 25 16 | 24 |
| 25 | 18 57 58 | 5 20 | 2 11 40 | 29 47 | 21 7 22 | 23 32 | 29 47 | 28 29 | 1 12 18 | 26 26 | 57 55 | 23 26 | 25 |
| 26 | 42 57 | 8 42 57 | 35 10 | 49 1 | 17 38 | 21 24 | 16 23 | 7 31 | 35 44 | 46 54 | 21 9 6 | 21 7 | 26 |
| 27 | 27 36 | 20 28 | 58 36 | 14 8 1 | 27 32 | 18 52 | 2 42 | 9 46 23 | 59 10 | 13 7 10 | 19 53 | 18 20 | 27 |
| 28 | 11 56 | 7 57 51 | 3 21 59 | 26 46 | 37 4 | 15 55 | 18 48 41 | 25 6 | 2 22 35 | 27 13 | 30 16 | 15 5 | 28 |
| 29 | 17 55 55 | 35 8 | 45 17 | 45 19 | 46 14 | 12 34 | 34 22 | 3 39 | 45 58 | 47 4 | 40 15 | 11 22 | 29 |
| 30 | 39 35 | | 4 8 31 | 15 3 37 | 55 1 | 8 48 | 19 43 | 8 42 4 | 3 9 20 | 14 6 42 | 49 48 | 7 12 | 30 |
| 31 | 22 58 | | 31 42 | | 22 3 25 | | 4 48 | 20 21 | | 26 6 | | 2 33 | 31 |

### 309. TABLE of the SUN's DECLINATION for the Years 1789, 1793, and 1797, being the firſt after Leap Year.

| Days. | January. | | | February. | | | March. | | | April. | | | May. | | | June. | | | July. | | | Auguſt. | | | September. | | | October. | | | Novemb. | | | December | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | South. | | | South. | | | South. | | | North. | | | North. | | | North. | | | North | | | North. | | | North. | | | South. | | | South. | | | South. | | | |
| | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | |
| 1 | 22 | 57 | 27 | 16 | 52 | 59 | 7 | 17 | 51 | 4 | 49 | 12 | 15 | 17 | 17 | 22 | 9 | 32 | 23 | 5 | 41 | 17 | 53 | 17 | 8 | 3 | 47 | 3 | 27 | 0 | 14 | 40 | 39 | 21 | 56 | 45 | 1 |
| 2 | | 51 | 54 | | 35 | 31 | 6 | 54 | 56 | 5 | 12 | 14 | | 35 | 8 | | 17 | 15 | | 1 | 12 | | 37 | 51 | 7 | 41 | 50 | | 50 | 17 | | 59 | 37 | 22 | 5 | 34 | 2 |
| 3 | | 45 | 52 | | 17 | 45 | | 31 | 55 | | 35 | 10 | | 52 | 44 | | 24 | 35 | 22 | 56 | 18 | | 22 | 6 | | 19 | 44 | 4 | 13 | 33 | 15 | 18 | 22 | | 13 | 58 | 3 |
| 4 | | 39 | 24 | 15 | 59 | 43 | | 8 | 49 | | 58 | 0 | 16 | 10 | 4 | | 31 | 32 | | 51 | 4 | | 6 | 5 | 6 | 57 | 33 | | 36 | 45 | | 36 | 54 | | 21 | 56 | 4 |
| 5 | | 32 | 28 | | 41 | 25 | 5 | 45 | 37 | 6 | 20 | 44 | | 27 | 8 | | 38 | 6 | | 45 | 24 | 16 | 49 | 48 | | 35 | 14 | | 59 | 54 | | 55 | 5 | | 29 | 27 | 5 |
| 6 | | 25 | 7 | | 22 | 50 | | 22 | 22 | | 43 | 21 | | 43 | 55 | | 44 | 15 | | 39 | 19 | | 33 | 14 | | 12 | 48 | 5 | 22 | 59 | 16 | 13 | 4 | | 36 | 32 | 6 |
| 7 | | 17 | 20 | | 3 | 59 | 4 | 59 | 0 | 7 | 5 | 52 | 17 | 0 | 26 | | 50 | 1 | | 32 | 52 | | 16 | 24 | 5 | 50 | 18 | | 46 | 1 | | 30 | 45 | | 43 | 10 | 7 |
| 8 | | 9 | 5 | 14 | 44 | 54 | | 35 | 37 | | 28 | 15 | | 16 | 40 | | 55 | 23 | | 26 | 1 | 15 | 59 | 18 | | 27 | 39 | 6 | 8 | 56 | | 48 | 10 | | 49 | 22 | 8 |
| 9 | | 0 | 24 | | 25 | 34 | | 12 | 10 | | 50 | 32 | | 32 | 36 | 23 | 0 | 20 | | 18 | 47 | | 41 | 57 | | 4 | 56 | | 31 | 48 | 17 | 5 | 17 | | 55 | 6 | 9 |
| 10 | 21 | 51 | 19 | | 5 | 59 | 3 | 48 | 41 | 8 | 12 | 39 | | 48 | 15 | | 4 | 54 | | 11 | 10 | | 24 | 21 | 4 | 42 | 8 | | 54 | 34 | | 22 | 9 | 23 | 0 | 23 | 10 |
| 11 | | 41 | 46 | 13 | 46 | 12 | | 25 | 7 | | 34 | 37 | 18 | 3 | 37 | | 9 | 2 | | 3 | 10 | | 6 | 29 | | 19 | 14 | 7 | 17 | 16 | | 38 | 40 | | 5 | 14 | 11 |
| 12 | | 31 | 50 | | 26 | 10 | | 1 | 32 | | 56 | 29 | | 18 | 40 | | 12 | 46 | 21 | 54 | 47 | 14 | 48 | 23 | 3 | 56 | 16 | | 39 | 50 | | 54 | 57 | | 9 | 36 | 12 |
| 13 | | 21 | 28 | | 5 | 55 | 2 | 37 | 53 | 9 | 18 | 12 | | 33 | 25 | | 16 | 7 | | 46 | 1 | | 30 | 5 | | 33 | 14 | 8 | 2 | 19 | 18 | 10 | 49 | | 13 | 31 | 13 |
| 14 | | 10 | 42 | 12 | 45 | 27 | | 14 | 15 | | 39 | 46 | | 47 | 51 | | 19 | 2 | | 36 | 54 | | 11 | 30 | | 10 | 8 | | 24 | 41 | | 26 | 25 | | 16 | 58 | 14 |
| 15 | 20 | 59 | 31 | | 24 | 48 | 1 | 50 | 34 | 10 | 1 | 9 | 19 | 1 | 59 | | 21 | 33 | | 27 | 24 | 13 | 52 | 43 | 2 | 56 | 59 | | 46 | 55 | | 41 | 42 | | 19 | 58 | 15 |
| 16 | | 47 | 55 | | 3 | 56 | | 26 | 52 | | 22 | 22 | | 15 | 47 | | 23 | 39 | | 17 | 32 | | 33 | 43 | | 23 | 47 | 9 | 9 | 3 | | 56 | 38 | | 22 | 28 | 16 |
| 17 | | 35 | 57 | 11 | 42 | 52 | | 3 | 11 | | 43 | 26 | | 29 | 15 | | 25 | 20 | | 7 | 18 | | 14 | 29 | | 0 | 31 | | 31 | 3 | 19 | 11 | 15 | | 24 | 31 | 17 |
| 18 | | 23 | 35 | | 21 | 38 | 0 | 39 | 29 | 11 | 4 | 19 | | 42 | 24 | | 26 | 37 | 20 | 56 | 44 | 12 | 55 | 1 | 1 | 37 | 13 | | 52 | 54 | | 25 | 31 | | 26 | 5 | 18 |
| 19 | | 10 | 50 | | 0 | 14 | S. | 15 | 48 | | 25 | 1 | | 55 | 12 | | 27 | 29 | | 45 | 47 | | 35 | 25 | | 13 | 52 | 10 | 14 | 36 | | 39 | 24 | | 27 | 12 | 19 |
| 20 | 19 | 57 | 43 | 10 | 38 | 39 | N. | 7 | 53 | | 45 | 32 | 20 | 7 | 41 | | 27 | 56 | | 34 | 30 | | 15 | 34 | 0 | 50 | 30 | | 36 | 10 | | 53 | 0 | | 27 | 50 | 20 |
| 21 | | 44 | 13 | | 16 | 57 | | 31 | 33 | 12 | 5 | 51 | | 19 | 49 | | 27 | 58 | | 22 | 52 | 11 | 55 | 31 | | 27 | 5 | | 57 | 34 | 20 | 6 | 9 | | 28 | 0 | 21 |
| 22 | | 30 | 22 | 9 | 55 | 0 | | 55 | 12 | | 25 | 58 | | 31 | 36 | | 27 | 35 | | 10 | 53 | | 35 | 18 | N. | 3 | 41 | 11 | 18 | 47 | | 18 | 58 | | 27 | 41 | 22 |
| 23 | | 16 | 8 | | 32 | 56 | 1 | 18 | 49 | | 45 | 53 | | 43 | 2 | | 26 | 48 | 19 | 58 | 35 | | 14 | 53 | S. | 19 | 45 | | 39 | 51 | | 31 | 24 | | 26 | 53 | 23 |
| 24 | | 1 | 34 | | 10 | 45 | | 42 | 26 | 13 | 5 | 36 | | 54 | 6 | | 25 | 36 | | 45 | 56 | 10 | 54 | 17 | | 43 | 11 | 12 | 0 | 45 | | 43 | 28 | | 25 | 39 | 24 |
| 25 | 18 | 46 | 38 | 8 | 48 | 24 | 2 | 5 | 58 | | 25 | 6 | 21 | 4 | 40 | | 23 | 59 | | 32 | 57 | | 33 | 32 | 1 | 6 | 37 | | 21 | 27 | | 55 | 8 | | 23 | 55 | 25 |
| 26 | | 31 | 22 | | 25 | 57 | | 29 | 29 | | 44 | 23 | | 15 | 11 | | 21 | 57 | | 19 | 39 | | 12 | 37 | | 30 | 4 | | 41 | 57 | 21 | 6 | 25 | | 21 | 44 | 26 |
| 27 | | 15 | 46 | | 3 | 22 | | 52 | 56 | 14 | 3 | 26 | | 25 | 11 | | 19 | 31 | | 6 | 2 | 9 | 51 | 31 | | 53 | 29 | 13 | 2 | 16 | | 17 | 18 | | 19 | 13 | 27 |
| 28 | 17 | 59 | 51 | 7 | 40 | 39 | 3 | 16 | 19 | | 22 | 16 | | 34 | 48 | | 16 | 40 | 18 | 52 | 5 | | 30 | 16 | 2 | 16 | 54 | | 22 | 23 | | 27 | 47 | | 15 | 55 | 28 |
| 29 | | 43 | 35 | | | | | 39 | 39 | | 40 | 51 | | 44 | 3 | | 13 | 25 | | 37 | 51 | | 8 | 51 | | 40 | 17 | | 42 | 16 | | 37 | 53 | | 12 | 19 | 29 |
| 30 | | 27 | 2 | | | | 4 | 2 | 54 | | 59 | 12 | | 52 | 56 | | 9 | 45 | | 23 | 18 | 8 | 47 | 18 | 3 | 3 | 40 | 14 | 1 | 57 | | 47 | 31 | | 8 | 15 | 30 |
| 31 | | 10 | 9 | | | | | 26 | 6 | | | | 22 | 1 | 23 | | | | | 8 | 27 | | 25 | 36 | | | | | 21 | 25 | | | | | 3 | 43 | 31 |

309. A TABLE of the SUN's DECLINATION for the Years 1790, 1794, and 1798, being the ſecond after Leap Year.

| Days. | January. | | | February. | | | March. | | | April. | | | May. | | | June. | | | July. | | | Auguſt. | | | September. | | | October. | | | Novemb. | | | December | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | South. | | | South. | | | South. | | | North. | | | North. | | | North. | | | North. | | | North. | | | North. | | | South. | | | South. | | | South. | | | |
| | ° | ′ | ′ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | ° | ′ | ″ | |
| 1 | 22 | 58 | 44 | 16 | 57 | 11 | 7 | 23 | 23 | 4 | 43 | 37 | 15 | 12 | 56 | 22 | 7 | 36 | 23 | 6 | 42 | 17 | 56 | 59 | 8 | 9 | 5 | 3 | 21 | 21 | 14 | 35 | 59 | 21 | 54 | 33 | 1 |
| 2 | | 53 | 16 | | 39 | 48 | 6 | 0 | 29 | 5 | 6 | 40 | | 30 | 51 | | 15 | 25 | | 2 | 20 | | 41 | 36 | 7 | 47 | 10 | | 44 | 40 | | 55 | 3 | 22 | 3 | 28 | 2 |
| 3 | | 47 | 22 | | 22 | 6 | | 37 | 28 | | 29 | 37 | | 48 | 29 | | 22 | 51 | 22 | 57 | 33 | | 25 | 57 | | 25 | 7 | 4 | 7 | 56 | 15 | 13 | 51 | | 11 | 58 | 3 |
| 4 | | 41 | 1 | | 4 | 7 | | 14 | 25 | | 52 | 28 | 16 | 5 | 53 | | 29 | 54 | | 52 | 23 | | 10 | 0 | | 2 | 56 | | 31 | 9 | | 32 | 24 | | 20 | 2 | 4 |
| 5 | | 34 | 13 | 15 | 45 | 53 | 5 | 51 | 16 | 6 | 15 | 14 | | 23 | 1 | | 36 | 33 | | 46 | 48 | 16 | 53 | 46 | 6 | 40 | 38 | | 54 | 18 | | 50 | 41 | | 27 | 40 | 5 |
| 6 | | 26 | 57 | | 27 | 22 | | 28 | 1 | | 37 | 53 | | 39 | 53 | | 42 | 48 | | 40 | 49 | | 37 | 16 | | 18 | 14 | 5 | 17 | 23 | 16 | 8 | 43 | | 34 | 51 | 6 |
| 7 | | 19 | 15 | | 8 | 36 | | 4 | 42 | 7 | 0 | 25 | | 56 | 28 | | 48 | 40 | | 34 | 28 | | 20 | 30 | 5 | 55 | 43 | | 40 | 26 | | 26 | 29 | | 41 | 36 | 7 |
| 8 | | 11 | 8 | 14 | 49 | 35 | 4 | 41 | 19 | | 22 | 51 | 17 | 12 | 46 | | 54 | 7 | | 27 | 43 | | 3 | 28 | | 33 | 8 | 6 | 3 | 24 | | 43 | 58 | | 47 | 54 | 8 |
| 9 | | 2 | 34 | | 30 | 19 | | 17 | 53 | | 45 | 8 | | 28 | 46 | | 59 | 10 | | 20 | 34 | 15 | 46 | 11 | | 10 | 27 | | 26 | 16 | 17 | 1 | 11 | | 53 | 45 | 9 |
| 10 | 21 | 53 | 33 | | 10 | 47 | 3 | 54 | 23 | 8 | 7 | 18 | | 44 | 30 | 23 | 3 | 50 | | 13 | 2 | | 28 | 38 | 4 | 47 | 39 | | 49 | 4 | | 18 | 5 | | 59 | 9 | 10 |
| 11 | | 44 | 8 | 13 | 51 | 2 | | 30 | 50 | | 29 | 20 | | 59 | 56 | | 8 | 4 | | 5 | 7 | | 10 | 51 | | 24 | 45 | 7 | 11 | 47 | | 34 | 41 | 23 | 4 | 6 | 11 |
| 12 | | 34 | 18 | | 31 | 3 | | 7 | 15 | | 51 | 13 | 18 | 15 | 3 | | 11 | 55 | 21 | 56 | 51 | 14 | 52 | 48 | | 1 | 50 | | 34 | 23 | | 51 | 0 | | 8 | 35 | 12 |
| 13 | | 24 | 1 | | 10 | 52 | 2 | 43 | 38 | 9 | 12 | 58 | | 29 | 52 | | 15 | 21 | | 48 | 10 | | 34 | 32 | 3 | 38 | 47 | | 56 | 53 | 18 | 6 | 59 | | 12 | 37 | 13 |
| 14 | | 13 | 21 | 12 | 50 | 27 | | 19 | 58 | | 34 | 34 | | 44 | 23 | | 18 | 22 | | 39 | 9 | | 16 | 1 | | 15 | 44 | 8 | 19 | 17 | | 22 | 40 | | 16 | 11 | 14 |
| 15 | | 2 | 16 | | 29 | 50 | 1 | 56 | 18 | | 55 | 59 | | 58 | 36 | | 21 | 0 | | 29 | 43 | 13 | 57 | 17 | 2 | 52 | 36 | | 41 | 34 | | 38 | 1 | | 19 | 16 | 15 |
| 16 | 20 | 50 | 46 | | 9 | 1 | | 32 | 37 | 10 | 17 | 15 | 19 | 12 | 28 | | 23 | 12 | | 19 | 57 | | 38 | 20 | | 29 | 24 | 9 | 3 | 43 | | 53 | 3 | | 21 | 54 | 16 |
| 17 | | 38 | 54 | 11 | 48 | 1 | | 8 | 56 | | 38 | 21 | | 26 | 1 | | 24 | 59 | | 9 | 49 | | 19 | 9 | | 6 | 9 | | 25 | 43 | 19 | 7 | 44 | | 24 | 4 | 17 |
| 18 | | 26 | 38 | | 26 | 49 | 0 | 45 | 14 | | 59 | 16 | | 39 | 15 | | 26 | 21 | 20 | 59 | 19 | 12 | 59 | 47 | 1 | 42 | 51 | | 47 | 37 | | 22 | 4 | | 25 | 45 | 18 |
| 19 | | 13 | 58 | | 5 | 26 | S. | 21 | 30 | 11 | 20 | 1 | | 52 | 8 | | 27 | 18 | | 48 | 28 | | 40 | 10 | | 19 | 31 | 10 | 9 | 21 | | 36 | 4 | | 26 | 58 | 19 |
| 20 | | 0 | 57 | 10 | 43 | 53 | N. | 2 | 9 | | 40 | 34 | 20 | 4 | 42 | | 27 | 51 | | 37 | 16 | | 20 | 22 | 0 | 56 | 9 | | 30 | 57 | | 49 | 43 | | 27 | 43 | 20 |
| 21 | 19 | 47 | 33 | | 22 | 10 | | 25 | 49 | 12 | 0 | 57 | | 16 | 55 | | 28 | 0 | | 25 | 43 | | 0 | 24 | | 32 | 47 | | 52 | 23 | 20 | 3 | 0 | | 28 | 0 | 21 |
| 22 | | 33 | 46 | | 0 | 18 | | 49 | 29 | | 21 | 7 | | 28 | 47 | | 27 | 43 | | 13 | 49 | 11 | 40 | 12 | N. | 9 | 21 | 11 | 13 | 38 | | 15 | 54 | | 27 | 49 | 22 |
| 23 | | 19 | 37 | 9 | 38 | 18 | 1 | 13 | 6 | | 41 | 6 | | 40 | 18 | | 27 | 1 | | 1 | 36 | | 19 | 50 | S. | 14 | 4 | | 34 | 46 | | 28 | 25 | | 27 | 8 | 23 |
| 24 | | 5 | 8 | | 16 | 8 | | 36 | 42 | 13 | 0 | 51 | | 51 | 27 | | 25 | 55 | 19 | 49 | 1 | 10 | 59 | 18 | | 37 | 30 | | 55 | 41 | | 40 | 35 | | 26 | 0 | 24 |
| 25 | 18 | 50 | 18 | 8 | 53 | 50 | 2 | 0 | 16 | | 20 | 24 | 21 | 2 | 16 | | 24 | 25 | | 36 | 7 | | 38 | 36 | 1 | 0 | 57 | 12 | 16 | 27 | | 52 | 20 | | 24 | 23 | 25 |
| 26 | | 35 | 6 | | 31 | 23 | | 23 | 47 | | 39 | 44 | | 12 | 43 | | 22 | 29 | | 22 | 54 | | 17 | 42 | | 24 | 23 | | 37 | 0 | 21 | 3 | 43 | | 22 | 18 | 26 |
| 27 | | 19 | 35 | | 8 | 50 | | 47 | 14 | | 58 | 50 | | 22 | 48 | | 20 | 9 | | 9 | 22 | 9 | 56 | 39 | | 47 | 49 | | 57 | 22 | | 14 | 42 | | 19 | 45 | 27 |
| 28 | | 3 | 45 | 7 | 46 | 10 | 3 | 10 | 37 | 14 | 17 | 43 | | 32 | 31 | | 17 | 24 | 18 | 55 | 30 | | 35 | 25 | 2 | 11 | 14 | 13 | 17 | 32 | | 25 | 17 | | 16 | 43 | 28 |
| 29 | 17 | 47 | 34 | | | | | 33 | 59 | | 36 | 22 | | 41 | 51 | | 14 | 15 | | 41 | 19 | | 14 | 2 | | 34 | 38 | | 37 | 29 | | 35 | 28 | | 13 | 14 | 29 |
| 30 | | 31 | 5 | | | | | 57 | 17 | | 54 | 47 | | 50 | 49 | | 10 | 41 | | 26 | 49 | 8 | 52 | 32 | | 58 | 0 | | 57 | 12 | | 45 | 13 | | 9 | 17 | 30 |
| 31 | | 14 | 17 | | | | 4 | 20 | 30 | | | | | 59 | 24 | | | | | 12 | 4 | | 30 | 53 | | | | 14 | 16 | 43 | | | | | 4 | 52 | 31 |

## 309. A TABLE of the SUN'S DECLINATION for the Years 1791, 1795, and 1799, being the third after Leap Year.

| | January | February. | March. | April. | May. | June. | July. | Auguſt. | September. | October. | Novemb. | December | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | South. | South. | South. | North. | North | North. | North. | North. | North. | South. | South. | South. | |
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | |
| 1 | 23 0 0 | 17 1 22 | 7 28 55 | 4 38 1 | 15 8 34 | 22 5 39 | 23 7 42 | 17 0 39 | 8 14 23 | 3 15 42 | 14 31 20 | 21 52 19 | 1 |
| 2 | 22 54 39 | 44 3 | 6 3 | 5 1 5 | 26 32 | 13 34 | 3 25 | 45 21 | 7 52 29 | 39 1 | 50 26 | 22 1 21 | 2 |
| 3 | 48 51 | 16 26 25 | 6 43 4 | 24 4 | 44 15 | 21 5 | 22 58 45 | 29 45 | 30 27 | 4 2 17 | 15 9 18 | 9 57 | 3 |
| 4 | 42 30 | 8 31 | 20 1 | 46 57 | 16 1 43 | 28 14 | 53 39 | 13 52 | 8 18 | 25 31 | 27 55 | 18 7 | 4 |
| 5 | 35 57 | 15 50 2 | 5 56 53 | 6 9 43 | 18 54 | 34 59 | 48 11 | 16 57 42 | 6 46 3 | 48 41 | 46 15 | 25 51 | 5 |
| 6 | 28 45 | 31 53 | 33 38 | 32 25 | 35 50 | 41 20 | 42 19 | 41 16 | 23 40 | 5 11 48 | 16 4 22 | 33 9 | 6 |
| 7 | 21 1 | 13 11 | 10 22 | 54 58 | 52 28 | 47 17 | 36 2 | 24 34 | 1 11 | 34 51 | 22 12 | 40 0 | 7 |
| 8 | 13 3 | 14 54 13 | 4 47 0 | 7 17 25 | 17 8 50 | 52 50 | 29 23 | 7 37 | 5 38 37 | 57 50 | 39 45 | 46 25 | 8 |
| 9 | 4 41 | 35 0 | 23 34 | 39 45 | 24 55 | 57 59 | 22 20 | 15 50 22 | 15 56 | 6 20 45 | 57 2 | 52 23 | 9 |
| 10 | 21 55 47 | 15 32 | 3 0 5 | 8 1 56 | 40 43 | 23 2 44 | 14 53 | 32 54 | 4 53 11 | 43 32 | 17 14 0 | 57 53 | 10 |
| 11 | 46 28 | 13 55 51 | 50 33 | 24 0 | 56 13 | 7 5 | 7 5 | 15 11 | 30 20 | 7 6 16 | 30 41 | 23 2 56 | 11 |
| 12 | 26 43 | 35 55 | 12 59 | 45 56 | 18 11 26 | 11 2 | 21 58 53 | 14 57 11 | 7 24 | 28 55 | 47 4 | 7 32 | 12 |
| 13 | 20 34 | 15 47 | 2 49 22 | 9 7 42 | 26 19 | 14 33 | 50 19 | 38 58 | 3 44 24 | 51 26 | 18 3 8 | 11 41 | 13 |
| 14 | 15 50 | 12 55 25 | 25 44 | 29 21 | 40 55 | 17 41 | 41 21 | 20 31 | 21 20 | 8 13 51 | 18 54 | 15 21 | 14 |
| 15 | 4 56 | 34 51 | 2 3 | 50 49 | 55 11 | 20 23 | 32 3 | 1 50 | 2 58 12 | 36 10 | 34 20 | 18 33 | 15 |
| 16 | 20 53 31 | 14 6 | 1 38 22 | 10 12 7 | 19 9 8 | 22 41 | 22 21 | 13 42 56 | 35 1 | 58 20 | 49 25 | 21 18 | 16 |
| 17 | 41 40 | 11 53 7 | 14 41 | 33 15 | 22 46 | 24 34 | 12 18 | 23 48 | 11 48 | 9 20 24 | 19 4 12 | 23 35 | 17 |
| 18 | 29 30 | 31 58 | 0 50 58 | 54 14 | 36 6 | 26 3 | 1 54 | 4 28 | 1 48 31 | 42 9 | 18 37 | 25 23 | 18 |
| 19 | 17 5 | 10 38 | 27 16 | 11 15 0 | 49 4 | 27 7 | 20 51 8 | 12 44 56 | 25 10 | 10 4 5 | 32 42 | 26 43 | 19 |
| 20 | 4 9 | 10 49 8 | S. 3 35 | 35 37 | 20 1 42 | 27 46 | 40 0 | 25 12 | 1 50 | 25 43 | 46 27 | 27 35 | 20 |
| 21 | 19 50 59 | 27 28 | N. 20 5 | 56 2 | 14 0 | 28 0 | 28 32 | 5 15 | 0 38 27 | 47 11 | 59 48 | 27 58 | 21 |
| 22 | 37 8 | 5 39 | 43 44 | 12 16 16 | 25 57 | 27 49 | 16 44 | 11 45 6 | N. 15 3 | 11 8 30 | 20 12 47 | 27 53 | 22 |
| 23 | 23 5 | 9 43 41 | 1 7 23 | 36 16 | 37 33 | 27 14 | 4 35 | 24 47 | S. 8 23 | 29 39 | 25 25 | 27 20 | 23 |
| 24 | 8 41 | 21 32 | 30 58 | 56 6 | 48 48 | 26 14 | 19 52 5 | 4 17 | 31 49 | 50 37 | 37 40 | 26 19 | 24 |
| 25 | 18 53 56 | 8 59 15 | 54 33 | 13 15 42 | 59 42 | 24 48 | 39 16 | 10 43 36 | 55 16 | 12 11 25 | 49 31 | 24 49 | 25 |
| 26 | 38 50 | 36 51 | 2 18 5 | 35 4 | 21 10 13 | 22 59 | 26 8 | 22 45 | 1 18 43 | 32 1 | 21 0 59 | 22 51 | 26 |
| 27 | 23 23 | 14 20 | 41 34 | 54 14 | 20 23 | 20 45 | 12 40 | 1 44 | 42 9 | 52 26 | 12 3 | 20 24 | 27 |
| 28 | 7 38 | 7 51 41 | 5 0 | 14 13 10 | 30 11 | 18 5 | 18 58 52 | 9 40 34 | 2 5 33 | 13 12 39 | 22 45 | 17 30 | 28 |
| 29 | 17 51 32 | | 3 28 22 | 31 52 | 39 37 | 15 2 | 44 46 | 19 14 | 28 57 | 32 38 | 33 12 | 14 8 | 29 |
| 30 | 35 7 | | 51 38 | 50 20 | 48 41 | 11 34 | 30 22 | 8 57 45 | 52 21 | 52 26 | 42 53 | 10 17 | 30 |
| 31 | 18 21 | | 4 14 56 | | 57 22 | | 15 40 | 36 8 | | 14 12 0 | | 5 59 | 31 |

## 310. A Table of the Sun's Right Ascension for the Years 1788, 1792, and 1796, being Leap Years.

| Days. | January. | | | February. | | | March. | | | April. | | | May. | | | June. | | | July. | | | August. | | | Septemb. | | | October. | | | Novemb. | | | December | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | s. |
| 1 | 18 | 47 | 13 | 20 | 59 | 30 | 22 | 52 | 17 | 0 | 45 | 41 | 2 | 37 | 1 | 4 | 39 | 56 | 6 | 44 | 14 | 8 | 48 | 49 | 10 | 44 | 42 | 12 | 32 | 49 | 14 | 29 | 24 | 16 | 33 | 39 | 1 |
| 2 | | 51 | 38 | 21 | 3 | 33 | | 56 | 0 | | 49 | 19 | | 40 | 51 | | 44 | 2 | | 48 | 21 | | 52 | 42 | | 48 | 19 | | 36 | 27 | | 33 | 20 | | 37 | 59 | 2 |
| 3 | | 56 | 2 | | 7 | 37 | | 59 | 44 | | 52 | 58 | | 44 | 41 | | 48 | 9 | | 52 | 28 | | 56 | 33 | | 51 | 56 | | 40 | 5 | | 37 | 18 | | 42 | 20 | 3 |
| 4 | 19 | 0 | 27 | | 11 | 39 | 23 | 3 | 27 | | 56 | 36 | | 48 | 32 | | 52 | 15 | | 56 | 35 | 9 | 0 | 25 | | 55 | 33 | | 43 | 43 | | 41 | 16 | | 46 | 42 | 4 |
| 5 | | 4 | 50 | | 15 | 41 | | 7 | 9 | 1 | 0 | 15 | | 52 | 23 | | 56 | 22 | 7 | 0 | 42 | | 4 | 15 | | 59 | 10 | | 47 | 22 | | 45 | 14 | | 51 | 4 | 5 |
| 6 | | 9 | 13 | | 19 | 41 | | 10 | 51 | | 3 | 54 | | 56 | 15 | 5 | 0 | 29 | | 4 | 49 | | 8 | 5 | 11 | 2 | 47 | | 51 | 2 | | 49 | 14 | | 55 | 26 | 6 |
| 7 | | 13 | 36 | | 23 | 41 | | 14 | 33 | | 7 | 34 | 3 | 0 | 7 | | 4 | 37 | | 8 | 55 | | 11 | 54 | | 6 | 23 | | 54 | 42 | | 53 | 15 | | 59 | 49 | 7 |
| 8 | | 17 | 58 | | 27 | 40 | | 18 | 14 | | 11 | 13 | | 4 | 0 | | 8 | 45 | | 13 | 1 | | 15 | 43 | | 9 | 58 | | 58 | 21 | | 57 | 17 | 17 | 4 | 13 | 8 |
| 9 | | 22 | 20 | | 31 | 39 | | 21 | 55 | | 14 | 53 | | 7 | 53 | | 12 | 53 | | 17 | 6 | | 19 | 31 | | 13 | 35 | 13 | 2 | 3 | 15 | 1 | 39 | | 8 | 36 | 9 |
| 10 | | 26 | 41 | | 35 | 36 | | 25 | 35 | | 18 | 33 | | 11 | 47 | | 17 | 2 | | 21 | 11 | | 23 | 19 | | 17 | 11 | | 5 | 44 | | 5 | 23 | | 13 | 1 | 10 |
| 11 | | 31 | 2 | | 39 | 32 | | 29 | 15 | | 22 | 13 | | 15 | 42 | | 21 | 11 | | 25 | 16 | | 27 | 6 | | 20 | 47 | | 9 | 26 | | 9 | 26 | | 17 | 25 | 11 |
| 12 | | 35 | 22 | | 43 | 29 | | 32 | 55 | | 25 | 53 | | 19 | 37 | | 25 | 19 | | 29 | 20 | | 30 | 52 | | 24 | 22 | | 13 | 8 | | 13 | 31 | | 21 | 50 | 12 |
| 13 | | 39 | 41 | | 47 | 24 | | 36 | 35 | | 29 | 34 | | 23 | 33 | | 29 | 28 | | 33 | 23 | | 34 | 39 | | 27 | 58 | | 16 | 51 | | 17 | 37 | | 26 | 16 | 13 |
| 14 | | 44 | 0 | | 51 | 18 | | 40 | 14 | | 33 | 16 | | 27 | 30 | | 33 | 38 | | 37 | 26 | | 38 | 24 | | 31 | 34 | | 20 | 34 | | 21 | 44 | | 30 | 42 | 14 |
| 15 | | 48 | 18 | | 55 | 12 | | 43 | 53 | | 36 | 57 | | 31 | 26 | | 37 | 46 | | 41 | 29 | | 42 | 9 | | 35 | 9 | | 24 | 17 | | 25 | 51 | | 35 | 7 | 15 |
| 16 | | 52 | 35 | | 59 | 5 | | 47 | 31 | | 40 | 39 | | 35 | 24 | | 41 | 56 | | 45 | 32 | | 45 | 53 | | 38 | 45 | | 28 | 2 | | 30 | 0 | | 39 | 31 | 16 |
| 17 | | 56 | 52 | 22 | 2 | 57 | | 51 | 9 | | 44 | 21 | | 39 | 22 | | 46 | 5 | | 49 | 33 | | 49 | 37 | | 42 | 20 | | 31 | 47 | | 34 | 9 | | 44 | 0 | 17 |
| 18 | 20 | 1 | 9 | | 6 | 49 | | 54 | 47 | | 48 | 4 | | 43 | 21 | | 50 | 14 | | 53 | 34 | | 53 | 20 | | 45 | 56 | | 35 | 32 | | 38 | 19 | | 48 | 27 | 18 |
| 19 | | 5 | 24 | | 10 | 39 | | 58 | 25 | | 51 | 47 | | 47 | 20 | | 54 | 24 | | 57 | 35 | | 57 | 3 | | 49 | 31 | | 39 | 19 | | 42 | 29 | | 52 | 53 | 19 |
| 20 | | 9 | 38 | | 14 | 29 | 0 | 2 | 6 | | 55 | 31 | | 51 | 20 | | 58 | 34 | 8 | 1 | 35 | 10 | 0 | 45 | | 53 | 7 | | 43 | 6 | | 46 | 41 | | 57 | 20 | 20 |
| 21 | | 13 | 52 | | 18 | 19 | | 5 | 44 | | 59 | 15 | | 55 | 20 | 6 | 2 | 43 | | 5 | 34 | | 4 | 27 | | 56 | 42 | | 46 | 53 | | 50 | 53 | 18 | 1 | 46 | 21 |
| 22 | | 18 | 5 | | 22 | 8 | | 9 | 21 | 2 | 2 | 59 | | 59 | 21 | | 6 | 53 | | 9 | 33 | | 8 | 9 | 12 | 0 | 18 | | 50 | 41 | | 55 | 6 | | 6 | 13 | 22 |
| 23 | | 22 | 17 | | 25 | 56 | | 13 | 0 | | 6 | 44 | 4 | 3 | 22 | | 11 | 3 | | 13 | 32 | | 11 | 50 | | 3 | 55 | | 54 | 30 | | 59 | 20 | | 10 | 40 | 23 |
| 24 | | 26 | 28 | | 29 | 43 | | 16 | 38 | | 10 | 30 | | 7 | 24 | | 15 | 12 | | 17 | 29 | | 15 | 30 | | 7 | 31 | | 58 | 19 | 16 | 3 | 35 | | 15 | 7 | 24 |
| 25 | | 30 | 39 | | 33 | 31 | | 20 | 15 | | 14 | 15 | | 11 | 26 | | 19 | 21 | | 21 | 26 | | 19 | 10 | | 11 | 7 | 14 | 2 | 10 | | 7 | 50 | | 19 | 33 | 25 |
| 26 | | 34 | 49 | | 37 | 17 | | 23 | 53 | | 18 | 2 | | 15 | 29 | | 23 | 30 | | 25 | 23 | | 22 | 50 | | 14 | 43 | | 6 | 1 | | 12 | 7 | | 24 | 0 | 26 |
| 27 | | 38 | 57 | | 41 | 2 | | 27 | 31 | | 21 | 49 | | 19 | 33 | | 27 | 39 | | 29 | 19 | | 26 | 29 | | 18 | 20 | | 9 | 53 | | 16 | 24 | | 28 | 26 | 27 |
| 28 | | 43 | 6 | | 44 | 48 | | 31 | 9 | | 25 | 37 | | 23 | 37 | | 31 | 48 | | 33 | 14 | | 30 | 8 | | 21 | 57 | | 13 | 45 | | 20 | 42 | | 32 | 52 | 28 |
| 29 | | 47 | 13 | | 48 | 33 | | 34 | 47 | | 29 | 24 | | 27 | 41 | | 35 | 56 | | 37 | 8 | | 33 | 47 | | 25 | 33 | | 17 | 38 | | 25 | 0 | | 37 | 18 | 29 |
| 30 | | 51 | 19 | | | | | 38 | 25 | | 33 | 12 | | 31 | 45 | | 40 | 5 | | 41 | 2 | | 37 | 26 | | 29 | 11 | | 21 | 33 | | 29 | 20 | | 41 | 43 | 30 |
| 31 | | 55 | 25 | | | | | 42 | 3 | | | | | 35 | 50 | | | | | 44 | 56 | | 41 | 4 | | | | | 25 | 28 | | | | | 46 | 9 | 31 |

310. A TABLE of the SUN'S RIGHT ASCENSION for the Years 1789, 1793, and 1797, being the firſt after Leap Year.

| Days. | January. | | | February. | | | March | | | April. | | | May. | | | June. | | | July. | | | Auguſt. | | | Septemb. | | | October. | | | Novemb. | | | December | | | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | h. | m. | s. | |
| 1 | 18 | 50 | 34 | 21 | 2 | 35 | 22 | 51 | 23 | 0 | 44 | 48 | 2 | 36 | 6 | 4 | 38 | 56 | 6 | 43 | 13 | 8 | 47 | 53 | 10 | 43 | 49 | 12 | 31 | 55 | 14 | 28 | 26 | 16 | 32 | 36 | 1 |
| 2 | | 54 | 58 | | 6 | 38 | | 55 | 6 | | 48 | 26 | | 39 | 55 | | 43 | 3 | | 47 | 21 | | 51 | 45 | | 47 | 27 | | 35 | 34 | | 32 | 22 | | 36 | 56 | 2 |
| 3 | | 59 | 23 | | 10 | 41 | | 58 | 49 | | 52 | 5 | | 43 | 45 | | 47 | 9 | | 51 | 28 | | 55 | 37 | | 51 | 4 | | 39 | 12 | | 36 | 20 | | 41 | 16 | 3 |
| 4 | 19 | 3 | 46 | | 14 | 42 | 23 | 2 | 32 | | 55 | 44 | | 47 | 36 | | 51 | 15 | | 55 | 35 | | 59 | 29 | | 54 | 41 | | 42 | 50 | | 40 | 18 | | 45 | 38 | 4 |
| 5 | | 8 | 10 | | 18 | 43 | | 6 | 15 | | 59 | 22 | | 51 | 27 | | 55 | 22 | | 59 | 42 | 9 | 3 | 19 | | 58 | 18 | | 46 | 30 | | 44 | 16 | | 50 | 0 | 5 |
| 6 | | 12 | 32 | | 22 | 44 | | 9 | 57 | 1 | 3 | 1 | | 55 | 19 | | 59 | 29 | 7 | 3 | 49 | | 7 | 9 | 11 | 1 | 54 | | 50 | 9 | | 48 | 16 | | 54 | 22 | 6 |
| 7 | | 16 | 55 | | 26 | 43 | | 13 | 39 | | 6 | 40 | | 59 | 11 | 5 | 3 | 37 | | 7 | 56 | | 10 | 59 | | 5 | 30 | | 53 | 49 | | 52 | 17 | | 58 | 45 | 7 |
| 8 | | 21 | 17 | | 30 | 41 | | 17 | 20 | | 10 | 20 | 3 | 3 | 3 | | 7 | 45 | | 12 | 1 | | 14 | 47 | | 9 | 7 | | 57 | 29 | | 56 | 18 | 17 | 3 | 8 | 8 |
| 9 | | 25 | 38 | | 34 | 38 | | 21 | 1 | | 14 | 0 | | 6 | 57 | | 11 | 53 | | 16 | 7 | | 18 | 36 | | 12 | 43 | 13 | 1 | 9 | 15 | 0 | 20 | | 7 | 32 | 9 |
| 10 | | 29 | 59 | | 38 | 36 | | 24 | 42 | | 17 | 39 | | 10 | 51 | | 16 | 1 | | 20 | 12 | | 22 | 24 | | 16 | 19 | | 4 | 50 | | 4 | 23 | | 11 | 58 | 10 |
| 11 | | 34 | 19 | | 42 | 31 | | 28 | 22 | | 21 | 19 | | 14 | 45 | | 20 | 10 | | 24 | 16 | | 26 | 11 | | 19 | 55 | | 8 | 32 | | 8 | 27 | | 16 | 21 | 11 |
| 12 | | 38 | 39 | | 46 | 27 | | 32 | 2 | | 25 | 0 | | 18 | 40 | | 24 | 20 | | 28 | 21 | | 29 | 58 | | 23 | 30 | | 12 | 14 | | 12 | 31 | | 20 | 51 | 12 |
| 13 | | 42 | 58 | | 50 | 22 | | 35 | 42 | | 28 | 40 | | 22 | 36 | | 28 | 28 | | 32 | 26 | | 33 | 44 | | 27 | 5 | | 15 | 56 | | 16 | 37 | | 25 | 11 | 13 |
| 14 | | 47 | 16 | | 54 | 15 | | 39 | 22 | | 32 | 22 | | 26 | 33 | | 32 | 37 | | 36 | 28 | | 37 | 29 | | 30 | 42 | | 19 | 39 | | 20 | 43 | | 29 | 37 | 14 |
| 15 | | 51 | 33 | | 58 | 8 | | 43 | 0 | | 36 | 4 | | 30 | 29 | | 36 | 46 | | 40 | 31 | | 41 | 14 | | 34 | 17 | | 23 | 24 | | 24 | 50 | | 34 | 3 | 15 |
| 16 | | 55 | 59 | 22 | 2 | 1 | | 46 | 39 | | 39 | 46 | | 34 | 27 | | 40 | 56 | | 44 | 33 | | 44 | 59 | | 37 | 52 | | 27 | 7 | | 28 | 59 | | 38 | 28 | 16 |
| 17 | 20 | 0 | 6 | | 5 | 52 | | 50 | 18 | | 43 | 28 | | 38 | 24 | | 45 | 5 | | 48 | 35 | | 48 | 42 | | 41 | 28 | | 30 | 52 | | 33 | 8 | | 42 | 55 | 17 |
| 18 | | 4 | 22 | | 9 | 43 | | 53 | 56 | | 47 | 10 | | 42 | 23 | | 49 | 15 | | 52 | 36 | | 52 | 26 | | 45 | 4 | | 34 | 38 | | 37 | 18 | | 47 | 21 | 18 |
| 19 | | 8 | 36 | | 13 | 34 | | 57 | 35 | | 50 | 53 | | 46 | 22 | | 53 | 23 | | 56 | 36 | | 56 | 9 | | 48 | 39 | | 38 | 25 | | 41 | 29 | | 51 | 48 | 19 |
| 20 | | 12 | 50 | | 17 | 24 | 0 | 1 | 12 | | 54 | 37 | | 50 | 21 | | 57 | 33 | 8 | 0 | 36 | | 59 | 12 | | 52 | 15 | | 42 | 11 | | 45 | 40 | | 56 | 15 | 20 |
| 21 | | 17 | 3 | | 21 | 12 | | 4 | 51 | | 58 | 20 | | 54 | 22 | 6 | 1 | 42 | | 4 | 36 | 10 | 3 | 34 | | 55 | 50 | | 45 | 58 | | 49 | 52 | 18 | 0 | 41 | 21 |
| 22 | | 21 | 15 | | 25 | 0 | | 8 | 29 | 2 | 2 | 6 | | 58 | 22 | | 5 | 52 | | 8 | 35 | | 7 | 15 | | 59 | 26 | | 49 | 46 | | 54 | 5 | | 5 | 8 | 22 |
| 23 | | 25 | 27 | | 28 | 48 | | 12 | 7 | | 5 | 50 | 4 | 2 | 24 | | 10 | 2 | | 12 | 34 | | 10 | 56 | 12 | 3 | 1 | | 53 | 34 | | 58 | 19 | | 9 | 34 | 23 |
| 24 | | 29 | 38 | | 32 | 15 | | 15 | 45 | | 9 | 35 | | 6 | 25 | | 14 | 11 | | 16 | 31 | | 14 | 37 | | 6 | 38 | | 57 | 25 | 16 | 2 | 33 | | 14 | 1 | 24 |
| 25 | | 33 | 48 | | 36 | 22 | | 19 | 23 | | 13 | 21 | | 10 | 27 | | 18 | 20 | | 20 | 29 | | 18 | 17 | | 10 | 14 | 14 | 1 | 15 | | 6 | 48 | | 18 | 28 | 25 |
| 26 | | 37 | 57 | | 40 | 8 | | 23 | 0 | | 17 | 7 | | 14 | 30 | | 22 | 28 | | 24 | 25 | | 21 | 57 | | 13 | 50 | | 5 | 5 | | 11 | 4 | | 22 | 55 | 26 |
| 27 | | 42 | 6 | | 43 | 53 | | 26 | 38 | | 20 | 53 | | 18 | 34 | | 26 | 39 | | 28 | 22 | | 25 | 36 | | 17 | 27 | | 8 | 57 | | 15 | 21 | | 27 | 21 | 27 |
| 28 | | 46 | 13 | | 47 | 38 | | 30 | 17 | | 24 | 41 | | 22 | 37 | | 30 | 48 | | 32 | 18 | | 29 | 16 | | 21 | 4 | | 12 | 49 | | 19 | 39 | | 31 | 47 | 28 |
| 29 | | 50 | 20 | | | | | 33 | 55 | | 28 | 29 | | 26 | 42 | | 34 | 56 | | 36 | 12 | | 32 | 55 | | 24 | 41 | | 16 | 42 | | 23 | 57 | | 36 | 13 | 29 |
| 30 | | 54 | 25 | | | | | 37 | 32 | | 32 | 17 | | 30 | 46 | | 38 | 4 | | 40 | 7 | | 36 | 33 | | 28 | 18 | | 20 | 36 | | 28 | 16 | | 40 | 39 | 30 |
| 31 | | 58 | 30 | | | | | 41 | 10 | | | | | 34 | 51 | | | | | 44 | 0 | | 40 | 11 | | | | | 24 | 31 | | | | | 45 | 4 | 31 |

## 310. A TABLE of the SUN's RIGHT ASCENSION for the Years 1790, 1794, 1798, being the ſecond after Leap Year.

| Days. | January. | February. | March. | April. | May. | June. | July. | Auguſt. | Septemb. | October. | Novemb. | December | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s | h. m. s. | h. m. s. | h. m. s. | h. m. s. | |
| 1 | 18 49 29 | 21 1 35 | 22 50 28 | 0 43 55 | 2 35 11 | 4 37 56 | 6 42 13 | 8 46 57 | 10 42 56 | 12 31 3 | 14 27 29 | 16 31 33 | 1 |
| 2 | 53 54 | 5 39 | 54 12 | 47 33 | 39 0 | 42 3 | 46 21 | 50 49 | 46 34 | 34 41 | 31 25 | 35 53 | 2 |
| 3 | 58 18 | 9 41 | 57 56 | 51 11 | 42 49 | 46 9 | 50 29 | 54 41 | 50 11 | 38 19 | 35 22 | 40 13 | 3 |
| 4 | 19 2 42 | 13 44 | 23 1 39 | 54 48 | 46 40 | 50 16 | 54 36 | 58 33 | 53 48 | 41 58 | 39 20 | 44 34 | 4 |
| 5 | 7 5 | 17 45 | 5 21 | 58 28 | 50 31 | 54 23 | 58 43 | 9 2 23 | 57 25 | 45 37 | 43 18 | 48 56 | 5 |
| 6 | 11 29 | 21 45 | 9 4 | 1 2 8 | 54 22 | 58 29 | 7 2 50 | 6 14 | 11 1 2 | 49 16 | 47 16 | 53 18 | 6 |
| 7 | 15 51 | 25 44 | 12 45 | 5 48 | 58 14 | 5 2 38 | 6 56 | 10 4 | 4 38 | 52 55 | 51 18 | 57 41 | 7 |
| 8 | 20 13 | 29 43 | 16 27 | 9 26 | 3 2 7 | 6 45 | 11 2 | 13 53 | 8 14 | 56 35 | 55 19 | 17 2 4 | 8 |
| 9 | 24 35 | 33 41 | 20 8 | 13 6 | 6 0 | 10 53 | 15 8 | 17 41 | 11 50 | 13 0 16 | 59 22 | 6 28 | 9 |
| 10 | 28 55 | 37 38 | 23 48 | 16 46 | 9 54 | 15 2 | 19 13 | 21 29 | 15 26 | 3 57 | 15 3 24 | 10 53 | 10 |
| 11 | 33 16 | 41 34 | 27 28 | 20 26 | 13 48 | 19 10 | 23 18 | 25 16 | 19 2 | 7 38 | 7 28 | 15 17 | 11 |
| 12 | 37 35 | 45 30 | 31 8 | 24 7 | 17 43 | 23 19 | 27 22 | 29 3 | 22 38 | 11 20 | 11 33 | 19 41 | 12 |
| 13 | 41 55 | 49 24 | 34 48 | 27 47 | 21 39 | 27 28 | 31 25 | 32 49 | 26 14 | 15 2 | 15 38 | 24 7 | 13 |
| 14 | 46 13 | 53 17 | 38 28 | 31 28 | 25 35 | 31 36 | 35 29 | 36 35 | 29 49 | 18 45 | 19 44 | 28 32 | 14 |
| 15 | 50 31 | 57 11 | 42 7 | 35 10 | 29 32 | 35 44 | 39 22 | 40 20 | 33 25 | 22 29 | 23 51 | 32 58 | 15 |
| 16 | 54 48 | 22 1 4 | 45 46 | 38 51 | 33 29 | 39 55 | 43 34 | 44 5 | 37 0 | 26 13 | 27 59 | 37 24 | 16 |
| 17 | 59 4 | 4 56 | 49 24 | 42 34 | 37 27 | 44 5 | 47 36 | 47 49 | 40 36 | 29 58 | 32 7 | 41 50 | 17 |
| 18 | 20 3 19 | 8 47 | 53 3 | 46 16 | 41 25 | 48 14 | 51 38 | 51 32 | 44 11 | 33 43 | 36 17 | 46 17 | 18 |
| 19 | 7 36 | 12 38 | 56 42 | 49 59 | 45 24 | 52 23 | 55 39 | 55 15 | 47 47 | 37 29 | 40 28 | 50 43 | 19 |
| 20 | 11 49 | 16 28 | 0 0 19 | 53 42 | 49 24 | 56 33 | 59 39 | 58 58 | 51 23 | 41 16 | 44 39 | 55 10 | 20 |
| 21 | 16 2 | 20 16 | 3 58 | 57 26 | 53 24 | 6 0 42 | 8 3 38 | 10 2 40 | 54 58 | 45 3 | 48 51 | 59 37 | 21 |
| 22 | 20 14 | 24 5 | 7 36 | 2 1 10 | 57 24 | 4 52 | 7 38 | 6 21 | 58 34 | 48 51 | 53 4 | 18 4 4 | 22 |
| 23 | 24 26 | 27 53 | 11 15 | 4 55 | 4 1 25 | 9 1 | 11 36 | 10 3 | 12 2 9 | 52 39 | 57 17 | 8 30 | 23 |
| 24 | 28 37 | 31 41 | 14 52 | 8 40 | 5 27 | 13 10 | 15 34 | 13 44 | 5 42 | 56 28 | 16 1 31 | 12 57 | 24 |
| 25 | 32 47 | 35 27 | 18 30 | 12 26 | 9 29 | 17 19 | 19 31 | 17 24 | 9 21 | 14 0 18 | 5 46 | 17 23 | 25 |
| 26 | 36 57 | 39 13 | 22 7 | 16 12 | 13 32 | 21 28 | 23 29 | 21 4 | 12 58 | 4 9 | 10 2 | 21 49 | 26 |
| 27 | 41 5 | 42 58 | 25 45 | 19 59 | 17 35 | 25 37 | 27 25 | 24 43 | 16 35 | 8 0 | 14 19 | 26 16 | 27 |
| 28 | 45 13 | 46 44 | 29 23 | 23 46 | 21 38 | 29 46 | 31 20 | 28 23 | 20 11 | 11 53 | 18 36 | 30 43 | 28 |
| 29 | 49 20 | | 33 1 | 27 34 | 25 43 | 33 55 | 35 15 | 32 2 | 23 48 | 15 46 | 22 55 | 35 9 | 29 |
| 30 | 53 25 | | 36 39 | 31 21 | 29 47 | 38 4 | 39 10 | 35 40 | 27 25 | 19 39 | 27 14 | 39 34 | 30 |
| 31 | 57 31 | | 40 17 | | 33 52 | | 43 4 | 39 18 | | 23 34 | | 44 0 | 31 |

## 310. A TABLE of the SUN's RIGHT ASCENSION for the Years 1791, 1795, 1799, being the third after Leap Year.

| Days | January. | February. | March. | April. | May. | June. | July. | Auguſt. | Septemb. | October. | Novemb. | December | Days. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | h. m. s. | |
| 1 | 18 48 26 | 21 0 36 | 22 49 33 | 0 43 2 | 2 34 15 | 4 36 58 | 6 41 13 | 8 46 0 | 10 42 3 | 12 30 11 | 14 26 32 | 30 30 | 1 |
| 2 | 52 50 | 4 40 | 53 17 | 46 40 | 38 4 | 41 4 | 45 21 | 49 53 | 45 41 | 33 48 | 30 28 | 34 50 | 2 |
| 3 | 57 14 | 8 43 | 57 1 | 50 19 | 41 54 | 45 9 | 49 29 | 53 46 | 49 19 | 37 27 | 34 25 | 39 10 | 3 |
| 4 | 19 1 38 | 12 45 | 23 0 45 | 53 57 | 45 44 | 49 16 | 53 36 | 57 37 | 52 56 | 41 5 | 38 23 | 43 31 | 4 |
| 5 | 6 2 | 16 46 | 4 27 | 57 36 | 49 35 | 53 23 | 57 43 | 9 1 28 | 56 33 | 44 43 | 42 20 | 47 52 | 5 |
| 6 | 10 25 | 20 47 | 8 10 | 1 1 15 | 53 27 | 57 29 | 7 1 50 | 5 18 | 11 0 9 | 48 22 | 46 19 | 52 15 | 6 |
| 7 | 14 47 | 24 46 | 11 51 | 4 54 | 57 19 | 5 1 37 | 5 56 | 9 8 | 3 46 | 52 2 | 50 19 | 56 37 | 7 |
| 8 | 19 10 | 28 45 | 15 32 | 8 33 | 3 1 11 | 5 45 | 10 3 | 12 57 | 7 22 | 55 42 | 54 20 | 17 1 0 | 8 |
| 9 | 23 31 | 32 43 | 19 14 | 12 13 | 5 4 | 9 53 | 14 9 | 16 45 | 10 58 | 59 22 | 58 22 | 5 23 | 9 |
| 10 | 27 52 | 36 40 | 22 55 | 15 53 | 8 57 | 14 1 | 18 14 | 20 34 | 14 34 | 13 3 3 | 15 2 25 | 9 48 | 10 |
| 11 | 32 13 | 40 37 | 26 35 | 19 33 | 12 51 | 18 9 | 22 18 | 24 22 | 18 10 | 6 45 | 6 28 | 14 12 | 11 |
| 12 | 36 33 | 44 33 | 30 15 | 23 13 | 16 46 | 22 19 | 26 22 | 28 8 | 21 45 | 10 26 | 10 33 | 18 37 | 12 |
| 13 | 40 52 | 48 27 | 33 55 | 26 54 | 20 41 | 26 28 | 30 27 | 31 54 | 25 21 | 14 9 | 14 38 | 23 2 | 13 |
| 14 | 45 10 | 52 22 | 37 35 | 30 35 | 24 38 | 30 36 | 34 30 | 35 40 | 28 57 | 17 51 | 18 44 | 27 28 | 14 |
| 15 | 49 28 | 56 15 | 41 14 | 34 16 | 28 35 | 34 45 | 38 33 | 39 25 | 32 32 | 21 35 | 22 51 | 31 54 | 15 |
| 16 | 53 45 | 22 0 8 | 44 53 | 37 58 | 32 33 | 38 55 | 42 36 | 43 10 | 36 8 | 25 19 | 26 58 | 36 19 | 16 |
| 17 | 58 2 | 4 0 | 48 31 | 41 39 | 36 29 | 43 4 | 46 38 | 46 54 | 39 44 | 29 3 | 31 7 | 40 46 | 17 |
| 18 | 20 2 18 | 7 51 | 52 10 | 45 21 | 40 28 | 47 14 | 50 39 | 50 38 | 43 19 | 32 48 | 35 16 | 45 12 | 18 |
| 19 | 6 33 | 11 42 | 55 49 | 49 5 | 44 26 | 51 23 | 54 40 | 54 22 | 46 54 | 36 34 | 39 26 | 49 39 | 19 |
| 20 | 10 47 | 15 32 | 59 27 | 52 48 | 48 26 | 55 33 | 58 41 | 58 4 | 50 30 | 40 20 | 43 37 | 54 6 | 20 |
| 21 | 15 0 | 19 21 | 0 3 5 | 56 32 | 52 26 | 59 43 | 8 2 41 | 10 1 46 | 54 6 | 44 8 | 47 49 | 58 32 | 21 |
| 22 | 19 13 | 23 10 | 6 43 | 2 0 16 | 56 26 | 6 3 52 | 6 40 | 5 28 | 57 42 | 47 55 | 52 2 | 18 2 59 | 22 |
| 23 | 23 25 | 26 58 | 10 21 | 4 1 | 4 0 27 | 8 2 | 10 33 | 9 9 | 12 1 17 | 51 43 | 56 15 | 7 25 | 23 |
| 24 | 27 36 | 30 46 | 14 1 | 7 46 | 4 29 | 12 11 | 14 37 | 12 50 | 4 53 | 55 33 | 16 0 30 | 11 51 | 24 |
| 25 | 31 47 | 34 32 | 17 37 | 11 31 | 8 31 | 16 20 | 18 34 | 16 31 | 8 29 | 59 22 | 4 44 | 16 19 | 25 |
| 26 | 35 56 | 38 18 | 21 15 | 15 17 | 12 33 | 20 29 | 22 31 | 20 11 | 12 6 | 14 3 13 | 9 0 | 20 45 | 26 |
| 27 | 40 5 | 42 4 | 24 53 | 19 4 | 16 36 | 24 38 | 26 28 | 23 50 | 15 42 | 7 4 | 13 16 | 25 12 | 27 |
| 28 | 44 12 | 45 49 | 28 31 | 22 51 | 20 39 | 28 47 | 30 23 | 27 30 | 19 19 | 10 56 | 17 34 | 29 38 | 28 |
| 29 | 48 19 | | 32 8 | 26 38 | 24 43 | 32 56 | 34 18 | 31 8 | 22 57 | 14 48 | 21 52 | 34 3 | 29 |
| 30 | 52 26 | | 35 46 | 30 26 | 28 47 | 37 4 | 38 13 | 34 47 | 26 33 | 18 42 | 26 10 | 38 30 | 30 |
| 31 | 56 32 | | 39 24 | | 32 53 | | 42 7 | 38 26 | | 22 37 | | 42 55 | 31 |

311. TABLE for fitting the TABLES of the SUN's LONG. DECL. or Rt. ASCEN. to any meridian: Or, for finding either quantity at any hour. Degrees of longitude from the Meridian of London: Or, time before and after noon.

| Daily diff. | 5 Deg. | 10 D. | 15 D. | 20 D. | 25 D. | 30 D. | 35 D. | 40 D. | 45 D. | 50 D. | 55 D. | 60 D. | 65 D. | 70 D. | 75 D. | 80 D. | 85 D. | 90 D. | 95 D. | 100D. | 105D. | 110D. | 115D. | 120D. | Daily diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | h. m.<br>0 20 | h. m.<br>0 40 | h. m.<br>1 0 | h. m.<br>1 20 | h. m.<br>1 40 | h. m.<br>2 0 | h. m.<br>2 20 | h. m.<br>2 40 | h. m.<br>3 0 | h. m.<br>3 20 | h. m.<br>3 40 | h. m.<br>4 0 | h. m.<br>4 20 | h. m.<br>4 40 | h. m.<br>5 0 | h. m.<br>5 20 | h. m.<br>5 40 | h. m.<br>6 0 | h. m.<br>6 20 | h. m.<br>6 40 | h. m.<br>7 0 | h. m.<br>7 20 | h. m.<br>7 40 | h. m.<br>8 0 | |
| ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ |
| 0 20 | 0 0 | 0 0 | 0 1 | 0 1 | 0 1 | 0 2 | 0 2 | 0 2 | 0 2 | 0 3 | 0 3 | 0 3 | 0 4 | 0 4 | 0 4 | 0 4 | 0 5 | 0 5 | 0 5 | 0 5 | 0 6 | 0 6 | 0 6 | 0 7 | 0 20 |
| 40 | 0 0 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 7 | 7 | 8 | 8 | 9 | 9 | 10 | 10 | 11 | 12 | 12 | 13 | 13 | 40 |
| 1 0 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 12 | 13 | 14 | 15 | 16 | 17 | 17 | 19 | 19 | 20 | 1 0 |
| 20 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 27 | 20 |
| 40 | 1 | 3 | 4 | 5 | 7 | 8 | 10 | 11 | 12 | 14 | 15 | 17 | 18 | 19 | 21 | 22 | 24 | 25 | 26 | 28 | 29 | 30 | 32 | 33 | 40 |
| 2 0 | 2 | 3 | 5 | 7 | 8 | 10 | 12 | 13 | 15 | 17 | 18 | 20 | 22 | 23 | 25 | 27 | 28 | 30 | 32 | 33 | 35 | 37 | 38 | 40 | 2 0 |
| 20 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 | 31 | 33 | 35 | 38 | 39 | 40 | 43 | 45 | 47 | 20 |
| 40 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 18 | 20 | 22 | 24 | 27 | 29 | 31 | 33 | 35 | 38 | 40 | 42 | 44 | 47 | 49 | 51 | 53 | 40 |
| 3 0 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 | 25 | 27 | 30 | 32 | 35 | 37 | 40 | 42 | 45 | 47 | 50 | 52 | 55 | 57 | 1 0 | 3 0 |
| 20 | 3 | 5 | 8 | 11 | 14 | 17 | 19 | 22 | 25 | 28 | 30 | 33 | 36 | 39 | 42 | 44 | 47 | 50 | 53 | 55 | 58 | 1 1 | 1 4 | 1 7 | 20 |
| 40 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 55 | 58 | 1 1 | 1 4 | 1 7 | 1 10 | 1 13 | 40 |
| 4 0 | 3 | 7 | 10 | 13 | 17 | 20 | 23 | 27 | 30 | 33 | 37 | 40 | 43 | 47 | 50 | 53 | 57 | 1 0 | 1 3 | 1 7 | 1 10 | 1 13 | 1 16 | 1 20 | 4 0 |
| 20 | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 | 36 | 40 | 43 | 47 | 50 | 54 | 58 | 1 1 | 1 5 | 1 9 | 1 12 | 1 16 | 1 19 | 1 23 | 1 27 | 20 |
| 40 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 | 39 | 43 | 47 | 50 | 54 | 58 | 1 2 | 1 6 | 1 10 | 1 14 | 1 18 | 1 22 | 1 25 | 1 29 | 1 33 | 40 |
| 5 0 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 | 42 | 46 | 50 | 54 | 58 | 1 2 | 1 7 | 1 10 | 1 15 | 1 19 | 1 23 | 1 27 | 1 32 | 1 36 | 1 40 | 5 0 |
| 20 | 4 | 9 | 13 | 18 | 22 | 26 | 31 | 35 | 40 | 44 | 49 | 53 | 58 | 1 2 | 1 6 | 1 11 | 1 15 | 1 20 | 1 24 | 1 29 | 1 33 | 1 38 | 1 42 | 1 47 | 20 |
| 40 | 5 | 9 | 14 | 19 | 24 | 28 | 33 | 38 | 42 | 47 | 52 | 57 | 1 1 | 1 6 | 1 11 | 1 15 | 1 20 | 1 25 | 1 30 | 1 34 | 1 39 | 1 44 | 1 49 | 1 53 | 40 |
| 6 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 1 0 | 1 5 | 1 10 | 1 15 | 1 20 | 1 25 | 1 30 | 1 39 | 1 40 | 1 45 | 1 50 | 1 55 | 2 0 | 6 0 |
| 20 | 5 | 10 | 16 | 21 | 26 | 32 | 37 | 42 | 47 | 53 | 58 | 1 3 | 1 9 | 1 14 | 1 19 | 1 24 | 1 30 | 1 35 | 1 40 | 1 45 | 1 51 | 1 56 | 2 1 | 2 7 | 20 |
| 40 | 5 | 11 | 17 | 22 | 28 | 33 | 39 | 44 | 50 | 55 | 1 1 | 1 7 | 1 12 | 1 18 | 1 23 | 1 29 | 1 34 | 1 40 | 1 45 | 1 51 | 1 57 | 2 2 | 2 8 | 2 13 | 40 |
| 7 0 | 6 | 12 | 17 | 23 | 29 | 35 | 41 | 47 | 52 | 58 | 1 4 | 1 10 | 1 16 | 1 22 | 1 27 | 1 33 | 1 39 | 1 45 | 1 51 | 1 57 | 2 2 | 2 8 | 2 14 | 2 20 | 7 0 |
| 20 | 6 | 12 | 18 | 24 | 30 | 37 | 43 | 49 | 55 | 1 1 | 1 7 | 1 13 | 1 19 | 1 25 | 1 32 | 1 38 | 1 44 | 1 50 | 1 56 | 2 2 | 2 8 | 2 14 | 2 20 | 2 27 | 20 |
| 40 | 6 | 13 | 19 | 25 | 32 | 38 | 45 | 51 | 57 | 1 4 | 1 10 | 1 17 | 1 23 | 1 29 | 1 36 | 1 42 | 1 49 | 1 55 | 2 1 | 2 8 | 2 14 | 2 20 | 2 27 | 2 33 | 40 |
| 8 0 | 7 | 13 | 20 | 27 | 33 | 40 | 47 | 53 | 1 0 | 1 7 | 1 13 | 1 20 | 1 27 | 1 33 | 1 40 | 1 47 | 1 53 | 2 0 | 2 7 | 2 13 | 2 20 | 2 27 | 2 33 | 2 40 | 8 0 |
| 20 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 55 | 1 2 | 1 9 | 1 16 | 1 23 | 1 30 | 1 37 | 1 44 | 1 51 | 1 58 | 2 5 | 2 12 | 2 19 | 2 26 | 2 33 | 2 40 | 2 47 | 20 |
| 40 | 7 | 14 | 21 | 29 | 36 | 43 | 50 | 58 | 1 5 | 1 12 | 1 19 | 1 27 | 1 34 | 1 41 | 1 48 | 1 55 | 2 3 | 2 10 | 2 17 | 2 24 | 2 32 | 2 39 | 2 46 | 2 53 | 40 |
| 9 0 | 7 | 15 | 22 | 30 | 37 | 45 | 52 | 1 0 | 1 7 | 1 15 | 1 22 | 1 30 | 1 37 | 1 45 | 1 52 | 2 0 | 2 7 | 2 15 | 2 22 | 2 30 | 2 37 | 2 45 | 2 52 | 3 0 | 9 0 |
| 20 | 8 | 15 | 23 | 31 | 39 | 47 | 54 | 1 2 | 1 10 | 1 18 | 1 25 | 1 33 | 1 41 | 1 49 | 1 57 | 2 4 | 2 12 | 2 20 | 2 28 | 2 35 | 2 43 | 2 51 | 2 59 | 3 7 | 20 |
| 40 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 1 4 | 1 12 | 1 20 | 1 29 | 1 37 | 1 45 | 1 53 | 2 1 | 2 9 | 2 17 | 2 25 | 2 33 | 2 41 | 2 49 | 2 57 | 3 5 | 3 13 | 40 |
| 10 0 | 8 | 17 | 25 | 33 | 41 | 50 | 58 | 1 7 | 1 15 | 1 23 | 1 32 | 1 40 | 1 48 | 1 57 | 2 5 | 2 13 | 2 22 | 2 30 | 2 38 | 2 47 | 2 55 | 3 3 | 3 12 | 3 20 | 10 0 |
| 20 | 9 | 17 | 26 | 34 | 43 | 52 | 1 0 | 1 9 | 1 17 | 1 26 | 1 35 | 1 43 | 1 52 | 2 0 | 2 9 | 2 18 | 2 26 | 2 35 | 2 44 | 2 52 | 3 1 | 3 9 | 3 18 | 3 27 | 20 |
| 40 | 9 | 18 | 27 | 35 | 44 | 53 | 1 3 | 1 11 | 1 20 | 1 29 | 1 38 | 1 47 | 1 55 | 2 4 | 2 13 | 2 22 | 2 31 | 2 40 | 2 49 | 2 58 | 3 7 | 3 15 | 3 24 | 3 33 | 40 |
| 11 0 | 9 | 18 | 27 | 37 | 46 | 55 | 1 4 | 1 13 | 1 22 | 1 32 | 1 41 | 1 50 | 1 59 | 2 8 | 2 17 | 2 27 | 2 36 | 2 45 | 2 54 | 3 3 | 3 12 | 3 22 | 3 31 | 3 40 | 11 0 |
| 20 | 10 | 19 | 28 | 38 | 47 | 57 | 1 6 | 1 16 | 1 24 | 1 35 | 1 44 | 1 54 | 2 3 | 2 12 | 2 22 | 2 31 | 2 41 | 2 50 | 2 59 | 3 9 | 3 18 | 3 28 | 3 38 | 3 47 | 20 |
| 40 | 10 | 19 | 29 | 39 | 49 | 58 | 1 8 | 1 18 | 1 27 | 1 37 | 1 47 | 1 57 | 2 6 | 2 16 | 2 26 | 2 35 | 2 45 | 2 55 | 3 5 | 3 14 | 3 24 | 3 34 | 3 44 | 3 53 | 40 |

## 311. TABLE for fitting the SUN's LONGITUDE, DECLINATION, or Right ASCENSION, to any meridian, continued.

| Daily diff. | 5 Deg. | 10 D. | 15 D. | 20 D. | 25 D. | 30 D. | 35 D. | 40 D. | 45 D. | 50 D. | 55 D. | 60 D. | 65 D. | 70 D. | 75 D. | 80 D. | 85 D. | 90 D. | 95 D. | 100 D. | 105 D. | 110 D. | 115 D. | 120 D. | Daily diff. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  | h. m.<br>0 20 | h. m.<br>0 40 | h. m.<br>1 0 | h. m.<br>1 20 | h. m.<br>1 40 | h. m.<br>2 0 | h. m.<br>2 20 | h. m.<br>2 40 | h. m.<br>3 0 | h. m.<br>3 20 | h. m.<br>3 40 | h. m.<br>4 0 | h. m.<br>4 20 | h. m.<br>4 40 | h. m.<br>5 0 | h. m.<br>5 20 | h. m.<br>5 40 | h. m.<br>6 0 | h. m.<br>6 20 | h. m.<br>6 40 | h. m.<br>7 0 | h. m.<br>7 20 | h. m.<br>7 40 | h. m.<br>8 0 |  |
| ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ |
| 12 0 | 10 | 20 | 30 | 40 | 50 | 1 0 | 1 10 | 1 20 | 1 30 | 1 40 | 1 50 | 2 0 | 2 10 | 2 20 | 2 30 | 2 40 | 2 50 | 3 0 | 3 10 | 3 20 | 3 30 | 3 40 | 3 50 | 4 0 | 12 0 |
| 20 | 10 | 20 | 31 | 41 | 52 | 1 2 | 1 12 | 1 23 | 1 32 | 1 43 | 1 53 | 2 3 | 2 14 | 2 24 | 2 34 | 2 45 | 2 55 | 3 5 | 3 15 | 3 25 | 3 35 | 3 46 | 3 57 | 4 7 | 20 |
| 40 | 11 | 21 | 31 | 42 | 53 | 1 3 | 1 14 | 1 25 | 1 34 | 1 45 | 1 56 | 2 7 | 2 18 | 2 28 | 2 38 | 2 49 | 2 59 | 3 10 | 3 20 | 3 31 | 3 41 | 3 52 | 4 3 | 4 14 | 40 |
| 13 0 | 11 | 22 | 32 | 43 | 54 | 1 5 | 1 16 | 1 27 | 1 37 | 1 48 | 1 59 | 2 10 | 2 21 | 2 32 | 2 42 | 2 53 | 3 4 | 3 15 | 3 26 | 3 37 | 3 47 | 3 58 | 4 9 | 4 20 | 13 0 |
| 20 | 11 | 22 | 33 | 44 | 55 | 1 7 | 1 18 | 1 20 | 1 40 | 1 51 | 2 2 | 2 13 | 2 24 | 2 35 | 2 47 | 2 58 | 3 9 | 3 20 | 3 31 | 3 42 | 3 53 | 4 4 | 4 15 | 4 27 | 20 |
| 40 | 11 | 23 | 34 | 45 | 57 | 1 8 | 1 20 | 1 31 | 1 43 | 1 54 | 2 5 | 2 16 | 2 28 | 2 39 | 2 51 | 3 3 | 3 14 | 3 25 | 3 37 | 3 47 | 3 59 | 4 11 | 4 22 | 4 34 | 40 |
| 14 0 | 12 | 23 | 35 | 47 | 58 | 1 10 | 1 22 | 1 33 | 1 45 | 1 57 | 2 8 | 2 20 | 2 32 | 2 43 | 2 55 | 3 7 | 3 18 | 3 30 | 3 42 | 3 53 | 4 5 | 4 17 | 4 28 | 4 40 | 14 0 |
| 20 | 12 | 24 | 36 | 48 | 59 | 1 11 | 1 24 | 1 35 | 1 48 | 1 59 | 2 11 | 2 24 | 2 35 | 2 47 | 2 59 | 3 11 | 3 23 | 3 35 | 3 46 | 3 59 | 4 11 | 4 23 | 4 35 | 4 47 | 20 |
| 40 | 12 | 24 | 37 | 49 | 1 0 | 1 13 | 1 25 | 1 38 | 1 50 | 2 2 | 2 14 | 2 27 | 2 39 | 2 51 | 3 3 | 3 15 | 3 28 | 3 40 | 3 52 | 4 4 | 4 17 | 4 29 | 4 41 | 4 53 | 40 |
| 15 0 | 12 | 25 | 37 | 50 | 1 2 | 1 15 | 1 27 | 1 40 | 1 52 | 2 5 | 2 17 | 2 30 | 2 42 | 2 55 | 3 7 | 3 20 | 3 32 | 3 45 | 3 57 | 4 10 | 4 22 | 4 35 | 4 47 | 5 0 | 15 0 |
| 20 | 13 | 25 | 38 | 51 | 1 3 | 1 16 | 1 29 | 1 43 | 1 55 | 2 8 | 2 20 | 2 33 | 2 46 | 2 59 | 3 12 | 3 25 | 3 37 | 3 50 | 4 3 | 4 16 | 4 28 | 4 41 | 4 54 | 5 7 | 20 |
| 40 | 13 | 26 | 39 | 52 | 1 5 | 1 18 | 1 31 | 1 45 | 1 58 | 2 11 | 2 24 | 2 37 | 2 50 | 3 3 | 3 16 | 3 29 | 3 42 | 3 55 | 4 8 | 4 21 | 4 34 | 4 47 | 5 0 | 5 14 | 40 |
| 16 0 | 13 | 27 | 40 | 53 | 1 7 | 1 20 | 1 33 | 1 47 | 2 0 | 2 13 | 2 27 | 2 40 | 2 53 | 3 7 | 3 20 | 3 33 | 3 47 | 4 0 | 4 13 | 4 27 | 4 40 | 4 53 | 5 7 | 5 20 | 16 0 |
| 20 | 14 | 27 | 41 | 54 | 1 8 | 1 22 | 1 35 | 1 49 | 2 2 | 2 16 | 2 30 | 2 43 | 2 57 | 3 10 | 3 24 | 3 38 | 3 51 | 4 5 | 4 19 | 4 32 | 4 46 | 4 59 | 5 13 | 5 27 | 20 |
| 40 | 14 | 28 | 41 | 56 | 1 10 | 1 23 | 1 37 | 1 51 | 2 5 | 2 19 | 2 33 | 2 47 | 3 1 | 3 14 | 3 28 | 3 42 | 3 56 | 4 10 | 4 24 | 4 38 | 4 52 | 5 6 | 5 20 | 5 34 | 40 |
| 17 0 | 14 | 29 | 42 | 57 | 1 11 | 1 25 | 1 39 | 1 53 | 2 7 | 2 21 | 2 36 | 2 50 | 3 4 | 3 18 | 3 32 | 3 47 | 4 0 | 4 15 | 4 29 | 4 44 | 4 58 | 5 12 | 5 26 | 5 40 | 17 0 |
| 20 | 14 | 29 | 43 | 58 | 1 12 | 1 27 | 1 41 | 1 55 | 2 10 | 2 24 | 2 39 | 2 53 | 3 8 | 3 22 | 3 37 | 3 51 | 4 5 | 4 20 | 4 34 | 4 49 | 5 3 | 5 18 | 5 32 | 5 47 | 20 |
| 40 | 15 | 30 | 44 | 59 | 1 14 | 1 28 | 1 43 | 1 58 | 2 12 | 2 27 | 2 42 | 2 57 | 3 11 | 3 26 | 3 41 | 3 56 | 4 10 | 4 25 | 4 39 | 4 55 | 5 9 | 5 24 | 5 38 | 5 54 | 40 |
| 18 0 | 15 | 30 | 45 | 1 0 | 1 15 | 1 30 | 1 45 | 2 0 | 2 15 | 2 30 | 2 45 | 3 0 | 3 15 | 3 30 | 3 45 | 4 0 | 4 15 | 4 30 | 4 45 | 5 0 | 5 15 | 5 30 | 5 45 | 6 0 | 18 0 |
| 20 | 15 | 30 | 46 | 1 1 | 1 16 | 1 32 | 1 47 | 2 2 | 2 17 | 2 33 | 2 48 | 3 3 | 3 19 | 3 34 | 3 49 | 4 4 | 4 20 | 4 35 | 4 50 | 5 6 | 5 21 | 5 36 | 5 51 | 6 7 | 20 |
| 40 | 15 | 31 | 47 | 1 2 | 1 18 | 1 33 | 1 49 | 2 4 | 2 20 | 2 35 | 2 51 | 3 7 | 3 22 | 3 38 | 3 53 | 4 9 | 4 24 | 4 40 | 4 55 | 5 11 | 5 27 | 5 42 | 5 58 | 6 13 | 40 |
| 19 0 | 16 | 31 | 47 | 1 3 | 1 19 | 1 35 | 1 51 | 2 7 | 2 23 | 2 38 | 2 54 | 3 10 | 3 26 | 3 42 | 3 58 | 4 14 | 4 29 | 4 45 | 5 0 | 5 17 | 5 33 | 5 49 | 6 5 | 6 20 | 19 0 |
| 20 | 16 | 32 | 48 | 1 4 | 1 20 | 1 36 | 1 53 | 2 9 | 2 25 | 2 41 | 2 57 | 3 13 | 3 29 | 3 45 | 4 2 | 4 18 | 4 34 | 4 50 | 5 6 | 5 22 | 5 38 | 5 55 | 6 11 | 6 27 | 20 |
| 40 | 16 | 32 | 49 | 1 5 | 1 21 | 1 38 | 1 55 | 2 11 | 2 27 | 2 44 | 3 0 | 3 16 | 3 33 | 3 49 | 4 6 | 4 22 | 4 39 | 4 55 | 5 12 | 5 28 | 5 44 | 6 1 | 6 17 | 6 34 | 40 |
| 20 0 | 17 | 33 | 50 | 1 7 | 1 23 | 1 40 | 1 57 | 2 13 | 2 30 | 2 47 | 3 3 | 3 20 | 3 37 | 3 53 | 4 10 | 4 27 | 4 43 | 5 0 | 5 17 | 5 33 | 5 50 | 6 7 | 6 23 | 6 40 | 20 0 |
| 20 | 17 | 33 | 50 | 1 8 | 1 25 | 1 42 | 1 59 | 2 16 | 2 33 | 2 50 | 3 6 | 3 23 | 3 40 | 3 57 | 4 14 | 4 31 | 4 48 | 5 5 | 5 22 | 5 39 | 5 56 | 6 13 | 6 29 | 6 47 | 20 |
| 40 | 17 | 34 | 51 | 1 9 | 1 26 | 1 43 | 2 1 | 2 18 | 2 35 | 2 52 | 3 9 | 3 26 | 3 43 | 4 1 | 4 18 | 4 35 | 4 53 | 5 10 | 5 27 | 5 44 | 6 1 | 6 19 | 6 36 | 6 54 | 40 |
| 21 0 | 17 | 35 | 52 | 1 10 | 1 27 | 1 45 | 2 2 | 2 20 | 2 37 | 2 55 | 3 12 | 3 30 | 3 47 | 4 5 | 4 22 | 4 40 | 4 57 | 5 15 | 5 32 | 5 50 | 6 7 | 6 25 | 6 42 | 7 0 | 21 0 |
| 20 | 18 | 35 | 53 | 1 11 | 1 29 | 1 47 | 2 4 | 2 22 | 2 40 | 2 58 | 3 15 | 3 33 | 3 51 | 4 9 | 4 27 | 4 44 | 5 2 | 5 20 | 5 38 | 5 55 | 6 13 | 6 31 | 6 49 | 7 7 | 20 |
| 40 | 18 | 36 | 54 | 1 12 | 1 30 | 1 49 | 2 6 | 2 24 | 2 43 | 3 0 | 3 19 | 3 36 | 3 53 | 4 13 | 4 31 | 4 48 | 5 7 | 5 25 | 5 43 | 6 1 | 6 19 | 6 37 | 6 55 | 7 13 | 40 |
| 22 0 | 18 | 37 | 55 | 1 13 | 1 32 | 1 50 | 2 8 | 2 27 | 2 45 | 3 3 | 3 22 | 3 40 | 3 58 | 4 17 | 4 35 | 4 53 | 5 12 | 5 30 | 5 48 | 6 7 | 6 25 | 6 43 | 7 2 | 7 20 | 22 0 |
| 20 | 18 | 37 | 56 | 1 14 | 1 33 | 1 52 | 2 10 | 2 29 | 2 48 | 3 6 | 3 25 | 3 44 | 4 2 | 4 21 | 4 39 | 4 58 | 5 17 | 5 35 | 5 54 | 6 13 | 6 31 | 6 49 | 7 8 | 7 27 | 20 |
| 40 | 19 | 38 | 57 | 1 15 | 1 34 | 1 53 | 2 12 | 2 31 | 2 50 | 3 9 | 3 28 | 3 47 | 4 5 | 4 24 | 4 43 | 5 2 | 5 21 | 5 40 | 5 59 | 6 18 | 6 37 | 6 55 | 7 14 | 7 33 | 40 |
| 23 0 | 19 | 38 | 58 | 1 17 | 1 36 | 1 55 | 2 14 | 2 33 | 2 52 | 3 12 | 3 31 | 3 50 | 4 9 | 4 28 | 4 47 | 5 7 | 5 26 | 5 45 | 6 4 | 6 23 | 6 42 | 7 2 | 7 21 | 7 40 | 23 0 |
| 27 | 19 | 39 | 59 | 1 18 | 1 38 | 1 57 | 2 17 | 2 36 | 2 55 | 3 15 | 3 35 | 3 54 | 4 14 | 4 33 | 4 53 | 5 12 | 5 32 | 5 52 | 6 11 | 6 30 | 6 49 | 7 9 | 7 29 | 7 40 | 27 |
| 54 | 20 | 40 | 1 0 | 1 20 | 1 39 | 1 59 | 2 19 | 2 39 | 2 59 | 3 19 | 3 39 | 3 59 | 4 19 | 4 39 | 4 59 | 5 19 | 5 38 | 5 58 | 6 18 | 6 38 | 6 58 | 7 18 | 7 38 | 7 58 | 54 |

## 312. TABLE of the RIGHT ASCENSIONS and DECLINATIONS of ſixty STARS in the Northern Hemiſphere; for the Year 1780.

| Conſtellations. | Places of the Stars in the Conſtellations. | Names. | Marks | Magn. | Rt. Aſcen. in time. h. m. ſ. | Year. Var. ſ. | Declinat. North. ° ′ ″ | Yearly Var. ″ |
|---|---|---|---|---|---|---|---|---|
| Pegaſus | Ending of the Wing | | γ | 2 | 0 1 56 | 3,08 | 13 57 36 | 20,04+ |
| Caſſiopea | Breaſt | Schedar | α | 3 | 0 28 8 | 3,31 | 55 19 46 | 19,91+ |
| Pole Star | | Alruccabah | α | 2 | 0 47 43 | 10,05 | 88 7 57 | 19,69+ |
| Andromeda | Girdle | Mirach | β | 2 | 0 57 29 | 3,30 | 34 26 56 | 19,45+ |
| Aries | Preceding Horn | | β | 3 | 1 42 32 | 3,28 | 19 43 32 | 18,10+ |
| Andromeda | Foot | Almaach | γ | 2 | 1 50 29 | 3,62 | 41 15 54 | 17,80+ |
| Aries | Following Horn | | α | 3 | 1 54 49 | 3,34 | 22 24 52 | 17,64+ |
| Whale | Following in the Cheek | | γ | 3 | 2 31 56 | 3,12 | 2 17 59 | 15,86+ |
| | Jaw | Menkar | α | 2 | 2 50 48 | 3,13 | 3 12 56 | 14,80+ |
| Meduſa | Head | Algol | β | 2 | 2 53 56 | 3,85 | 40 5 37 | 14,63+ |
| Perſeus | Brighteſt | Algenib | α | 2 | 3 8 43 | 4,20 | 49 3 44 | 13,72+ |
| Taurus | Brighteſt in | Pleiades | η | 3 | 3 34 27 | 3,55 | 23 24 38 | 12,00+ |
| Perſeus | Knee | | ε | 3 | 3 43 08 | 3,94 | 38 21 24 | 11,41+ |
| Taurus | Firſt of the | Hyades | γ | 3 | 4 7 18 | 3,39 | 15 4 51 | 9,60+ |
| | Northern Eye | | ε | 3 | 4 15 48 | 3,48 | 18 40 32 | 8,90+ |
| | Southern Eye | Aldebaran | α | 1 | 4 23 19 | 3,43 | 16 3 6 | 8,32+ |
| Auriga | In the Goat | Capella | α | 1 | 5 0 28 | 4,40 | 45 44 48 | 5,30+ |
| Taurus | Northern Horn | | β | 2 | 5 12 24 | 3,79 | 28 24 13 | 4,26+ |
| Orion | Weſt Shoulder | Bellatrix | γ | 2 | 5 13 21 | 3,22 | 6 8 2 | 4,15+ |
| | Eaſt Shoulder | Betelgueſe | α | 1 | 5 43 16 | 3,25 | 7 20 56 | 1,58+ |
| Auriga | In the Hand | | θ | 3 | 5 44 43 | 4,09 | 37 10 36 | 1,40+ |
| Gemini | Foot of Pollux | | γ | 3 | 6 25 00 | 3,47 | 16 34 9 | 2,10— |
| | Knee of Pollux | | ζ | 3 | 6 51 3 | 3,58 | 20 52 31 | 4,33— |
| | Brighteſt in the Head | Caſtor | α | 2 | 7 20 33 | 3,88 | 32 21 8 | 6,78— |
| Little Dog | Brighteſt | Procyon | α | 1 | 7 27 48 | 3,20 | 5 46 56 | 7,40— |
| Gemini | Head of Pollux | Pollux | β | 2 | 7 31 51 | 3,75 | 28 32 29 | 7,70— |
| Great Bear | North Paw | | ι | 3 | 8 44 4 | 4,25 | 48 53 29 | 13,00— |
| Cancer | In the Claw | Acubens | α | 3 | 8 46 27 | 3,24 | 12 41 59 | 13,30— |
| Great Bear | Preceding Knee | | θ | 3 | 9 18 5 | 4,23 | 52 40 21 | 15,18— |
| Leo | North in the Head | | μ | 3 | 9 40 13 | 3,47 | 27 1 58 | 16,33— |
| | In the Heart | Regulus | α | 1 | 9 56 39 | 3,24 | 13 2 6 | 17,16— |
| Great Bear | Lower Pointer | | β | 2 | 10 48 27 | 3,74 | 57 33 25 | 19,05— |
| | Upper Pointer | Dubhe | α | 2 | 10 50 0 | 3,88 | 62 56 05 | 19,09— |
| Leo | In the Tail | | β | 2 | 11 37 50 | 3,10 | 15 48 8 | 19,95— |
| Great Bear | S following in the Square | | γ | 2 | 11 42 10 | 3,24 | 54 55 4 | 19,99— |
| | Laſt in the Square | | δ | 2 | 12 4 27 | 3,05 | 58 15 25 | 20,05— |
| | Firſt in the Tail | Alioth | ε | 2 | 12 44 17 | 2,69 | 57 9 25 | 19,69— |
| | Middle of the Tail | | ζ | 2 | 13 15 2 | 2,44 | 56 4 46 | 19,01— |
| | Laſt in the Tail | Benetnach | η | 2 | 13 38 52 | 2,41 | 50 25 03 | 18,24— |
| Dragon | In the Tail | | α | 2 | 13 58 27 | 1,63 | 65 25 54 | 17,46— |
| Bootes | Skirt of the Coat | Arcturus | α | 1 | 14 5 40 | 2,82 | 20 20 39 | 17,16— |
| | In the following Thigh | Mirach | ε | 3 | 14 35 28 | 2,63 | 28 0 37 | 15,67— |
| Crown | The brighteſt | Alphacca | α | 2 | 15 25 23 | 2,54 | 27 28 00 | 12,60— |
| Serpent | In the Neck | | α | 2 | 15 33 27 | 2,94 | 7 7 48 | 12,03— |
| Hercules | Preceding Shoulder | | β | 3 | 16 20 48 | 2,59 | 21 58 52 | 8,51— |
| | Following at the Side | | ε | 3 | 16 51 52 | 2,29 | 31 15 43 | 5,96— |
| | The Head | Raſ. Algethi | α | 2 | 17 4 38 | 2,74 | 14 39 18 | 4,87— |
| Ophiucus | The Head | Raſ. Alhague | α | 2 | 17 24 43 | 2,75 | 12 44 8 | 3,15— |
| Dragon | In the Head | Raſtaben | γ | 2 | 17 51 31 | 1,37 | 51 31 21 | 0,78— |
| Harp | The brighteſt | Vega | α | 1 | 18 29 29 | 2,02 | 38 35 14 | 2,52+ |
| | Following in Lozange | | δ | 3 | 18 46 49 | 2,11 | 36 37 48 | 3,97+ |
| Eagle | Preceding Wing | | δ | 3 | 19 14 24 | 3,02 | 2 41 22 | 6,31+ |
| | The brighteſt | Atair | α | 2 | 19 40 02 | 2,90 | 8 17 53 | 8,40+ |
| Swan | The Breaſt | | γ | 3 | 20 14 20 | 2,16 | 39 33 43 | 11,00+ |
| | The Tail | Deneb | α | 2 | 20 33 56 | 2,05 | 44 30 7 | 12,44+ |
| Cepheus | Preceding Shoulder | Alderaimin | α | 3 | 21 13 19 | 1,44 | 61 39 32 | 14,95+ |
| Pegaſus | The Neck | | ζ | 3 | 22 30 29 | 2,99 | 9 41 18 | 18,46+ |
| | The Thigh | Scheat | β | 2 | 22 53 8 | 2,88 | 26 53 32 | 19,18+ |
| | The Wing | Markab | α | 2 | 22 53 49 | 2,98 | 14 1 30 | 19,20+ |
| Andromeda | The Head | | α | 2 | 23 57 3 | 3,07 | 27 52 22 | 20,05+ |

## 312. TABLE of the RIGHT ASCENSIONS and DECLINATIONS of ſixty STARS in the Southern Hemiſphere; for the Year 1780.

| Conſtellations | Places of the Stars in the Conſtellations. | Names. | Marks. | Magn. | Rt. Aſcen. in time. | | | Year. Var. | Declinat. South. | | | Yearly Var. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | h. | m. | ſ. | ſ. | ° | ′ | ″ | ″ |
| Phenix | The Head | | α | 2 | 0 | 15 | 22 | 3,01 | 43 | 29 | 43 | 20,00— |
| Whale | Brighteſt in the Tail | | β | 2 | 0 | 32 | 32 | 3,01 | 19 | 11 | 50 | 19,86— |
| Phenix | Thigh | | β | 3 | 0 | 56 | 15 | 2,73 | 47 | 54 | 58 | 19,46— |
| | Following Wing | | γ | 3 | 1 | 18 | 49 | 2,67 | 44 | 26 | 52 | 18,90— |
| Eridanus | Source of the River | Achernar | α | 1 | 1 | 29 | 31 | 2,25 | 58 | 21 | 35 | 18,56— |
| Whale | Preceding Jaw | | δ | 3 | 2 | 28 | 13 | 3,07 | 0 | 37 | 50 | 16,00— |
| Eridanus | Near the Whale | | ζ | 3 | 3 | 5 | 10 | 2,91 | 9 | 38 | 55 | 13,92— |
| | The following | | δ | 3 | 3 | 32 | 44 | 2,88 | 10 | 31 | 30 | 12,08— |
| | The fourth Bend | | γ | 3 | 3 | 47 | 46 | 2,80 | 14 | 8 | 47 | 11,01— |
| Goldfiſh | In the Tail | | α | 3 | 4 | 29 | 15 | 1,28 | 55 | 30 | 20 | 7,76— |
| Orion | Bright Foot | Rigel | β | 1 | 5 | 3 | 58 | 2,89 | 8 | 28 | 11 | 4,94— |
| | Preceding in Belt | | δ | 2 | 5 | 20 | 47 | 3,07 | 0 | 28 | 40 | 3,50— |
| | Middle of Belt | | ε | 2 | 5 | 25 | 4 | 3,05 | 1 | 21 | 30 | 3,13— |
| | Laſt in the Belt | | ζ | 2 | 5 | 29 | 41 | 3,04 | 2 | 4 | 27 | 2,77— |
| Dove | Preceding of the brighteſt | | α | 2 | 5 | 31 | 43 | 2,20 | 34 | 12 | 6 | 2,56— |
| Orion | In the Knee | | κ | 3 | 5 | 37 | 20 | 2,85 | 9 | 45 | 35 | 2,10— |
| Dove | Following of brighteſt | | β | 3 | 5 | 43 | 13 | 2,11 | 35 | 51 | 51 | 1,65— |
| Argo | The brighteſt | Canopus | α | 1 | 6 | 19 | 5 | 1,34 | 52 | 34 | 57 | 1,60+ |
| Great Dog | The brighteſt | Syrius | α | 1 | 6 | 35 | 28 | 2,69 | 16 | 25 | 8 | 3,10+ |
| | In the Back | | δ | 2 | 6 | 59 | 27 | 2,45 | 26 | 3 | 37 | 5,08+ |
| | In the Tail | | η | 2 | 7 | 15 | 24 | 2,38 | 28 | 53 | 5 | 6,42+ |
| Argo | In the Poop | | ζ | 2 | 7 | 55 | 52 | 2,12 | 39 | 23 | 28 | 9,62+ |
| | Preceding in the Hull | | γ | 2 | 8 | 2 | 46 | 1,85 | 46 | 41 | 40 | 10,16+ |
| | Brighteſt in the Middle | | δ | 2 | 8 | 38 | 36 | 1,61 | 53 | 54 | 24 | 12,73+ |
| | Bright among the Oars | | β | 1 | 9 | 10 | 44 | 0,75 | 68 | 48 | 50 | 14,79+ |
| Female Hydra | The Heart | Alphard | α | 2 | 9 | 16 | 47 | 2,96 | 7 | 42 | 51 | 15,13+ |
| Argo | Northern in Section | | η | 2 | 10 | 36 | 34 | 2,27 | 58 | 31 | 59 | 18,68+ |
| Centaur | Preceding in the Crupper | | δ | 3 | 11 | 57 | 3 | 3,06 | 49 | 29 | 40 | 20,04+ |
| Croſs | Preceding Arm | | δ | 3 | 12 | 3 | 35 | 3,10 | 57 | 31 | 31 | 20,04+ |
| | The Foot | | α | 1 | 12 | 14 | 33 | 3,22 | 61 | 52 | 48 | 20,01+ |
| | The Head | | γ | 2 | 12 | 19 | 4 | 3,24 | 55 | 52 | 42 | 19,98+ |
| Centaur | Top of the Crupper | | γ | 2 | 12 | 29 | 30 | 3,27 | 47 | 44 | 50 | 19,89+ |
| Croſs | Following Arm | | β | 2 | 12 | 35 | 2 | 3,42 | 58 | 29 | 3 | 19,83+ |
| Female Hydra | The Tail | | γ | 3 | 13 | 7 | 00 | 3,22 | 22 | 0 | 22 | 19,22+ |
| Virgo | The Sheaf | Virgins Spike | α | 1 | 13 | 13 | 38 | 3,15 | 10 | 0 | 24 | 19,00+ |
| Centaur | Preceding Leg | | β | 2 | 13 | 48 | 30 | 4,09 | 59 | 17 | 59 | 17,91+ |
| | South in the Shield | | η | 3 | 14 | 21 | 37 | 3,75 | 41 | 10 | 42 | 16,43+ |
| | Bright in the Foot | | α | 1 | 14 | 25 | 2 | 4,41 | 59 | 55 | 17 | 16,26+ |
| Libra | Southern Scale | Zubeneſch | α | 2 | 14 | 38 | 45 | 3,31 | 15 | 6 | 55 | 15,50+ |
| Centaur | Following in the Head | | κ | 3 | 14 | 44 | 56 | 3,84 | 41 | 12 | 21 | 15,17+ |
| Libra | Northern Scale | Zubenelg. | β | 2 | 15 | 5 | 12 | 3,22 | 8 | 33 | 31 | 13,93+ |
| Southern Δ | The Vertex | | β | 3 | 15 | 35 | 56 | 5,12 | 62 | 43 | 31 | 11,88+ |
| Scorpio | Middle of the Forehead | | δ | 3 | 15 | 47 | 21 | 3,53 | 21 | 58 | 42 | 11,00+ |
| | N. in the Forehead | | β | 2 | 15 | 52 | 41 | 3,47 | 19 | 11 | 14 | 10,70+ |
| Ophiucus | Preceding Hand | | δ | 3 | 16 | 2 | 50 | 3,14 | 3 | 6 | 45 | 9,89+ |
| Scorpio | The Heart | Antares. | α | 1 | 16 | 15 | 57 | 3,66 | 25 | 55 | 32 | 8,90+ |
| | Firſt Joint in the Tail | | ε | 3 | 16 | 35 | 58 | 3,90 | 33 | 52 | 21 | 7,34+ |
| Ophiucus | Following Knee | | η | 2 | 16 | 57 | 47 | 3,44 | 15 | 26 | 14 | 5,52+ |
| Altar | In the Middle | | α | 3 | 17 | 14 | 52 | 4,61 | 49 | 40 | 36 | 4,12+ |
| Scorpio | Bright at Tail's End | | λ | 3 | 17 | 18 | 42 | 4,08 | 36 | 55 | 22 | 3,77+ |
| | Sixth Knot in the Tail | | ι | 3 | 17 | 32 | 13 | 4,18 | 40 | 1 | 7 | 2,60+ |
| Sagittarius | S. End of the Bow | | ζ | 2 | 18 | 9 | 35 | 4,00 | 34 | 27 | 58 | 0,72— |
| | Preceding Shoulder | | σ | 3 | 18 | 41 | 37 | 3,73 | 26 | 32 | 57 | 3,54— |
| | Following in the Head | | π | 3 | 18 | 56 | 41 | 3,58 | 21 | 21 | 17 | 4,80— |
| Capricorn | In the following Horn | | α | 3 | 20 | 5 | 54 | 3,35 | 13 | 12 | 45 | 10,40— |
| Peacock | The Eye | | α | 2 | 20 | 8 | 8 | 4,89 | 57 | 25 | 14 | 10,44— |
| Capricorn | In the Forehead | | β | 3 | 20 | 8 | 37 | 3,35 | 15 | 27 | 43 | 10,60— |
| Crane | Preceding Wing | | α | 2 | 21 | 54 | 18 | 3,92 | 48 | 0 | 51 | 17,00— |
| Aquarius | Following Arm | | γ | 3 | 22 | 10 | 17 | 3,11 | 2 | 29 | 23 | 17,76— |
| Southern Fiſh | The Brighteſt | Fomalhaut | α | 1 | 22 | 45 | 27 | 3,33 | 30 | 46 | 55 | 18,97— |

## 313. Table of the Sun's Right Aſcenſion, in Degrees, &c. to each Deg. of Long. And of the Diff. between Long. and Rt. Aſcen.

| | 0 S. or 6 S. | | 1 S. or 7 S. | | 2 S. or 8 S. | | |
|---|---|---|---|---|---|---|---|
| | Add to { 0 180 | Sub. from ☉'s Long. | Add to { 0 180 | Sub. from ☉'s Long. | Add to { 0 180 | Sub. from ☉'s Long. | |
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | |
| 0 | 0 0 0 | 0 0 0 | 27 54 20 | 2 5 40 | 57 48 48 | 2 11 12 | 30 |
| 1 | 0 55 2 | 0 4 58 | 28 51 43 | 2 8 17 | 58 51 21 | 2 8 39 | 29 |
| 2 | 1 50 5 | 0 9 55 | 29 49 15 | 2 10 45 | 59 54 4 | 2 5 56 | 28 |
| 3 | 2 45 8 | 0 14 52 | 30 46 56 | 2 13 4 | 60 56 57 | 2 3 3 | 27 |
| 4 | 3 40 12 | 0 19 48 | 31 44 46 | 2 15 14 | 62 0 0 | 2 0 0 | 26 |
| 5 | 4 35 18 | 0 24 42 | 32 42 45 | 2 17 15 | 63 3 12 | 1 56 48 | 25 |
| 6 | 5 30 25 | 0 29 35 | 33 40 54 | 2 19 6 | 64 6 34 | 1 53 26 | 24 |
| 7 | 6 25 34 | 0 34 26 | 34 39 13 | 2 20 47 | 65 10 4 | 1 49 56 | 23 |
| 8 | 7 20 45 | 0 39 15 | 35 37 41 | 2 22 19 | 66 13 43 | 1 46 17 | 22 |
| 9 | 8 15 59 | 0 44 1 | 36 36 19 | 2 23 41 | 67 17 31 | 1 42 29 | 21 |
| 10 | 9 11 16 | 0 48 44 | 37 35 8 | 2 24 52 | 68 21 27 | 1 38 33 | 20 |
| 11 | 10 6 35 | 0 53 25 | 38 34 7 | 2 25 53 | 69 25 31 | 1 34 29 | 19 |
| 12 | 11 1 58 | 0 58 2 | 39 33 16 | 2 26 44 | 70 29 42 | 1 30 18 | 18 |
| 13 | 11 57 25 | 1 2 35 | 40 32 36 | 2 27 24 | 71 34 1 | 1 25 59 | 17 |
| 14 | 12 52 56 | 1 7 4 | 41 32 7 | 2 27 53 | 72 38 27 | 1 21 33 | 16 |
| 15 | 13 48 32 | 1 11 28 | 42 31 48 | 2 28 12 | 73 42 59 | 1 17 1 | 15 |
| 16 | 14 44 12 | 1 15 48 | 43 31 40 | 2 28 20 | 74 47 38 | 1 12 22 | 14 |
| 17 | 15 39 57 | 1 20 3 | 44 31 43 | 2 28 17 | 75 52 23 | 1 7 37 | 13 |
| 18 | 16 35 47 | 1 24 13 | 45 31 56 | 2 28 4 | 76 57 13 | 1 2 47 | 12 |
| 19 | 17 31 43 | 1 28 17 | 46 32 21 | 2 27 39 | 78 2 9 | 0 57 51 | 11 |
| 20 | 18 27 45 | 1 32 15 | 47 32 57 | 2 27 3 | 79 7 9 | 0 52 51 | 10 |
| 21 | 19 23 53 | 1 36 7 | 48 33 43 | 2 26 17 | 80 12 13 | 0 47 47 | 9 |
| 22 | 20 20 7 | 1 39 53 | 49 34 41 | 2 25 19 | 81 17 21 | 0 42 39 | 8 |
| 23 | 21 16 28 | 1 43 32 | 50 35 49 | 2 24 11 | 82 22 33 | 0 37 27 | 7 |
| 24 | 22 12 55 | 1 47 5 | 51 37 8 | 2 22 52 | 83 27 49 | 0 32 11 | 6 |
| 25 | 23 9 30 | 1 50 30 | 52 38 38 | 2 21 22 | 84 33 7 | 0 26 53 | 5 |
| 26 | 24 6 12 | 1 53 48 | 53 40 19 | 2 19 41 | 85 38 27 | 0 21 33 | 4 |
| 27 | 25 3 2 | 1 56 58 | 54 42 11 | 2 17 49 | 86 43 48 | 0 16 12 | 3 |
| 28 | 26 0 0 | 2 0 0 | 55 44 13 | 2 15 47 | 87 49 11 | 0 10 49 | 2 |
| 29 | 26 57 6 | 2 2 54 | 56 46 25 | 2 13 35 | 88 54 35 | 0 5 25 | 1 |
| 30 | 27 54 20 | 2 5 40 | 57 48 48 | 2 11 12 | 90 0 0 | 0 0 0 | 0 |
| | Sub. from { 180 360 | Add to ☉'s Long. | Sub. from { 180 360 | Add to ☉'s Long. | Sub. from { 180 360 | Add to ☉'s Long. | |
| | 5 S. or 11 S. | | 4 S. or 10 S. | | 3 S or 9 S | | |

## 314. Table I. of the Equat. of Time on the Diff. betw. the Sun's Lon. and Rt. Aſc.

| Deg. | ♈ ♎ 0 6 m. ſ. | ♉ ♏ 1 7 m. ſ. | ♊ ♐ 2 8 m. ſ. | Deg. |
|---|---|---|---|---|
| 0 | 0 0,0 | 8 22,7 | 8 44,8 | 30 |
| 1 | 0 19,9 | 8 33,1 | 8 34,6 | 29 |
| 2 | 0 39,7 | 8 43,0 | 8 23,7 | 28 |
| 3 | 0 59,5 | 8 52,3 | 8 12,2 | 27 |
| 4 | 1 19,2 | 9 0,9 | 8 0,0 | 26 |
| 5 | 1 38,8 | 9 9,0 | 7 47,2 | 25 |
| 6 | 1 58,3 | 9 16,4 | 7 33,7 | 24 |
| 7 | 2 17,7 | 9 23,1 | 7 19,7 | 23 |
| 8 | 2 37,0 | 9 29,3 | 7 5,1 | 22 |
| 9 | 2 56,1 | 9 34,7 | 6 49,9 | 21 |
| 10 | 3 14,6 | 9 39,5 | 6 34,2 | 20 |
| 11 | 3 33,7 | 9 43,5 | 6 17,7 | 19 |
| 12 | 3 52,1 | 9 46,9 | 6 1,2 | 18 |
| 13 | 4 10,3 | 9 49,6 | 5 43,9 | 17 |
| 14 | 4 28,3 | 9 51,5 | 5 26,2 | 16 |
| 15 | 4 45,9 | 9 52,8 | 5 8,1 | 15 |
| 16 | 5 3,2 | 9 53,3 | 4 49,5 | 14 |
| 17 | 5 20,2 | 9 53,1 | 4 30,5 | 13 |
| 18 | 5 36,9 | 9 52,3 | 4 11,1 | 12 |
| 19 | 5 53,1 | 9 50,6 | 3 51,4 | 11 |
| 20 | 6 9,0 | 9 48,2 | 3 31,4 | 10 |
| 21 | 6 24,5 | 9 45,1 | 3 11,1 | 9 |
| 22 | 6 39,5 | 9 41,3 | 2 50,6 | 8 |
| 23 | 6 54,1 | 9 36,7 | 2 29,9 | 7 |
| 24 | 7 8,3 | 9 31,5 | 2 8,7 | 6 |
| 25 | 7 22,0 | 9 25,5 | 1 47,5 | 5 |
| 26 | 7 35,2 | 9 18,7 | 1 26,2 | 4 |
| 27 | 7 47,7 | 9 11,3 | 1 4,8 | 3 |
| 28 | 8 0,0 | 9 3,1 | 0 43,3 | 2 |
| 29 | 8 11,6 | 8 54,3 | 0 21,7 | 1 |
| 30 | 8 22,7 | 8 44,8 | 0 0,0 | 0 |
| Deg. | ♓ ♍ 11 5 | ♒ ♌ 10 4 | ♑ ♋ 9 3 | Deg. |

## 315. Table II. of the Equation of Time on the Sun's Anomaly; or the Equation of the Center turned into Time.

| ☉ Mea. An. | Sign 0 m. ſ. | Sign 1 m. ſ. | Sign 2 m. ſ. | Sign 3 m. ſ. | Sign 4 m. ſ. | Sign 5 m. ſ. | ☉ Mea. An. |
|---|---|---|---|---|---|---|---|
| 0 | 0 0 | 3 47 | 6 36 | 7 42 | 6 45 | 3 56 | 30 |
| 1 | 0 8 | 3 54 | 6 40 | 7 43 | 6 41 | 3 48 | 29 |
| 2 | 0 16 | 4 1 | 6 44 | 7 43 | 6 37 | 3 41 | 28 |
| 3 | 0 24 | 4 8 | 6 48 | 7 42 | 6 32 | 3 34 | 27 |
| 4 | 0 32 | 4 14 | 6 52 | 7 42 | 6 28 | 3 27 | 26 |
| 5 | 0 39 | 4 21 | 6 55 | 7 42 | 6 23 | 3 19 | 25 |
| 6 | 0 47 | 4 27 | 6 59 | 7 41 | 6 19 | 3 12 | 24 |
| 7 | 0 55 | 4 34 | 7 2 | 7 40 | 6 14 | 3 4 | 23 |
| 8 | 1 3 | 4 40 | 7 5 | 7 39 | 6 9 | 2 57 | 22 |
| 9 | 1 11 | 4 46 | 7 9 | 7 38 | 6 4 | 2 49 | 21 |
| 10 | 1 23 | 4 53 | 7 12 | 7 37 | 5 59 | 2 41 | 20 |
| 11 | 1 26 | 4 59 | 7 14 | 7 36 | 5 54 | 2 32 | 19 |
| 12 | 1 34 | 5 5 | 7 17 | 7 34 | 5 49 | 2 26 | 18 |
| 13 | 1 43 | 5 11 | 7 20 | 7 33 | 5 43 | 2 18 | 17 |
| 14 | 1 50 | 5 17 | 7 22 | 7 31 | 5-38 | 2 10 | 16 |
| 15 | 1 57 | 5 22 | 7 24 | 7 29 | 5 32 | 2 2 | 15 |
| 16 | 2 5 | 5 28 | 7 26 | 7 27 | 5 26 | 1 54 | 14 |
| 17 | 2 13 | 5 33 | 7 28 | 7 25 | 5 20 | 1 46 | 13 |
| 18 | 2 20 | 5 39 | 7 30 | 7 23 | 5 14 | 1 38 | 12 |
| 19 | 2 28 | 5 44 | 7 32 | 7 20 | 5 8 | 1 30 | 11 |
| 20 | 2 35 | 5 50 | 7 34 | 7 18 | 5 2 | 1 22 | 10 |
| 21 | 2 43 | 5 55 | 7 35 | 7 15 | 4 56 | 1 14 | 9 |
| 22 | 2 50 | 6 0 | 7 37 | 7 12 | 4 50 | 1 6 | 8 |
| 23 | 2 57 | 6 5 | 7 38 | 7 9 | 4 43 | 0 58 | 7 |
| 24 | 3 5 | 6 10 | 7 39 | 7 6 | 4 37 | 0 49 | 6 |
| 25 | 3 12 | 6 14 | 7 40 | 7 3 | 4 31 | 0 41 | 5 |
| 26 | 3 19 | 6 19 | 7 41 | 7 0 | 4 23 | 0 33 | 4 |
| 27 | 3 26 | 6 24 | 7 41 | 6 56 | 4 16 | 0 25 | 3 |
| 28 | 3 33 | 6 28 | 7 42 | 6 52 | 4 10 | 0 16 | 2 |
| 29 | 3 40 | 6 32 | 7 42 | 6 49 | 4 3 | 0 8 | 1 |
| 30 | 3 47 | 6 36 | 7 42 | 6 45 | 3 56 | 0 0 | 0 |
| | 11 Sign | 10 Sign | 9 Sign | 8 Sign | 7 Sign | 6 Sign | |

## 316. TABLE of the Abſolute Equation of Time, fitted to each Sign and Degree of the Ecliptic.

Place of the Apogee ♋ 9°. Obliquity of Ecliptic 23° 28′.

| Deg. | ♈ 0 | ♉ 1 | ♊ 2 | ♋ | ♌ 4 | ♍ 5 | ♎ 6 | ♏ 7 | ♐ 8 | ♑ 9 | ♒ 10 | ♓ 11 | Deg. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | + — | — | — + | + | + | + — | — | — | — | — + | + | + | |
| | m ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ. | m. ſ | |
| 0 | 7 36 | 1 9 | 3 51 | 1 13 | 5 57 | 2 20 | 7 38 | 15 31 | 13 33 | 1 11 | 11 28 | 14 19 | 0 |
| 1 | 7 17 | 1 23 | 3 47 | 1 26 | 5 59 | 2 4 | 7 58 | 15 39 | 13 17 | 0 42 | 11 45 | 14 13 | 1 |
| 2 | 6 58 | 1 36 | 3 41 | 1 40 | 6 0 | 1 48 | 8 19 | 15 46 | 13 0 | — 12 | 12 1 | 14 6 | 2 |
| 3 | 6 39 | 1 48 | 3 37 | 1 53 | 6 1 | 1 31 | 8 40 | 15 52 | 12 42 | + 17 | 12 17 | 13 59 | 3 |
| 4 | 6 20 | 2 0 | 3 32 | 2 7 | 6 1 | 1 14 | 9 1 | 15 57 | 12 23 | 0 46 | 12 32 | 13 51 | 4 |
| 5 | 6 1 | 2 11 | 3 26 | 2 20 | 6 0 | 0 56 | 9 21 | 16 2 | 12 4 | 1 16 | 12 46 | 13 43 | 5 |
| 6 | 5 42 | 2 22 | 3 19 | 2 33 | 5 59 | 0 38 | 9 41 | 16 6 | 11 44 | 1 45 | 12 59 | 13 34 | 6 |
| 7 | 5 24 | 2 32 | 3 12 | 2 45 | 5 57 | 0 20 | 10 1 | 16 9 | 11 23 | 2 14 | 13 12 | 13 24 | 7 |
| 8 | 5 5 | 2 42 | 3 4 | 2 58 | 5 54 | + 1 | 10 20 | 16 11 | 11 1 | 2 43 | 13 24 | 13 14 | 8 |
| 9 | 4 47 | 2 51 | 2 56 | 3 11 | 5 51 | — 18 | 10 39 | 16 13 | 10 39 | 3 11 | 13 35 | 13 3 | 9 |
| 10 | 4 28 | 3 0 | 2 47 | 3 23 | 5 47 | 0 37 | 10 57 | 16 13 | 10 16 | 3 39 | 13 45 | 12 51 | 10 |
| 11 | 4 9 | 3 8 | 2 38 | 3 35 | 5 42 | 0 57 | 11 15 | 16 13 | 9 53 | 4 7 | 13 54 | 12 39 | 11 |
| 12 | 3 50 | 3 16 | 2 29 | 3 46 | 5 37 | 1 17 | 11 33 | 16 12 | 9 29 | 4 35 | 14 2 | 12 27 | 12 |
| 13 | 3 32 | 3 23 | 2 19 | 3 58 | 5 31 | 1 38 | 11 51 | 16 10 | 9 5 | 5 2 | 14 9 | 12 14 | 13 |
| 14 | 3 13 | 3 30 | 2 8 | 4 9 | 5 24 | 1 58 | 12 8 | 16 7 | 8 40 | 5 29 | 14 16 | 12 0 | 14 |
| 15 | 2 55 | 3 36 | 1 57 | 4 19 | 5 17 | 2 19 | 12 25 | 16 4 | 8 14 | 5 56 | 14 22 | 11 46 | 15 |
| 16 | 2 37 | 3 41 | 1 46 | 4 29 | 5 9 | 2 40 | 12 41 | 16 0 | 7 48 | 6 22 | 14 27 | 11 31 | 16 |
| 17 | 2 19 | 3 46 | 1 35 | 4 39 | 5 1 | 3 1 | 12 57 | 15 55 | 7 22 | 6 48 | 14 31 | 11 16 | 17 |
| 18 | 2 1 | 3 50 | 1 23 | 4 48 | 4 52 | 3 22 | 13 12 | 15 49 | 6 55 | 7 13 | 14 35 | 11 1 | 18 |
| 19 | 1 43 | 3 53 | 1 11 | 4 57 | 4 43 | 3 44 | 13 27 | 15 42 | 6 28 | 7 37 | 14 38 | 10 46 | 19 |
| 20 | 1 26 | 3 56 | 0 59 | 5 5 | 4 33 | 4 5 | 13 42 | 15 35 | 6 0 | 8 1 | 14 40 | 10 30 | 20 |
| 21 | 1 9 | 3 58 | 0 46 | 5 13 | 4 22 | 4 26 | 13 56 | 15 26 | 5 32 | 8 24 | 14 41 | 10 14 | 21 |
| 22 | 0 52 | 4 0 | 0 34 | 5 20 | 4 11 | 4 47 | 14 9 | 15 17 | 5 4 | 8 47 | 14 42 | 9 58 | 22 |
| 23 | 0 36 | 4 1 | 0 21 | 5 27 | 3 59 | 5 9 | 14 21 | 15 7 | 4 36 | 9 9 | 14 41 | 9 41 | 23 |
| 24 | 0 20 | 4 1 | — 8 | 5 33 | 3 46 | 5 30 | 14 33 | 14 56 | 4 8 | 9 31 | 14 40 | 9 24 | 24 |
| 25 | + 4 | 4 1 | + 5 | 5 39 | 3 33 | 5 52 | 14 44 | 14 44 | 3 39 | 9 53 | 14 39 | 9 6 | 25 |
| 26 | — 11 | 4 0 | 0 19 | 5 44 | 3 19 | 6 13 | 14 53 | 14 31 | 3 10 | 10 14 | 14 37 | 8 48 | 26 |
| 27 | 0 26 | 3 59 | 0 31 | 5 48 | 3 4 | 6 35 | 15 5 | 14 17 | 2 41 | 10 34 | 14 34 | 8 30 | 27 |
| 28 | 0 40 | 3 57 | 0 46 | 5 52 | 2 50 | 6 56 | 15 14 | 14 3 | 2 11 | 10 53 | 14 30 | 8 12 | 28 |
| 29 | 0 53 | 3 54 | 0 59 | 5 55 | 2 35 | 7 17 | 15 23 | 13 48 | 1 41 | 11 11 | 14 25 | 7 54 | 29 |
| 30 | 1 9 | 3 51 | 1 13 | 5 57 | 2 20 | 7 3 | 15 31 | 13 33 | 1 11 | 11 28 | 14 19 | 7 36 | 30 |

The equations with +, are to be added to the apparent time, to have the mean time; thoſe with —, are to be ſubtracted from apparent for mean time.

The preceding mark, whether + or —, at the head of any column, belongs to all equations in that column until the ſign changes; and thoſe columns having two ſigns at the head, ſhew that the preceding ſign changes to the following ſomewhere in that column.

## 317. TABLE of Corrections for the Middle Time between the Equal Altitudes of the Sun.

| Deg. of Declin. | Latitude 30 D.N. | | | | | | Latitude 40 D.N. | | | | | | Latitude 50 D.N. | | | | | | Latitude 60 D.N. | | | | | | Deg. of Declin. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Hours bet. Obſer. | | | | | | Hours bet. Obſer. | | | | | | Hours bet. Obſer. | | | | | | Hours bet. Obſer. | | | | | | |
| | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | 6 | 5 | 4 | |
| | N. decl. | | | S. decl. | | | N. decl. | | | S. decl. | | | N. decl. | | | S. decl. | | | N. decl. | | | S. decl. | | | |
| | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | ſec. | |
| 0 | 9 | 9 | 8 | 9 | 9 | 8 | 14 | 13 | 13 | 14 | 13 | 13 | 20 | 19 | 18 | 20 | 19 | 18 | 28 | 28 | 27 | 28 | 28 | 27 | 0 |
| 1 | 9 | 9 | 8 | 9 | 9 | 9 | 14 | 13 | 12 | 14 | 13 | 13 | 19 | 19 | 18 | 20 | 19 | 18 | 28 | 27 | 27 | 29 | 28 | 27 | 1 |
| 2 | 9 | 8 | 8 | 9 | 9 | 9 | 13 | 13 | 12 | 14 | 14 | 13 | 19 | 18 | 18 | 20 | 19 | 18 | 28 | 27 | 26 | 29 | 28 | 27 | 2 |
| 3 | 8 | 8 | 7 | 10 | 9 | 9 | 13 | 12 | 12 | 14 | 14 | 13 | 18 | 18 | 17 | 20 | 19 | 18 | 28 | 27 | 26 | 29 | 28 | 27 | 3 |
| 4 | 8 | 8 | 7 | 10 | 10 | 9 | 13 | 12 | 12 | 14 | 14 | 13 | 18 | 17 | 17 | 20 | 19 | 19 | 27 | 26 | 25 | 29 | 29 | 28 | 4 |
| 5 | 8 | 7 | 7 | 10 | 10 | 10 | 12 | 12 | 11 | 15 | 14 | 14 | 18 | 17 | 17 | 20 | 19 | 19 | 27 | 26 | 25 | 29 | 29 | 28 | 5 |
| 6 | 8 | 7 | 6 | 10 | 10 | 10 | 12 | 11 | 11 | 15 | 14 | 14 | 17 | 17 | 16 | 20 | 20 | 19 | 27 | 25 | 24 | 29 | 29 | 28 | 6 |
| 7 | 7 | 7 | 6 | 10 | 10 | 10 | 12 | 11 | 11 | 15 | 14 | 14 | 17 | 16 | 16 | 20 | 20 | 19 | 26 | 25 | 24 | 29 | 28 | 27 | 7 |
| 8 | 7 | 7 | 6 | 10 | 10 | 10 | 11 | 11 | 10 | 15 | 14 | 14 | 17 | 16 | 16 | 20 | 20 | 19 | 26 | 24 | 23 | 29 | 28 | 27 | 8 |
| 9 | 7 | 6 | 6 | 10 | 10 | 10 | 11 | 10 | 10 | 15 | 15 | 14 | 16 | 15 | 15 | 20 | 20 | 19 | 25 | 23 | 23 | 28 | 28 | 27 | 9 |
| 10 | 7 | 6 | 5 | 10 | 10 | 10 | 11 | 10 | 10 | 15 | 15 | 14 | 16 | 15 | 15 | 20 | 19 | 19 | 24 | 23 | 22 | 28 | 28 | 27 | 10 |
| 11 | 6 | 6 | 5 | 10 | 10 | 10 | 10 | 10 | 9 | 14 | 14 | 14 | 15 | 15 | 14 | 19 | 19 | 18 | 23 | 22 | 22 | | 27 | 27 | 11 |
| 12 | 6 | 6 | 5 | 10 | 10 | 10 | 10 | 9 | 9 | 14 | 14 | 14 | 15 | 14 | 14 | 19 | 19 | 18 | 22 | 21 | 21 | | 27 | 26 | 12 |
| 13 | 6 | 5 | 5 | 10 | 10 | 9 | 9 | 9 | 8 | 14 | 14 | 14 | 14 | 14 | 13 | 19 | 19 | 18 | 21 | 21 | 20 | | 26 | 26 | 13 |
| 14 | 5 | 5 | 4 | 10 | 9 | 9 | 9 | 8 | 8 | 14 | 14 | 14 | 13 | 13 | 12 | 18 | 18 | 17 | 20 | 20 | 19 | | 25 | 25 | 14 |
| 15 | 5 | 5 | 4 | 9 | 9 | 9 | 8 | 8 | 7 | 14 | 14 | 13 | 13 | 12 | 12 | 18 | 18 | 17 | 19 | 19 | 18 | | | 24 | 15 |
| 16 | 5 | 4 | 4 | 9 | 9 | 9 | 8 | 7 | 7 | 13 | 13 | 13 | 12 | 12 | 11 | 17 | 17 | 17 | 18 | 18 | 17 | | | 23 | 16 |
| 17 | 4 | 4 | 3 | 9 | 9 | 8 | 7 | 7 | 6 | 12 | 12 | 12 | 11 | 11 | 10 | 16 | 16 | 16 | 17 | 17 | 16 | | | 22 | 17 |
| 18 | 4 | 3 | 3 | 9 | 8 | 8 | 6 | 6 | 5 | 12 | 12 | 12 | 10 | 10 | 9 | 15 | 15 | 15 | 16 | 15 | 15 | | | 21 | 18 |
| 19 | 3 | 3 | 2 | 8 | 8 | 8 | 6 | 5 | 5 | 11 | 11 | 11 | 9 | 9 | 8 | 14 | 14 | 14 | 15 | 14 | 13 | | | 19 | 19 |
| 20 | 3 | 2 | 2 | 8 | 7 | 7 | 5 | 5 | 4 | 10 | 10 | 10 | 8 | 8 | 7 | 13 | 13 | 13 | 14 | 13 | 12 | | | 17 | 20 |
| 21 | 2 | 2 | 1 | 7 | 7 | 7 | 4 | 3 | 3 | 9 | 9 | 9 | 7 | 6 | 6 | | 11 | 11 | 12 | 11 | 10 | | | 14 | 21 |
| 22 | 1 | 1 | 0 | 6 | 6 | 5 | 3 | 2 | 2 | 7 | 7 | 7 | 5 | 5 | 5 | | 9 | 9 | 8 | 8 | 8 | | | 11 | 22 |
| 23 | 0 | 0 | 0 | 3 | 3 | 3 | 2 | 1 | 0 | 4 | 4 | 4 | 3 | 2 | 2 | | 5 | 5 | 5 | 5 | 4 | | | 7 | 23 |

The correction is ſubtractive from December 22 to June 21, or in aſcending ſigns; and additive from June 21 to December 22, or in deſcending ſigns.

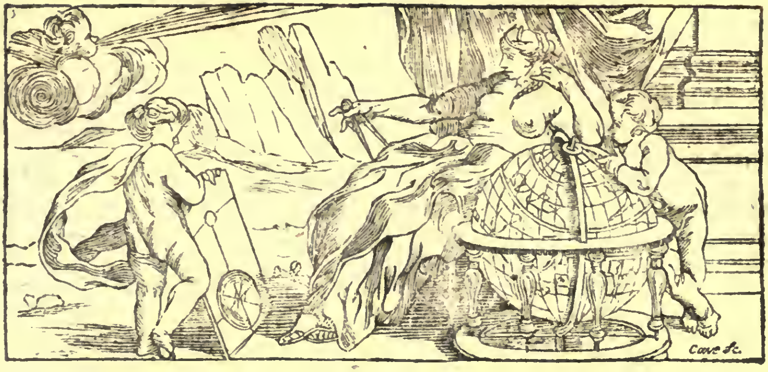
## 318. A TABLE of REFRACTIONS.

| Alt. Deg. | Refr. | | Alt. Deg. | Refr. | | Alt. Deg. | Refr. | | Alt. Deg. | Refr. | | Alt. Deg. | Refr. | | Alt. Deg. | Refr. | | Alt. Deg. | Refr. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | min. | ſec. | | min. | ſec. | | min. | ſec. | | min. | ſec. | | min. | ſec. | | min. | ſec. | | min. | ſec. |
| 0 | 33 | 00 | 4 | 11 | 51 | 14 | 3 | 45 | 26 | 1 | 56 | 38 | 1 | 13 | 50 | 0 | 48 | 62 | 0 | 30 |
| 0¼ | 30 | 35 | 4½ | 10 | 48 | 15 | 3 | 30 | 27 | 1 | 51 | 39 | 1 | 10 | 51 | 0 | 46 | 63 | 0 | 29 |
| 0½ | 28 | 22 | 5 | 9 | 54 | 16 | 3 | 17 | 28 | 1 | 47 | 40 | 1 | 8 | 52 | 0 | 44 | 64 | 0 | 28 |
| 0¾ | 26 | 21 | 5½ | 9 | 8 | 17 | 3 | 4 | 29 | 1 | 42 | 41 | 1 | 5 | 53 | 0 | 43 | 65 | 0 | 26 |
| 1 | 24 | 20 | 6 | 8 | 28 | 18 | 2 | 54 | 30 | 1 | 38 | 42 | 1 | 3 | 54 | 0 | 41 | 66 | 0 | 25 |
| 1¼ | 22 | 48 | 7 | 7 | 20 | 19 | 2 | 44 | 31 | 1 | 35 | 43 | 1 | 1 | 55 | 0 | 40 | 67 | 0 | 24 |
| 1½ | 21 | 15 | 8 | 6 | 29 | 20 | 2 | 35 | 32 | 1 | 31 | 44 | 0 | 59 | 56 | 0 | 38 | 68 | 0 | 23 |
| 1¾ | 19 | 51 | 9 | 5 | 48 | 21 | 2 | 27 | 33 | 1 | 28 | 45 | 0 | 57 | 57 | 0 | 37 | 69 | 0 | 22 |
| 2 | 18 | 35 | 10 | 5 | 15 | 22 | 2 | 20 | 34 | 1 | 24 | 46 | 0 | 55 | 58 | 0 | 35 | 70 | 0 | 21 |
| 2½ | 16 | 24 | 11 | 4 | 47 | 23 | 2 | 14 | 35 | 1 | 21 | 47 | 0 | 53 | 59 | 0 | 34 | 71 | 0 | 19 |
| 3 | 14 | 36 | 12 | 4 | 23 | 24 | 2 | 7 | 36 | 1 | 18 | 48 | 0 | 51 | 60 | 0 | 33 | 72 | 0 | 18 |
| 3½ | 13 | 6 | 13 | 4 | 3 | 25 | 2 | 2 | 37 | 1 | 16 | 49 | 0 | 49 | 61 | 0 | 32 | 73 | 0 | 17 |

## 319. *Of the Sun's Parallax.*

| Altitudes | 0°. | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90°. |
|---|---|---|---|---|---|---|---|---|---|---|
| Parallax | 9″ | 9″ | 8″ | 8″ | 6″ | 5″ | 3″ | 2″ | 1″ | 0″. |

END OF BOOK V.



# THE ELEMENTS OF NAVIGATION.

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## BOOK VI.

## OF GEOGRAPHY.

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## SECTION I.

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### *Definitions* and *Principles.*

1. GEOGRAPHY is the art of deſcribing the figure, magnitude, and poſitions of the ſeveral parts of the ſurface of the Earth.

2. The EARTH is a ſpherical or globular figure *, and is uſually called the terraqueous globe.

3. There are two points on the ſurface of the terraqueous globe, called the POLES OF THE EARTH, which are diametrically oppoſite to one another: one is called the *north pole*, and the other, the *ſouth pole*.

---

* For in ſhips at ſea the firſt parts of them that become viſible are the upper ſails; and as they approach nearer, the lower ſails appear; and ſo on until they ſhew their hulls.

Alſo ſhips in ſailing from high capes, or head lands, loſe ſight of thoſe eminences gradually from the lower parts, until the top vaniſhes.

Now as theſe appearances are the objects of our ſenſes in all parts of the Earth,

Therefore the ſurface of the Earth muſt be convex.

And this convexity is, at ſea, obſerved to be every where uniform.

But a body, the ſurface of which is every where uniformly convex, is a globe.

Therefore the figure of the Earth is globular.

In order to deſcribe the poſitions of places, Geographers have found it neceſſary to imagine certain circles drawn on the ſurface of the Earth, to which they have given the names of Equator, Meridian, Horizon, Parallels of latitude, *&c.*

4. The EQUATOR is a great circle on the Earth, equally diſtant from each pole, dividing the terraqueous globe into two equal parts; one called the *northern hemiſphere*, in which is the north pole; and the other, containing the ſouth pole, is called the *ſouthern hemiſphere.*

5. MERIDIANS are imaginary circles on the Earth paſſing through both the poles, and cutting the equator at right angles.

Every point on the ſurface of the Earth has its proper meridian.

6. LATITUDE is the diſtance of a place from the equator, reckoned in degrees and parts of degrees on a meridian.

On the north ſide of the equator it is north latitude; and on the ſouth ſide it is ſouth latitude.

As latitude begins at the equator, where it is nothing; ſo it ends at the poles, where the latitude is greateſt, or 90 degrees.

7. PARALLELS OF LATITUDE are circles parallel to the equator.

Every place on the Earth has its parallel of latitude.

DIFFERENCE OF LATITUDE is an arc of a meridian, or the leaſt diſtance of the parallels of latitude of two places; ſhewing how far one of them is to the northward, or ſouthward, of the other.

The difference of latitude can never exceed 180 degrees.

8. In north latitudes, if about the middle of the months of March and September a perſon looks towards the Sun at noon, the ſouth is before him, the north behind, the weſt on the right hand, and the eaſt on the left: and in ſouth latitudes, if the face is turned toward the Sun at the ſame times, the north is before, the ſouth behind, the eaſt to the right, and the weſt on the left.

In latitudes greater than 23½ degrees, theſe poſitions, found at noon, will hold good on any day of the year.

9. LONGITUDE of any place on the Earth is expreſſed by an arc of the equator, ſhewing the eaſt or weſt diſtance of the meridian of that place from ſome fixed meridian, where longitude is reckoned to begin.

10. DIFFERENCE OF LONGITUDE is an arc of the equator, intercepted between the meridians of two places, ſhewing how far one of them is to the eaſtward, or weſtward, of the other.

As longitude begins at the meridan of ſome place, and is counted from thence both eaſtward and weſtward, till they meet at the ſame meridian on the oppoſite point of the equator; therefore the difference of longitude can never exceed 180 degrees.

11. When two places have latitudes both north, or both ſouth; or have longitudes both eaſt, or both weſt, they are ſaid to be of the ſame, or of like name: but when one has north latitude, and the other ſouth; or if one has eaſt longitude, and the other weſt, then they are ſaid to have contrary, or different, or unlike names.

12. The HORIZON is that apparent circle which limits, or bounds, the view of a ſpectator on the ſea, or on an extended plain; the eye of the ſpectator being always ſuppoſed in the center of his horizon.

When the Planets or Stars come above the eaſtern part of the horizon, they are ſaid to riſe; and when they deſcend below the weſtern part, they are ſaid to ſet.

When a ſhip is under the equator, both the poles appear in the horizon; and in proportion as ſhe falls towards either, or increaſes her latitude, that pole is ſeen proportionally higher above the horizon, and the other diſappears as much: but when a ſhip is ſailing towards the equator, or decreaſes her latitude, ſhe depreſſes the elevated pole; that is, its diſtance from the horizon decreaſes.

### *Of the diviſion of the Earth into Zones.*

13. A ZONE is a broad ſpace on the Earth, included between two parallels of latitude.

There are five zones: namely, one *Torrid*, two *Frigid*, and two *Temperate*; theſe names ariſe from the degree of heat or cold, to which their ſituations are liable.

14. The TORRID ZONE is that portion of the Earth, over every part of which the Sun is perpendicular at one time of the year or other.

This zone is about 47 degrees in breadth, extending to about 23½ degrees on each ſide of the equator; the parallel of latitude terminating the limits in the northern hemiſphere, is called the *Tropic of Cancer*; and in the ſouthern hemiſphere, the limiting parallel is called the *Tropic of Capricorn.*

15. The FRIGID Zones are thoſe regions about the poles, where the Sun does not riſe for ſome days, nor ſet for ſome days, of the year.

Theſe zones extend round the poles to the diſtance of about 23½ degrees: that in the northern hemiſphere is called the north frigid zone, and is bounded by a parallel of latitude, called the *Arctic polar circle:* and the other, in the ſouthern hemiſphere, the ſouth frigid zone; the parallel of latitude bounding it, being called the *Antarctic polar circle.*

16. The TEMPERATE ZONES are the ſpaces between the Torrid and the Frigid zones.

### *Of the diviſion of the Earth by Climates.*

17. A CLIMATE, in a geographical ſenſe, is that ſpace of the Earth contained between two parallels of latitude, when the difference between the longeſt day in each parallel is half an hour.

Theſe climates are narrower the farther they are from the equator; therefore, ſuppoſing the equator to be the beginning of the firſt climate, the polar circle will be the end of the 24th climate; for afterwards the longeſt day does dot increaſe by half hours, but by days and months.

## SECTION II.

## *Of the natural diviſion of the Earth.*

18. By the natural diviſion of the Earth is meant the parts on its ſurface formed by nature; ſuch as *Continents*, *Oceans*, *Iſlands*, *Seas*, *Rivers*, *Mountains*, &c.

The ſurface of the Earth is naturally divided into Land and Water.

| | | |
|---|---|---|
| Land is divided into | 1. Continents. | 4. Iſthmuſes. |
| | 2. Iſlands. | 5. Promontaries. |
| | 3. Peninſulas. | 6. Mountains. |
| Water is divided into | 1. Oceans. | 4. Straits. |
| | 2. Seas. | 5. Lakes. |
| | 3. Gulfs. | 6. Rivers. |

19. A CONTINENT, or, as it is frequently called, the *main land*, is a very large track, comprehending ſeveral contiguous Countries, Kingdoms, and States.

20. An OCEAN is a vaſt collection of ſalt water, ſeparating the continents from one another.

21. An ISLAND is a part of dry land, ſurrounded with water.

22. A SEA is a branch of the Ocean, flowing between ſome parts of the Continent, or ſeparating an Iſland from the Continent.

23. A PENINSULA is a part of dry land encompaſſed by water, except a narrow neck which joins it to ſome other land.

24. An ISTHMUS is the neck joining the peninſula to the adjacent land, and forms the paſſage between them.

25. A MOUNTAIN is a part of the land more elevated than the adjacent country, and to be ſeen at a greater diſtance than the neighbouring lower lands.

26. A PROMONTORY is a mountain ſtretching itſelf into the ſea; the extremity of which is called a *Cape*, or *Head-land*.

27. A HILL is a ſmall kind of mountain: A *Cliff* is a ſteep ſhore, hill, or mountain: And *Rocks* are great ſtones, riſing like hills above the dry land, or above the bottom of the ſea.

28. A GULF, or BAY, is a part of the Ocean, or Sea, contained between two ſhores: and is every where environed with land, except at its entrance, where it communicates with other Bays, Seas, or Oceans.

29. A STRAIT is a narrow paſſage, by which there is a communication between a Gulf and its neighbouring ſea, or which joins one part of the ſea, or ocean, with another.

30. A LAKE is a collection of Waters contained in ſome hollow or cavity, in an inland place, of a large extent, and every where ſurrounded with land, having no viſible communication with the Ocean.

31. RIVERS are ſtreams of Water, flowing chiefly from the Mountains, and running in long narrow channels, or cavities, through the land, 'till they fall into the ſea, or into other rivers, which at laſt run into the ſea.

32. There are generally reckoned four Continents, namely, EUROPE, ASIA, AFRICA, and AMERICA.

To theſe may be added the *Terra arctica*, or nothern continent, and the *Terra antarctica*, or lands detached from *Aſia*, towards the ſouth.

The continent of *America* is uſually divided into two parts, called *North* and *South America*; they are joined together by the *Iſthmus* of *Darien*. Alſo the continents of *Aſia* and *Africa* are joined together by the *Iſthmus* of *Sues*.

The *Terra arctica*, *Europe*, and *Aſia*, lie all within the northern hemiſphere; and alſo part of *Africa* and *America*: The other parts of theſe two continents, together with the *Terra antarctica*, lie in the ſouthern hemiſphere.

33. There are five Oceans, namely, the NORTHERN, the ATLANTIC, the PACIFIC, the INDIAN, and the SOUTHERN.

The *Atlantic ocean* is uſually divided into two parts, one called the *north Atlantic ocean*, and the other the *ſouth Atlantic*, or *Ethiopic ocean*.

The *Northern ocean* ſtretches to the northward of Europe, Aſia, and America, towards the north pole.

The *Atlantic ocean* lies between the continents of Europe and Africa on the eaſt, and America on the weſt.

That part of the north Atlantic ocean, lying between Europe and America, is frequently called the *Weſtern ocean*.

The *Pacific ocean*, or, as it is ſometimes called, the *South Sea*, is bounded by the weſtern and north-weſt ſhores of America, and by the eaſtern and north-eaſt ſhores of Aſia.

The *Indian ocean* waſhes the ſhores of the eaſtern coaſts of Africa, and the ſouth of Aſia; and is bounded on the eaſt by the Indian iſlands, New Holland, and New Zeeland.

The *Southern ocean* extends to the ſouthward of Africa and America towards the ſouth pole.

The northern and ſouthern continents not being ſufficiently known to Geographers, all that need be ſaid of them is, that the Terra Arctica, or land to the northward of Hudſon's Bay and Greenland, is in general too cold for the reſidence of mankind; and that the lands formerly ſuppoſed to be parts of the ſouthern continent, are found to be very large iſlands: *viz*. New Zeland is much larger than Great Britain, and has a ſtrait dividing it into two iſlands. New Holland is an iſland as large as Europe. New Guinea is a very large iſland; and New Britain is a cluſter of large and ſmall iſlands, and are thought by ſome to be the iſlands hitherto called the Solomon's iſlands.

## SECTION III.

## *Of the Political diviſion of the Earth.*

34. By the political diviſion of the Earth is meant the different Countries, Empires, Kingdoms, States, and other denominations eſtabliſhed by men, either from the ambition of tyrants, or for the ſake of good government.

### OF EUROPE.

Europe is bounded on the north by the northern, or frozen, ocean; on the eaſt by Aſia; on the ſouth by the Mediterranean Sea, ſeparating Europe from Africa; and by the north Atlantic, or weſtern ocean, on the weſt. It lies between the latitudes of 36 and 72 degrees of north latitude; and between the longitudes of 10 degrees weſt, and 65 degrees eaſt from London; is about 3000 miles long, reckoning from the N. E. to the S. W. and about 2500 miles broad.

35. The countries, their poſition, with regard to the middle parts of Europe, the chief cities, principal rivers, with their courſes, and the moſt noted mountains, and what quarter of the country they are in, are exhibited in the following table; where E. ſtands for empire, K. for kingdom, R. for republic, Nd. for northward, *&c.*

| Countries. | Poſition. | Chief Cities. | Rivers. | Courſe. | Mountains. |
|---|---|---|---|---|---|
| E. Turky | S. E. | Conſtantinople | Danube | E. | Argentum Nd. |
| K. Poland | Mid. | Warſaw | Viſtula | N. N. W. | Carpathian Sd. |
| E. Muſcovy<br>E. Ruſſia | N. E. | Moſcow<br>Peterſburg | Volga<br>Niaper | E. to S.<br>S. | Boglowy Sd.<br>Riphean Wd. |
| K. Sweden | N. | Stockholm | Dalecarlia | E. | Dofrine Wd. |
| K. Norway | N. N. W. | Bergen | Glama | S. | Dofrine Ed. |
| K. Denmark | N. W. | Copenhagen | Eyder | W. | |
| K. Hungary | Mid. | Preſburg | Danube | S. E. | Carpathian Nd. |
| E. Germany | Mid. | Vienna | Danube | E. | Alps Sd. |
| Italy | S. | Rome | Po<br>Tyber | E.<br>S. | Alps Nd.<br>Apennine Mid. |
| R. Switzerland | Mid. | Bern | Rhine | W. | Alps Sd. |
| Netherlands | W. | Bruſſels | Maeſe | N. | |
| R. Holland | W. | Amſterdam | Rhine | N. N. W. | |
| K. France | W. | Paris | Loire<br>Rhone | N. to W.<br>S. | Pyrenees S. W.<br>Alps Ed. |
| K. Spain | S. W. | Madrid | Tagus | W. | Pyrenees N. E. |
| K. Portugal | S. W. | Liſbon | Tagus | W. | C. Rocca W. |
| K. England | W. | London | Thames | E. | Malvern N. W. |
| K. Scotland | W. | Edinburgh | Forth | E. | Grampian Nd. |
| K. Ireland | W. | Dublin | Shannon | S. W. | Knockpatric W |

There are in Europe four Kingdoms beſide thoſe enumerated above; but they are contained in the forenamed Countries.

The Kingdom of Pruſſia, which is part of Poland; the King's reſidence is at Berlin, a city in Germany.

The Kingdom of Bohemia, a part of Germany; the chief city is Prague.

The Kingdom of Sardinia, an Italian iſland; the King reſides at Turin, a city in Italy.

The Kingdom of the Sicilies, appending to Italy; the King reſides at Naples, a city in Italy.

In ſome of the forenamed countries are ſeveral dominions independent one of the other; particularly in Germany and Italy.

The principal ſtates in Germany are the following 12; where D. ſtands for duchy, El. for electorate, P. for principality.

| | | | | |
|---|---|---|---|---|
| States | D. Auſtria | K. Bohemia | El. Bavaria | El. Brandenburg |
| Ch. cities | Vienna | Prague | Munich | Berlin |
| States | El. Saxony | El. Hanover | El. Palatine | El. Mentz |
| Ch. cities | Dreſden | Hanover | Manheim | Mentz |
| States | El. Triers | El. Cologne | P. HeſſeCaſſel | D. Wurtemburg |
| Ch. cities | Triers | Cologne | Caſſel | Stutgard |

The principal ſtates in Italy are the following 12.

| | | | | |
|---|---|---|---|---|
| States | D. Savoy | P. Piedmont | D. Milaneſe | D. Parmeſan |
| Ch. cities | Chamberry | Turin | Milan | Parma |
| States | D. Modeneſe | D. Mantuan | R. Venice | R. Genoa |
| Ch. cities | Modena | Mantua | Venice | Genoa |
| States | D. Tuſcany | Patriarchate | R. Lucca | K. Naples |
| Ch. cities | Florence | Rome | Lucca | Naples |

36. *The principal Seas, Gulfs and Bays in Europe, are*

The *Mediterranean Sea*, having Europe on the N. and Africa on the S.

The *Adriatic Sea*, between Italy and Turky.

The *Euxine*, or *Black Sea*, in Turky, between Europe and Aſia.

The *White Sea*, in the N. N. W. parts of Muſcovy.

The *Baltic Sea*, between Sweden, Denmark, and Poland.

The *German Ocean*, or Sea, between Germany and Britain.

The *Engliſh Channel*, between England and France.

*St. George's Channel*, between Britain and Ireland.

The *Bay of Biſcay*, formed between France and Spain.

The *Gulf of Bothnia*, in the N. E. parts of Sweden.

The *Gulf of Finland*, between Sweden and Ruſſia.

The *Gulf of Venice*, the N. W. end of the Adriatic Sea

37. *The principal Iſlands in Europe, are*

The *Britiſh Iſles*, viz. Great Britain, Ireland, Orkneys, and Weſtern Iſles.

The *Spaniſh Iſles*; Majorca, Minorca, Ivica, in the Medit. Sea.

*Turkiſh Iſles*; Sicily, Sardinia, Corſica, Lipari, in the Medit. Sea.

*Italian Iſles*; Candia, Archipelago Iſles, in the Medit. Sea.

*Swediſh Iſles*; Gothland, Oeland, Alan, Rugen, in the Baltic Sea.

*Daniſh Iſles*; Zeland in the Baltic Sea; and Iceland, Faro Iſles, E. and W. Greenlands in the Northern ocean.

*Azores Iſles* in the Atlantic ocean, belonging to Portugal.

## OF ASIA.

38. The continent of Aſia is bounded on the north by the Northern or frozen ocean, on the eaſt by the Pacific ocean, on the ſouth by the Indian ocean, and by Africa and Europe on the weſt. It lies, including its Iſlands, between the latitudes of 10 degrees ſouth, and 72 degrees north; and is between the longitudes of 25 and 148 degrees eaſt of London; its length, excluſive of the iſles, being about 4800 miles, and breadth about 4300 miles.

39. The poſitions and names of the chief countries, cities, rivers, and mountains, are contained in the following table.

| Countries. | Poſition. | ChiefCities. | Rivers. | Courſe. | Mountains. |
|---|---|---|---|---|---|
| E. China | S. E. | Pekin | Yellow R. | S. to E. | Ottorocoran Nd. |
| | | Nankin | Kiam | E | Damaſian Wd. |
| | | Canton | Ta | S. E. | |
| K. Korea | E. | Kingkitau | Yalu | S. | Shanalin |
| ChineſeTartary | E. | Chynian | Yamour | N.E.to E. | Fongwanſhan |
| Mengalia | Mid. | Kudak | Yellow R. | N. to W. | |
| K. Thibet | Mid. | Eſkerdu | Yaru | E. | Kantes |
| Bukharia, or Uſbecs | Mid. | Samarkand | Amu | N. W. | Belurlag |
| Karazm | Mid. | Urjenz | Amu | S. W. | Irder |
| Kalmucks | Mid. | | Tekis | | Tubratubuſluk |
| E. Siberia | N. | Tobolſki | Oby | N. | Stolp |
| | | Aſtracan | Jeniſka | N. N.W. | |
| E. Turky | W. | Smryna | Euphrates | S. E. | Taurus |
| K. Syria | W. | Aleppo | Euphrates | S. | Lebanon |
| Arabia | S. W. | Medina | Euphrates | S. E. | Gabel el ared |
| E. Perſia | S. | Iſpahan | Oxus | W. | Caucaſus |
| | | | Araxes | S. W. | Taurus |
| India Weſt of the Ganges | S. | Agra | Indus | S. W. | Caucaſus |
| | | Delli | Ganges | S. W. | Balagate |
| India Eaſt of the Ganges | S. | Ava | Domea | S. | Damaſcene |
| | | Pegu | Mecon | S. | |
| | | Siam | Menan | S. | |
| | | Cambodia | Ava | S. | |

Some of theſe countries contain ſeveral others.

Aſiatic Turky contains

| | | | | |
|---|---|---|---|---|
| Countries | Georgia N. | Turcomania E. or Armenia | Curdiſtan E. or Aſſyria | Diarbec E. |
| Ch. cities | Teflis | Erzerum | Betlis | Mouſol |
| Countries | Eyraca S.E. | Arabia deſert S. | Natolia W. | Syria W. |
| Ch. cities | Bagdat | | Smyrna | Aleppo |

India weſt of the Ganges contains

| | | | | |
|---|---|---|---|---|
| Northern Parts | Countries | Indoſtan N. | Cambaya S. W.<br>or Guzarat | Bengala S.E. |
| | Ch. cities | Delli | Surat | Patna |
| Malabar Coaſt | Countries | Decan<br>or Viſapour | Biſnagar W.<br>or Carnate | |
| | Ch. cities | Goa | Calcut and Cochin | |
| Coromandel Coaſt | Countries | Biſnagar<br>Carnate, E. ſide | K. Golconda | K. Orixa |
| | Ch. cities | Madras | Golconda | Orixa |

India eaſt of the Ganges contains

| | | | | |
|---|---|---|---|---|
| Countries | K. Ava N. W. | K. Pegu W | K. Siam S. | K. Malacca S. |
| Ch. cities | Ava | Pegu | Siam | Malacca |
| Countries | K. Cambodia S. | K. Cochin China E. | K. Laos N. | K. Tonquin N. E. |
| Ch. cities | Cambodia | Thoanoa | Lanchan | Keccio |

40. *The principal Seas, Gulfs and Bays in Aſia, are*

*Caſpian Sea*, quite ſurrounded by Siberia on the north, Korazm eaſt, by Perſia on the ſouth, and by Georgia on the weſt.
*Korean Sea*, between Korea and the iſlands of Japan.
*Yellow Sea*, between China and the Japan iſles.
*Gulf of Cochin China*, on the borders of Tonquin and Cochin China.
*Bay of Siam*, formed by the countries of Siam and Malacca.
*Bay of Bengal*, between India eaſt, and India weſt of the Ganges.
*Gulf of Perſia*, having Perſia on the N. E. and Arabia on the S. W.

41. *The principal Iſlands belonging to Aſia, are*
*Ladrone*, or *Marian Iſles*, whoſe chief iſland is Guam.

| | | | | |
|---|---|---|---|---|
| *Japan Iſles* | Ch. iſles | Japan | Bongo | Tonſa |
| | Ch. cities | Jeddo | Bongo | Tonſa |
| *Philippines* | Ch. iſles | Luconia | Mindanao | Samar |
| | Ch. cities | Manilla | Mindanao | |
| *Chineſe Iſles* | Ch. iſles | Formoſa | Ainan | Makao |
| | Ch. cities | Taywanfu | Tan | Makau |
| *Moluccos* | Ch. iſles | Celebes | Gilolo | Ceram |
| | Ch. cities | Macaſſer | Gilolo | Ambay |
| *Sunda Iſles* | Ch. iſles | Borneo | Sumatra | Java |
| | Ch. cities | Banjar | Achin | Batavia |

The *Andaman Iſles* to the weſt of Siam.
*Nicobar Iſlands* weſt of Malacca.
*Maldive Iſlands* to the S. W. of Biſnagar.
The *Iſland of Ceylon* S. E. of Biſnagar; the chief city is Candy, or Candy Uta.

OF

## OF AFRICA.

42. This large continent is a peninſula, joined to Aſia by the Iſthmus of Suez. On the N. E. it is ſeparated from Aſia by the Red Sea; it has the Indian Ocean on the eaſt, the Southern on the ſouth, the Atlantic on the weſt, and the Mediterranean Sea on the north, which ſeparates it from Europe. It is ſituated between the latitudes of 37 degrees N. and 35 degrees S; and between the longitude of 18 degrees W. and 50 degrees E. from London; is about 4300 miles long, and 4200 miles broad.

43. The poſitions and names of the chief countries, cities, rivers, and mountains, are contained in the following table.

| Countries. | Poſition. | Chief Cities. | Rivers. | Courſe. | Mountains. |
|---|---|---|---|---|---|
| E. Morocco | N. W. | Fez | Mulvia | N. | Atlas |
| K. Algiers | N. | Algiers | Saffran | N. | Atlas |
| K. Tunis | N. | Tunis | Megrada | N. | Atlas |
| K. Tripoli | N. | Tripoli | Salines | N. E. | Atlas |
| K. Barca | N. | Docra | | | Meies |
| K. Egypt | N. E. | Cairo | Nile | N. | Gianadel |
| E. Abyſſinia or Ethiopia | E. | Ambamarjam | Nile | N. | |
| Ajan | E. | Adea | Madadoxa | S. | |
| Zanguebar | E. | Melinda | Cuama | E. S. E. | |
| Sofala | S. E. | Sofala | Amara | E. | Amara |
| Terra de Natal | S. E. | Natal | St. Eſprit | E. | Amara |
| Cafraria | S. | Cape Town | St. Chriſtopher | E. | Table |
| Mataman | S. S. W. | | Angri | W. | Sunda |
| K. Benguela | S. S. W. | Benguela | Negros | W. | Sunda |
| K. Angola | S. W. | Loando | Coanza | W. | Sunda |
| K. Congo | S. W. | St. Salvador | Zaara | S. W. | Sunda |
| K. Loango | S. W. | Loango | Zette | S. W. | St. Eſprit |
| Biafara | S. W. | Biafara | Camerones | S. W. | St. Eſprit |
| K. Benin | S. W. | Benin | Formoſa | S. W. | |
| Guinea | S. W. | Cape Coaſt | Volta | S. | Sierra Leon |
| Mandinga | W. | James Fort | Gambia | W. | C. Verd |
| Sanhaga | W. | Sanhaga | Senegal | N. W. | |
| Bildulgerid | Mid. N. | Dara | Dara | S. | Atlas |
| Zara | Mid. | Zuenzega | Nubia | E. | |
| Nubia | Mid. | Nubia | Nubia | E. | |
| Negroland | Mid. | Tombute | Niger | W. | |
| Ethiopia inter | Mid. | Chaxumo | Niger | W. | Luna |
| Monomugi | Mid. S. | Merango | Cuama | E. S. E | Luna |
| E. Monomotapa | Mid. S. | Morgar | Amara | S. E. | Amara |

Many parts of the coaſts of Africa are ſubject to the European nations: Thus the Kingdoms of Algiers, Tunis, Tripoli, Barca, and Egypt, are either ſubject to the Ottoman, or Turkiſh empire, or acknowledge themſelves under its protection.

Abyſſinia is governed by its own Emperor.

Ajan or Anian is peopled by a few wild Arabs.

In Zanguebar and Sofala, the Portugueſe have many black Princes tributary to them.

Cafraria, or the country of the Hottentots, belongs to the Dutch.

The ſea coaſts of Guinea are uſually diſtinguiſhed by the names of the *Slave Coaſt*, *Gold Coaſt*, *Ivory Coaſt*, *Grain Coaſt*, and *Sierra Leon.*

The Engliſh, Dutch, French, Portugueſe, and others, have ſeveral ſettlements along theſe coaſts, and even many miles up the country, particularly the Engliſh on the rivers Gambia and Senegal.

In the general table the countries are taken in a very large ſenſe; for many of them contain a great number of ſtates independent one of the other, the particulars of which are not known to Geographers.

44. *The principal Seas, Gulfs, and Bays in Africa, are*

The *Red Sea*, between Africa and Aſia: It waſhes the coaſt of Arabia on the Aſiatic ſide, and the coaſts of Egypt and Abyſſinia on the African ſide.

*Moſambique Sea*, between Africa and the iſland of Madagaſcar eaſtward.

*Saldanna Bay* in Cafraria, on the Ethiopic Ocean.

*Bight of Benin* on the coaſt of Guinea, in the Ethiopic Ocean.

45. *The principal African Iſlands, are*

| | Chief iſle. | Chief Town. | Situation. |
|---|---|---|---|
| *Madeira iſles* | Madeira | Funchal | N. Atlantic Ocean |
| *Canary iſles* | Canaria | Palma | N. Atlantic Ocean |
| *C. Verd iſles* | St. Jago | St. Jago | N. Atlantic Ocean |
| *Ethiopian iſles* | St. Helena | | Ethiopic Ocean |
| *Komora iſles* | Johanna | Demani | Indian Ocean |
| *Sokotora iſles* | Zocotora | Calanſia | Indian Ocean |
| *Almirante iſles* | But little | known | Indian Ocean |

The iſland of *Madagaſcar*, one of the largeſt in the world, lies in the Indian Ocean: It is divided into a multitude of little ſtates; ſome of them formed by the European privateers, and their ſucceſſors deſcended from a mixture with the natives.

The iſlands of *Bourbon* and *Mauritius* lie in the Indian Ocean, to the eaſt of Madagaſcar: theſe belong to the French.

The Madeiras, and Cape de Verd Iſles, belong to the Portugueſe.

The Canary Iſles to Spain; St. Helena to England.

## OF AMERICA.

46. This vaſt continent, called by ſome the new world, having been diſcovered by the Europeans ſince the year 1492, is uſually divided into two parts, one called North, and the other South America, being joined to one another by the Iſthmus of Darien.

North America lies between the latitudes of 10 degrees and 80 degrees north; and chiefly between the longitudes of 50 degrees and 130 degrees weſt of London; is about 4200 miles from north to ſouth, and about 4800 from eaſt to weſt. It is bounded on the eaſt by the north Atlantic Ocean, by the Gulf of Mexico on the ſouth, on the weſt by the Pacific Ocean, and by the Northern continent and ocean to the northward.

South America is bounded on the eaſt by the ſouth Atlantic Ocean, by the Southern Ocean to the ſouth, by the Pacific Ocean on the weſt, and on the north by the Caribbean Sea. It lies between the latitudes of 12 degrees north, and 56 degrees ſouth; and between the longitudes of 45 degrees and 83 degrees weſt from London; is about 4200 miles long, and about 2200 miles in breadth.

47. The poſitions and names of the chief countries, cities, rivers, and mountains, in North America, are in the following table.

| Countries. | Poſition. | Chief Cities. | Rivers. | Courſe. | Mountains. |
|---|---|---|---|---|---|
| California | W. | St. Juan. | | | |
| New Mexico | S. | Sante Fe | N. River | S. S. E. | |
| Old Mexico | S. W. | Mexico | Panuco | E. | |
| Louiſiana | S. | New Orleans | Miſſiſſipi | S. | |
| Florida | S. | St. Auguſtine | St. John | N. to E. | Apalachian |
| Georgia | S. S. E. | Savannah | Alatamaha | E. S. E. | Apalachian |
| Carolina | S. E. | Charles Town | Aſhly | S. E. | Apalachian |
| Virginia | E. | James Town | Powtomack | S. E. | Apalachian |
| Maryland | E. | Annapolis | Powtomack | S. E. | Apalachian |
| Penſylvania | E. | Philadelphia | Delawar | S. | Apalachian |
| Jerſeys | E. | New York | Albany | S. | |
| New England | E. | Boſton | Connecticut | S. | |
| Nova Scotia | N. E. | Halifax | St. John | S. S. E. | Ladies |
| Canada | Mid. | Quebec | St. Lawrence | N. E. | |
| New Britain | N. N. E. | Fort Rupert | Rupert | W. | |
| New Wales | N. | York Fort | Nelſon | W. | |

California, Old Mexico, and New Mexico, belong to Spain.

Louiſiana, to the weſt of the river Miſſiſſippi, was poſſeſſed by the French at the end of the late war; but is now transferred to Spain.

All the other countries are in the hands of the Engliſh.

In South America the poſition and names of the chief countries, cities, rivers, and mountains, are as follow:

| Countries. | Poſition. | Chief Cities. | Rivers. | Courſe. | Mountains. |
|---|---|---|---|---|---|
| Terra Firma. | N. | Panama | Oronoque | N. E. | |
| Peru | W. | Lima | Chuquimayo | W. N. W. | Andes |
| Chili | S. W. | St. Jago | Valpariſo | W. | Andes |
| Patagonia | S. | | Deſaguadero | S. | Andes |
| La Plata | S. E. | Buenos Ayres | La Plata | S. | Andes |
| Paraguay | Mid. | Aſſumption | Paragua | S. | |
| Braſil | E. | St. Salvador | Rio Real | N. E. | |
| Amazonia | Mid. | | Amazons | E. | |
| Guiana | N. E. | Surinam | Eſquebe | N. N. E. | |

Terra Firma, Peru, Chili, La Plata, and Paraguay, are in the poſſeſſion of the Spaniards.

Braſil belongs to the Portugueſe.

Patagonia, Amazonia, and Guiana, are poſſeſſed by the native Indians, except ſome parts of the coaſts of Guiana, in the hands of the Dutch and French.

48. *The principal Seas, Gulfs, and Bays in America.*

The *Caribbean Sea*, bounded by Terra Firma on the ſouth, and a range of iſlands on the north and eaſt.

*Gulf of Mexico*, formed by Old Mexico, Louiſiana, and Florida.

*Bay of Campeachy*, part of the Gulf of Mexico, on the Mexican coaſt.

*Bay of Honduras*, part of the Caribbean Sea, next to Mexico.

*Bay of Panama*, in the Pacific Ocean, next the Iſthmus of Darien.

*Bay of California*, in the Pacific Ocean, having California on the weſt.

*Bay of Fundy*, in Nova Scotia, north Atlantic Ocean.

*Gulf of St. Lawrence,* in the North Atlantic Ocean, bounded by Nova Scotia, New Britain, and ſome iſlands eaſtward and ſouth-eaſtward.

*Hudſon's Bay*, between New Britain E. and New Wales W.

49. *The chief American Iſlands in the Atlantic Ocean.*

*Newfoundland*, and *Cape Breton*, eaſt of the Gulf of St. Lawrence.

*Bermudas*, or *Summer Iſlands*, eaſt of Carolina.

*Bahama Iſles*, ſouth eaſt of Florida.

| | | | |
|---|---|---|---|
| *Great Antilles* | Cuba | ch. town Havanna | lying E. of the Mexican Gulf, and N. of the Car. Sea. |
| | Hiſpaniola | ch. town St. Domingo | |
| | Jamaica | ch. town Kingſton | |

*Caribbee Iſles*, bounding the Caribbean Sea on the E. and N. E.

*Leſſer Antilles*, on the N. N. E. of Terra Firma, in the Caribbee Sea.

*Terra del Fuego* on the ſouth of Patagonia, in the Southern Ocean.

*Gallipago Iſles*, lying N. W. of Peru in the Pacific Ocean.

## SECTION IV.

## *Geographical Problems.*

50. Problem I. *Given the latitudes of two places: Required their difference of latitude.*

Case I. When the latitudes of the given places have the ſame name:

Rule. Subtract the leſſer latitude from the greater, the remainder is the difference of latitude.

Exam. I. *What is the difference of latitude between London and Rome?*

| | |
|---|---|
| London's lat. | 51° 32′ N. |
| Rome's lat. | 41 54 N. |
| Diff. lat. | 9 38 |
| | 60 |
| | 578 miles. |

Exam. II. *What is the difference of latitude between the Lizard and the Iſland of Madeira?*

| | |
|---|---|
| Lizard's lat. | 49° 57′ N. |
| Madeira's lat. | 32 36 N. |
| Diff. lat. | 17 21=1041m. |

Exam. III. *What is the difference of latitude between the Iſland of St. Helena and the Cape of Good Hope?*

| | |
|---|---|
| C. Good Hope's lat. | 34° 29′ S. |
| St. Helena's lat. | 15 55 S. |
| Diff. lat. | 18 34=1114m. |

Exam. IV. *A ſhip from the latitude of 43° 18′ N. is come to the lat. of 34° 49′ N. Required the diff. of latitude.*

| | |
|---|---|
| Lat. from | 43° 18′ N. |
| Lat. in | 34 49 N. |
| Diff. lat. | 8 29=509m. |

Case II. When the latitudes of the given places have contrary names:

Rule. Add the latitudes together, and the ſum will be the difference of latitude.

Exam. I. *Required the diff. of lat. between C. Finiſterre and C. St. Roque.*

| | |
|---|---|
| C. Finiſterre lat. | 42° 57′ N. |
| C. St. Roque lat. | 5 00 S. |
| Diff. lat. | 47 57 |
| | 60 |
| | 2877 miles. |

Exam. II. *What is the difference of latitude between the Iſland of Barbadoes and C. Negro?*

| | |
|---|---|
| I. of Barbadoes's lat. | 13° 00′ N. |
| C. Negro's lat. | 16 30 S. |
| Diff. lat. | 29 30=1770m. |

Exam. III. *Required the difference of latitude between Cape Horn and Cape Corientes in Mexico.*

| | |
|---|---|
| Cape Horn's lat. | 55° 59′ S. |
| C. Corientes's lat. | 20 18 N. |
| Diff. lat. | 76 17=4577m. |

Exam. IV. *A ſhip from the lat. of 8° 28′ S. has ſailed north to the lat. 6° 45′ N. Required the diff. of latitude.*

| | |
|---|---|
| Lat. from | 8° 28′ S. |
| Lat. in | 6 45 N. |
| Diff. lat. | 15 13=913m. |

The ſituation of about 1500 particular places are contained in a Geographical Table, art. 137, at the end of this book; where the latitudes and longitudes of places are to be ſought, as they follow in alphabetical order.

### 51. PROBLEM II. *Given the latitude of one place and the difference of latitude between it and another place: Required the latitude of the latter place.*

CASE I. When the given latitude and difference of latitude have the ſame name:

RULE. To the given latitude add the degrees and minutes in the diff. of latitude, that ſum is the other latitude of the ſame name.

EXAM. I. *A ſhip from the latitude of* 38° 14′ *N. ſails north till her difference of latitude is* 12° 32′: *What latitude is ſhe come to?*

| | |
|---|---|
| Lat. from | 38° 14′ N. |
| Diff. lat. | 12 32 N. |
| Lat. in | 50 46 N. |

EXAM. II. *A ſhip from the iſland of Aſcenſion runs ſouth till her diff. of latitude is* 5° 37′: *What is the preſent latitude of the ſhip?*

| | |
|---|---|
| I. Aſcenſion's lat. | 7° 59′ S. |
| Diff. lat. | 5 37 S. |
| Ship's lat. | 13 36 S. |

EXAM. III. *A ſhip from the iſland of Madeira ſails N.* 675 *miles: What lat. is ſhe in?*

| | |
|---|---|
| I. Madeira's lat. | 32° 36′ N. |
| Diff. lat. $\frac{675}{60}$ (I. 22) = | 11 15 N. |
| Ship's lat. | 43 51 N. |

EXAM. IV. *Three days ago we were in the latitude of the Cape of Good Hope, and have run each day* 92 *miles directly S. What is our preſent latitude?*

| | |
|---|---|
| C. Good Hope's lat. | 34° 29′ S. |
| Diff. lat. $\frac{92 \times 3}{60}$ (I. 22) = | 4 36 S. |
| Preſent latitude | 39 05 S. |

CASE II. When the given latitude and difference of latitude have contrary names:

RULE. Take the difference between the given latitude and the degrees and minutes in the diff. of latitude, the remainder is the other latitude, of the ſame name with the greater.

EXAM. I. *A ſhip from the latitude of* 38° 14′ *N. ſails ſouth till her difference of latitude is* 12° 32′: *What latitude is ſhe come to?*

| | |
|---|---|
| Lat. from | 38° 14′ N. |
| Diff. lat. | 12 32 S. |
| Lat. in | 25 42 N. |

EXAM. II. *A ſhip from the iſland of Aſcenſion runs north till her diff. lat. is* 5° 37′: *What is the preſent latitude of the ſhip?*

| | |
|---|---|
| I. Aſcenſion's lat. | 7° 59′ S. |
| Diff. lat. | 5 37 N. |
| Ship's lat. | 2 22 S. |

EXAM. III. *A ſhip from Sierra Leon ſails S.* 839 *miles: What latitude is ſhe in?*

| | |
|---|---|
| Sierra Leon's lat. | 8° 30′ N. |
| Diff. lat. $\frac{839}{60}$ (I. 22) = | 13 59 S. |
| Ship's lat. | 5 29 S. |

EXAM. IV. *Four days ago we were in the latitude of the iſland of St. Matthew, and ſailed due north* 6 *miles an hour: What latitude is the ſhip in?*

| | |
|---|---|
| St. Matthew's lat. | 1° 23′ S. |
| Diff. lat. $\frac{6 \times 24 \times 4}{60}$ (I. 22) = | 9 36 N. |
| Ship's latitude | 8 13 N. |

52. PROBLEM. III. *Given the longitudes of two places: Required their difference of longitude.*

RULE. If the longitudes are of the ſame name, their difference is the difference of longitude required.

But if the longitudes are of different names; their ſum gives the difference of longitude.

And if this ſum exceeds 180 degrees, take it from 360 degrees, and there remains the difference of longitude.

EXAM. I. *Required the difference of longitude between London and Naples?*

| | |
|---|---|
| London's long. | 00° 00′ |
| Naples' long. | 14 19 E. |
| Diff. longitude | 14 19 |
| | 60 |
| | 859 |

EXAM. II. *A ſhip in longitude 14° 45′ W. is bound to a port in longitude 48° 18′ weſt: What diff. of longitude muſt ſhe make?*

| | |
|---|---|
| Ship's long. | 14° 45′ W. |
| Long. bound to | 48 18 W. |
| Diff. longitude | 33 33 |
| | 60 |
| | 2013 miles. |

EXAM. III. *What is the difference of longitude between Cape Gardafuir and Cape Comorin?*

| | |
|---|---|
| C. Gardafuir's long. | 50° 25′ E. |
| C. Comorin's long. | 78 17 E. |
| Diff. longitude | 27 52 |
| | 60 |
| | 1672 miles. |

EXAM. IV. *Required the difference of longitude between St. Chriſtopher's and Cape Negro.*

| | |
|---|---|
| St. Chriſtopher's long. | 62° 54′ W. |
| C. Negro's long. | 11 30 E. |
| Diff. longitude | 74 24 |
| | 60 |
| | 4464 miles. |

EXAM. V. *A ſhip in longitude 140° 20′ W. is bound to a place in longitude 139° 25′ E. what diff. of longitude muſt ſhe make?*

| | |
|---|---|
| Ship's long. | 140° 20′ W. |
| Long. bound to | 139 25 E. |
| | 279 55 |
| | 360 00 |
| Diff. longitude | 80 05 |

EXAM. VI. *What is the difference of longitude between Cape Horn and Manila?*

| | |
|---|---|
| Cape Horn's long. | 67° 26′ W. |
| Manila's long. | 120 25 E. |
| | 187 51 |
| | 360 00 |
| Diff. longitude | 172 09 |

Sometimes the diff. lon. between two places is eſtimated by the diff. of time, allowing an hour to every 15 degrees of longitude, and one min. of time for every 15 min. of a deg. or a deg. for every 4 min. of time.

EXAM. *At 6 h. 48 m. P. M. having obſerved at ſea a certain appearance in the heavens, which I knew was ſeen the ſame inſtant at 3 h. 25 m. P. M. in London: Required the diff. longitude between the places of obſervation.*

| | | | |
|---|---|---|---|
| From | 6 h. 48 m. | 3 h. = | 45 deg. |
| Take | 3 35 | 13 m. = | 3 15′ |
| Remain | 3 13 = diff. time. | Sum | 48 15 = Diff. long. |

And becauſe the hour of appearance at London was leaſt, therefore I know myſelf to be to the eaſtward of London.

53. **Problem IV.** *Given the longitude of one place, and the difference of longitude between that and another: Required the longitude of the ſecond place.*

**Rule.** If the given longitude and difference of longitude are of a contrary name, their difference is the longitude required; and is of the ſame name with the greater.

But if the given longitude and difference of longitude are of the ſame name, the ſum is the longitude ſought, of the ſame name with the given place.

And if the ſum is greater than 180 degrees, take it from 360 degrees, remains the longitude required, of a contrary name to that of the given place.

Exam. I. *A ſhip from the longitude of 41° 12′ E. ſails weſtward until her difference of longitude is 15° 47′: What is her preſent longitude?*

| | | |
|---|---|---|
| Ship's longitude | 41° 12′ | E. |
| Diff. long. | 15 47 | W. |
| Preſ. long. | 25 25 | E. |

Exam. II. *A ſhip from Cape Charles in Virginia ſails eaſtward until ſhe has altered her longitude 22° 53′: What longitude is ſhe in?*

| | | |
|---|---|---|
| C. Charles's long. | 76° 07′ | W. |
| Diff. long. | 22 53 | E. |
| Ship's long. | 53 14 | W. |

Exam. III. *Four days ago I departed from C. St. Sebaſtian in Madagaſcar, and I have made each day 75 miles of eaſt longitude: Required the longitude the ſhip is in?*

75
4
6,0)30,0
5°

| | | |
|---|---|---|
| C. Sebaſ. long. | 49° 13′ | E. |
| Diff. long. | 5 00 | E. |
| Ship's long. | 54 13 | E. |

Exam. IV. *A ſhip from Cape Finiſterre ſails weſtward, and finds ſhe has altered her longitude 587 miles: What longitude is ſhe arrived in?*

| | | |
|---|---|---|
| C. Finiſterre's long. | 9° 36′ | W. |
| Diff. lon. $\frac{587}{60}$ | 9 47 | W. |
| Long. in. | 19 23 | W. |

Exam. V. *A ſhip from Cape St. Lucar in California has made 87° 18′ of weſt longitude: What longitude is ſhe in?*

| | | |
|---|---|---|
| C. St. Lucar's long. | 109° 40′ | W. |
| Diff. long. | 87 18 | W. |
| | 196 58 | W. |
| | 360 00 | |
| Ship's longitude | 163 02 | E. |

Exam. VI. *Seven days ago my longitude was 172° 17′ W. and I have made each day 132 miles of weſt longitude: Required my preſent longitude?*

132
7
6,0)92,4
15 24

| | | |
|---|---|---|
| Departed long. | 172° 17′ | W. |
| Diff. long. | 15 24 | W. |
| | 187 41 | |
| | 360 00 | |
| Preſent long. | 172 19 | E. |

## SECTION V.

## 54. *Of the Uſe of the Globes.*

By the globes are here meant two ſpherical bodies, called the Terreſtrial and Celeſtial Globes, the convex ſurfaces of which are ſuppoſed to to give a true repreſentation of the earth and heavens.

The TERRESTRIAL GLOBE has delineated on its convexity the whole ſurface of the *earth* and *ſea* in their relative ſize, form, and ſituation.

The CELESTIAL GLOBE has drawn on its ſurface the images of the ſeveral conſtellations and ſtars; the relative magnitude and poſition which the ſtars are obſerved to have in the heavens, being preſerved on this globe.

The globes are fitted up with certain machinery, by means of which a great variety of uſeful problems are neatly ſolved.

The BRAZEN MERIDIAN is that ring, or hoop, in which the globe hangs on its axis; which is repreſented by two wires paſſing through its poles. This circle is divided into four quarters, of 90° each; in one ſemicircle the diviſions begin at each pole, and end at 90°, where they meet: In the other ſemicircle, the diviſions begin at the middle, and proceed thence towards each pole, where they end at 90 degrees. The graduated ſide of this brazen circle ſerves as a meridian for any point on the ſurface of the earth, the globe being turned about till that point comes under the circle.

The HOUR CIRCLE is a ſmall circle of braſs, which is divided into 24 hours, the quarters and half quarters. It is fixed on the brazen meridian, equally diſtant from the north end of the axis, to which an index is fitted, that points out the diviſions of the hour circle as the globe is turned about.

The HORIZON is repreſented by the upper ſurface of the wooden circular frame encompaſſing the globe about its middle. On this wooden frame is a kind of perpetual calender, contained in ſeveral concentric circles: The inner one is divided into four quarters, of 90 degrees each; the next circle is divided into the twelve months, with the days in each according to the new ſtyle; the next contains the 12 equal ſigns of the zodiac, each being divided into 30 degrees: the next is the 12 months and days according to the old ſtyle; and there is another circle, containing the 32 winds, with their halves and quarters. Although theſe circles are on all horizons, yet their diſpoſition is not always the ſame.

The QUADRANT of ALTITUDE is a thin ſtreight ſlip of braſs, one edge of which is graduated into 90 degrees and their quarters, equal to thoſe of the meridian. To one end of this is fixed a braſs nut and ſcrew, by which it is put on, and faſtened to the meridian: and if it is fixed to the zenith, or pole of the horizon, then the graduated edge repreſents a vertical circle paſſing through any point.

Beſides theſe, there are ſeveral circles deſcribed on the ſurfaces of both globes; ſuch as the equinoctial, ecliptic, circles of longitude and right aſcenſion, the tropics, polar circles, parallels of lat. and decl., on the celeſtial globe; and on the terreſtrial, the equator, ecliptic, tropics, polar circles, parallels of latitude, hour circles, or meridians to every 15 degrees, and the ſpiral rhumbs flowing from ſeveral centers, called Flies.

55. PRO-

### 55. PROBLEM I.

*To find the latitude and longitude of any place on the terreſtrial globe.*

1ſt. Bring the given place under that ſide of the graduated brazen meridian where the degrees begin at the equator, by turning the globe about.
2d. Then the degree of the meridian over it ſhews the latitude.
3d. And the degree of the equator under the merid., ſhews the long.

On ſome globes the longitude is reckoned on the equator from the meridian where it begins, eaſtward only, until it ends at 360°: On ſuch globes, when the longitude of a place exceeds 180°, take it from 360, and call the remainder the longitude weſtward.

### 56. PROBLEM II.

*To find any place on the globe, the latitude and longitude of which are given.*

1ſt. Bring the given longitude, found on the equator, to the meridian.
2d. Then under the given latitude, found on the meridian, is the place ſought.

### 57. PROBLEM III.

*To find the diſtance and bearing of any two given places on the globe.*

1ſt. Lay the graduated edge of the quadrant of altitude over both places, the beginning, or 0 degree, being on one of them, and the degrees between them ſhew their diſtance; theſe degrees multipled by 60 give ſea miles, and by 70 give the diſtance in land miles nearly; or multiplied by 20 give leagues.
2d. Obſerve, while the quadrant lies in this poſition, what rhumb of the neareſt fly, or compaſs, runs moſtly parallel to the edge of the quadrant, and that rhumb ſhews the bearing ſought, nearly.

### 58. PROBLEM IV.

*To find the Sun's place and declination on any day.*

1ſt. Seek the given day in the circle of months on the horizon, and right againſt it in the circle of ſigns is the Sun's place.
Thus it will be found that the Sun enters
*The ſpring ſigns*, Aries, March 20. Taurus, April 20. Gemini, May 21.
*The ſummer ſigns*, Cancer. June 21. Leo, July 23. Virgo, Aug. 23.
*Autumnal ſigns*, Libra, Sept. 22. Scorpio, Oct. 23. Sagittar. Nov. 22.
*The winter ſigns*, Capric. Dec. 21. Aquarius, Jan. 20. Piſces, Feb. 18.
2d. Seek the Sun's place in the ecliptic on the globe, bring that place to the meridian, and the diviſion it ſtands under is the Sun's declination on the given day.

On the globes, the ecliptic is readily diſtinguiſhed from the equator, not only by the different colours they are ſtained with, but alſo by the ecliptic's approaching towards the poles, after its interſection with the equator. The marks of the ſigns are alſo put along the ecliptic, one at the beginning of every ſucceſſive 30 degrees.

## 59. PROBLEM V.

*To rectify the globe for the latitude, zenith, and noon.*

1ſt. Set the globe upon an horizontal plane with its parts anſwering to thoſe of the world; move the meridian in its notches, by raiſing or depreſſing the pole, until the degrees of latitude cut the horizon; then is the globe rectified for the latititude.

2d. Reckon the latitude from the equator towards the elevated pole, there ſcrew the bevil edge of the nut belonging to the quadrant of altitude, and the rectification is made for the zenith.

3d. Bring the Sun's place (found by the laſt problem) to the meridian, ſet the index to the XII at noon, or upper XII, and the globe is rectified for the Sun's ſouthing, or noon.

## 60. PROBLEM VI.

*To find where the Sun is vertical, at any given time, in a given place.*

1ſt. Bring the Sun's place, found for the given day (58), to the meridian, and note the degree over it.

2d. Bring the place, for which the time is given, to the meridian, and ſet the index to the given hour.

3d. Turn the globe till the index comes to 12 at noon, then the place under the ſaid noted degree has the Sun in the zenith at that time.

4th. All the places that paſs under that degree, while the globe is turned round, will have the Sun vertical to them on that day.

## 61. PROBLEM VII.

*To find on what days the Sun will be vertical, at any given place, in the torrid zone.*

1ſt. Note the latitude of the given place on the meridian.

2d. Turn the globe, and note what two points of the ecliptic paſs under the latitude noted on the meridian.

3d. Seek thoſe points of the ecliptic in the circle of ſigns on the horizon, and right againſt them, in the circle of months, ſtand the days required.

In this manner it will be found, that the Sun will be vertical to the Iſland of St. Helena on the 6th of November, and on the 4th of February. And at Barbadoes on the 24th of April, and the 18th of Auguſt.

## 62. PROBLEM VIII.

*At any given hour in a given place, to find what hour it is in any other place.*

1ſt. Bring the place where the time is given to the meridian, and ſet the index to the given hour.

2d. Bring the other given place to the meridian, and the index ſhews the hour correſponding to the given time.

### 63. PROBLEM IX.

*At any given time to find all thoſe places of the Earth where the Sun is then riſing or ſetting, and where it is mid-day or midnight.*

Find the place where the Sun is vertical at the given time (60), rectify the globe for the latitude of that place, and bring it to the meridian.

Then all thoſe places, that are in the weſtern half of the horizon, have the Sun riſing; and thoſe in the eaſtern half have it ſetting.

Thoſe under the meridian, above the horizon, have the Sun culminating, or noon; and thoſe under the meridian, below the horizon, have midnight.

Thoſe above the horizon have day; thoſe below it have night.

### 64. PROBLEM X.

*To find the angle of poſition of two places, or the angle made by the meridian of one place, and a great circle paſſing through both places.*

Rectify for the latitude of one of the given places, and bring it to the meridian; there fix the quadrant of altitude, and ſet its graduated edge to the other place: then will that edge of the quadrant cut the horizon in the degree of poſition ſought.

Thus, the angle of poſition at the Land's End to Barbadoes is ſouth $71\frac{1}{2}^{\circ}$ weſterly: but the angle of poſition at Barbadoes to the Land's End is north $37\frac{1}{2}$ degrees eaſterly.

Hence neither of thoſe poſitions can be the true bearing; for the rhumb paſſing through both places, will be oppoſite one way to what it is the other.

### 65. PROBLEM XI.

*The latitude of any place not within the polar circle being given, to find the time of ſun-riſing and ſetting, and the length of the day and night.*

Rectify for the latitude and the noon; bring the Sun's place to the eaſtern ſide of the horizon, and the index ſhews the time of riſing: the Sun's place being brought to the weſtern ſide of the horizon, the index gives the ſetting.

Or, the time of riſing taken from 12 hours gives the time of ſetting.

The time of ſetting being doubled gives the length of the day.

And the time of riſing being doubled gives the length of the night.

Thus, at London, on April 15th, the day is $13\frac{1}{2}$ hours; the night $10\frac{1}{2}$ hours.

### 66. PROBLEM XII.

*To find the length of the longeſt and ſhorteſt days in any given place.*

Rectify for the latitude; bring the ſolſtitial point of that hemiſphere to the eaſtern part of the horizon, ſet the index to 12 at noon, turn the globe till the ſolſtitial point comes to the weſtern ſide of the horizon, the hours paſt over by the index give the length of the longeſt day, or night; and its complement to 24 hours gives the length of the ſhorteſt night, or day.

## 67. PROBLEM XIII.

*A place being given in either frigid or frozen zone, to find the time when the Sun begins to appear at, or depart from, that place: alſo how many ſucceſſive days he is preſent to, or abſent from, that place.*

Rectify for the latitude, turn the globe, and obſerve what degrees in the firſt and ſecond quadrants of the ecliptic are cut by the north point of the horizon, the latitude being ſuppoſed to be north.

Find thoſe degrees in the circle of ſigns on the horizon, and their correſponding days of the month; and all the time between thoſe days the Sun will not ſet in that place.

Again. Obſerve what degree in the third and fourth quadrants of the ecliptic will be cut by the ſouth point of the horizon, and the days anſwering: then the Sun will be quite abſent from the given place during the intermediate days; that day in the third quadrant ſhews when he begins to diſappear; and that in the fourth quadrant ſhews when he begins to ſhine in the place propoſed.

Thus at the North cape, in lat. 71°, the Sun never ſets from May 15 to July 28, which is 74 days; and never riſes from November 16 to January 24, which is 69 days.

## 68. PROBLEM XIV.

*To find the antæci, periœci, and antipodes of any place.*

Bring the given place to the meridian, tell as many degrees of latitude on the contrary ſide of the equator, and it gives the place of the antæci; that is, of thoſe who have oppoſite ſeaſons of the year, but the ſame times of the day.

The given place being under the meridian, ſet the index to 12 at noon, turn the globe until the index points to 12 at night, and the point under the meridian in the given latitude is the place of the periœci; that is, of thoſe who have the ſame ſeaſons of the year, but oppoſite times of the day.

The globe remaining in this poſition, ſeek on the contrary ſide of the equator for the degrees of latitude given, and the point under the meridian, thus found, will be the antipodes to the given place; that is, there the ſeaſons of the year and times of the day are directly oppoſite to thoſe of the given place.

## 69. PROBLEM XV.

*To find the beginning and end of the twilight in any place.*

Rectify the globe for the latitude, zenith, and noon. (59)

Seek the point of the ecliptic oppoſite to the Sun's place, turn the globe and quadrant of altitude, till the ſaid oppoſite point of the ecliptic ſtands againſt 18 degrees on the quadrant of altitude; then will the index ſhew the beginning or end of the twilight; that is, the beginning in the morning, when thoſe points meet in the weſtern hemiſphere; or the end in the evening, when the ſaid points meet in the eaſtern hemiſphere.

## 70. PROBLEM XVI.

*The latitude of a place and day of the month being given, to find the Sun's declination, meridian altitude, right aſcenſion, amplitude, oblique aſcenſion, aſcenſional difference; and thence the time of riſing, ſetting, length of the day and night.*

Rectify for the latitude and noon. Then,

The degree of the meridian over the Sun's place is the declination.

The meridian altitude is ſhewn by the degrees the Sun is above the horizon; and is equal to the ſum or diff. of the co-lat. and decl.

The Sun's right aſcen. is that degree of the equator under the meridian.

Bring the Sun's place to the eaſtern part of the horizon. Then,

The amplitude is that degree of the horizon oppoſite the Sun.

The oblique aſcenſion is that degree of the equator cut by the horizon.

The aſcen. diff. is the diff. between the right and oblique aſcenſions.

The aſcen. diff. converted into time, will give the time the Sun riſes before or after the hour of ſix, according as his amplitude is to the northward or ſouthward of the eaſt point of the horizon.

## 71. PROBLEM XVII.

*Given the latitude of the place and day of the month, to find the Sun's altitude and azimuth, either when he is due eaſt or weſt, at 6 o'clock, or at any other hour while he is above the horizon.*

Rectify the globe for the latitude, zenith, and noon.

Set the quadrant of altitude to the eaſt point of the horizon, turn the globe till the Sun's place comes to the quadrant's edge, and it ſhews the altitude, his azimuth being now 90°, and the index ſhews the hour.

Turn the globe till the index points at 6, there ſtay it, and move the quadrant until its edge cuts the Sun's place; then the degrees at the Sun ſhew its altitude, and the degrees cut by the quadrant in the horizon ſhew the azimuth, reckoning from the north.

In like manner, the globe being turned till the index is againſt any other hour, ſuppoſe 10 in the forenoon; then,

The graduated edge of the quadrant of altitude being turned to cut the Sun's place, will give both the altitude and azimuth at that time.

## 72. PROBLEM XVIII.

*Given the latitude, day of the month, and Sun's altitude, to find the azimuth and hour of the day.*

Rectify the globe for the latitude, zenith, and noon.

Turn the globe and quadrant, until the Sun's place coincide with the altitude on the graduated edge of the quadrant.

Then will that edge of the quadrant cut the degrees of azimuth on the horizon, reckoned from the north; and the index will ſhew the hour of the day.

## 73. PROBLEM XIX.

*To repreſent the appearance of the heavens at any time in a given place.*

Rectify the celeſtial globe for the latitude, zenith, and noon, and turn the globe till the index points at the given hour. Then,

The ſtars in the eaſtern half of the horizon are riſing; thoſe in the weſtern are ſetting: and thoſe on the meridian are culminating.

The quadrant being ſet to any given ſtar will ſhew its altitude, and at the ſame time its azimuth, reckoned on the horizon.

Now by turning the globe round it will readily appear, what ſtars never ſet in that place, and what never riſe: thoſe of perpetual apparition never go below the horizon, thoſe of perpetual abſence never come above it.

## 74. PROBLEM XX.

*To find the latitude and longitude of any ſtar.*

Put the center of the quadrant of altitude on the pole of the ecliptic, and its graduated edge on the given ſtar. Then,

The latitude is ſhewn by the degrees between the ecliptic and ſtar.

The longitude is the degrees cut on the ecliptic by the quadrant.

## 75. PROBLEM XXI.

*To find the declination and right aſcenſion of a ſtar.*

Bring the ſtar to the meridian, the degree over it is the declination; and the degree of the equator under the meridian is the right aſcenſion.

## 76. PROBLEM XXII.

*On any day, and in any given place, to find when a propoſed ſtar riſes, ſets, or culminates.*

Rectify the globe for the latitude and noon.

Bring the ſtar to the eaſtern ſide of the horizon, and the index ſhews the time of its riſing.

Turn the globe till the ſtar comes to the meridian, and the index ſhews the time of its culminating; and in like manner when it ſets, the time will be ſhewn by the index.

Its meridian altitude, oblique aſcenſion, and aſcenſional difference, are found in the ſame manner as for the Sun, at art. 70.

# SECTION VI.

## *Of Winds.*

77. A FLUID is a body, the particles of which readily give way to any impreſſed force; and by this readineſs of yielding, the particles are eaſily put into motion.

Thus, not only liquids, but ſtreams or vapours, ſmoke or fumes, and others of the like kind, are reckoned as fluids.

From all parts of the Earth vapours and fumes are conſtantly ariſing to ſome diſtance from its ſurface.

This is known by obſervation; it is cauſed chiefly by the heat of the Sun, and ſometimes by ſubterraneous fires ariſing from the accidental mixing of ſome bodies.

78. AIR is a fine inviſible fluid ſurrounding the globe of the Earth, and extended to ſome miles above its ſurface.

The ATMOSPHERE is that collection of air, and of bodies contained in it, which circumſcribes the Earth.

79. From a multitude of experiments, air is found to be both heavy and ſpringy.

By its weight it is capable of ſupporting other bodies, ſuch as vapours and fumes, in the ſame manner as wood is ſupported by water.

By its ſpringineſs or elaſticity a quantity of air is capable of being expanded, or of ſpreading itſelf ſo as to fill a larger ſpace *; and of being compreſſed or confined in a ſmaller compaſs †.

80. Air is compreſſed or condenſed by cold, and expanded or rarefied by heat.

This is evident from a multitude of experiments.

An alteration being made by heat or cold in any part of the atmoſphere, its neighbouring parts will be put in motion, by the endeavour which the air always makes to reſtore itſelf to its former ſtate.

For experiments ſhew, that condenſed or rarefied air will return to its natural ſtate, when the cauſe of that condenſation or rarefaction is removed.

81. WIND is a ſtream or current of air which may be felt; it uſually blows from one part of the horizon to its oppoſite part.

82. The *horizon*, beſide being divided into 360 degrees, like all other circles, is by mariners ſuppoſed to be divided into four *quadrants*, called the north-eaſt, north-weſt, ſouth-eaſt and ſouth-weſt quarters; each of theſe quarters they divide into eight equal parts, called points, and each point into four equal parts, called quarter-points.

So that the horizon is divided into 32 points, which are called *rhumbs*, or *winds*; to each wind is aſſigned a name, which ſhews from what point of the horizon the wind blows.

The points of North, South, Eaſt, and Weſt, are called *cardinal points*; and are at the diſtance of 90 degrees, or 8 points, from one another.

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* Near 14000 times. *Wallis*'s Hydroſ. p. 13.

† Into the [illegible] part. Phil. Tranſ. N° 181.

83. Winds are either conſtant or variable, general or particular.

*Conſtant winds* are ſuch as blow the ſame way, at leaſt for one or more days; and *variable winds* are ſuch as frequently ſhift within a day.

A *general wind* is that which blows the ſame way over a large tract of the Earth, almoſt the whole year.

A *particular wind* is that which blows in any place, ſometimes one way, and ſometimes another, indifferently.

If the wind blows gently, it is called a breeze; if it blows harder, it is called a gale, or a ſtiff gale; and if it blows very hard, it is called a ſtorm[*].

84. The following obſervations on the wind have been made by ſkillful ſeamen; and particularly by the great Dr. Halley.

1ſt. Between the limits of 60 degrees, namely, from 30° of north latitude to 30° of ſouth latitude, there is a conſtant eaſterly wind throughout the year, blowing on the Atlantic and Pacific Oceans; and this is called the *trade-wind.*

For as the Sun, in moving from eaſt to weſt, heats the air more immediately under him, and thereby expands it; the air to the eaſtward is conſtantly ruſhing towards the weſt to reſtore the equilibrium, or natural ſtate of the atmoſphere; and this occaſions a perpetual eaſterly wind in thoſe latitudes.

2d. The trade-winds near the northern limits blow between the north and eaſt; and near the ſouthern limits they blow between the ſouth and eaſt.

For as the air is expanded by the heat of the Sun near the equator; therefore the air from the northward and ſouthward will both tend towards the equator to reſtore the equilibrium. Now thoſe motions from the north and ſouth, joined with the foregoing eaſterly motion, will produce the motions obſerved near the ſaid limits between the north and eaſt, and between the ſouth and eaſt.

3d. Theſe general motions of the wind are diſturbed on the continents, and near their coaſts.

For the nature of the ſoil may cauſe the air to be either heated or cooled; and hence will ariſe motions that may be contrary to the foregoing general one.

4th. In ſome parts of the Indian ocean there are periodical winds, called Monsoons; that is, ſuch as blow one half the year one way, and the other half-year the contrary way.

For air that is cool and denſe, will force the warm and rarefied air in a continual ſtream upwards, where it muſt ſpread itſelf to preſerve the equilibrium: So that the upper courſe or current of the air ſhall be contrary to the under current; for the upper air muſt move from thoſe parts where the greateſt heat is; and ſo, by a kind of circulation, the N. E. trade-wind below will be attended with a S. W. above; and a S. E. below with a N. W. above: And this is confirmed by the experience of ſeamen, who, as ſoon as they get out of the trade-winds, immediately find a wind blowing from the oppoſite quarter.

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[*] The ſwiftneſs of the wind in a ſtorm is not more than 50 or 60 miles in an hour; and a common briſk gale is about 15 miles an hour.

5th. In the Atlantic Ocean near the coaſts of Africa, at about 100 leagues from the ſhore, between the latitudes of 28° and 10° north, ſeamen conſtantly meet with a freſh gale of wind blowing from the N. E.

6th. Thoſe bound to the Caribee Iſlands, acroſs the Atlantic Ocean, find, as they approach the American ſide, that the ſaid N. E. wind becomes eaſterly; or ſeldom blows more than a point from the eaſt, either to the northward or ſouthward.

Theſe trade-winds, on the American ſide, are extended to 30, 31, or even to 32° of N. latitude; which is about 4° farther than what they extend to on the African ſide. Alſo to the ſouthward of the equator, the trade-winds extend 3 or 4 degrees farther towards the coaſt of Braſil on the American ſide, than they do near the Cape of Good Hope on the African ſide.

7th. Between the latitudes of 4° North, and 4° South, the wind always blows between the ſouth and eaſt: On the African ſide the winds are neareſt to the ſouth; and on the American ſide neareſt to the eaſt. In theſe ſeas Dr. Halley obſerved, that when the wind was eaſtward, the weather was gloomy, dark, and rainy, with hard gales of wind: but when the wind veered to the ſouthward, the weather generally became ſerene with gentle breezes next to a calm.

Theſe winds are ſomewhat changed by the ſeaſons of the year; for when the Sun is far northward, the Braſil S. E. wind gets to the ſouth, and the N. E. wind to the eaſt; and when the Sun is far ſouth, the S. E. wind gets to the eaſt, and the N. E. winds on this ſide of the equator veer more to the north.

8th. Along the coaſt of Guinea, from Sierra Leon to the Iſland of St. Thomas (under the equator) which is above 500 leagues, the ſoutherly and ſouth-weſt winds blow perpetually. For the S. E. trade-wind having paſſed the equator, and approaching the Guinea coaſt within 80 or 100 leagues, inclines towards the ſhore, and becomes ſouth, then S. E. and by degrees, as it comes near the land, it veers about to ſouth, S. S. W. and in with the land it is S. W. and ſometimes W. S. W. This tract is troubled with frequent calms, violent ſudden guſts of wind, called Tornadoes, blowing from all points of the horizon.

The reaſon why the wind ſets in weſt on the coaſt of Guinea, is, in all probability, owing to the nature of the coaſt, which being greatly heated by the Sun, rarefies the air exceedingly; and conſequently the cool air from off the ſea will keep ruſhing in to reſtore the equilibrium.

9th. Between the 4th and 10th degrees of north latitude, and between the longitudes of Cape Verd, and the eaſtermoſt of the Cape Verd Iſles, there is a tract of ſea which ſeems to be condemned to perpetual calms, attended with terrible thunder and lightnings, and ſuch frequent rains, that this part of the ſea is called the *Rains*. Ships in ſailing theſe 6 degrees have been ſometimes detained whole months, as it is reported.

The cauſe of this ſeems to be, that the weſterly winds ſetting in on this coaſt, and meeting the general eaſterly wind in this tract, balance each other, and ſo cauſe the calms; and the vapours carried thither by each wind meeting and condenſing, occaſion the almoſt conſtant rains.

The laſt three obſervations ſhew the reaſon of two things which mariners experience in ſailing from Europe to India, and in the Guinea trade

First. The difficulty which ſhips in going to the ſouthward, eſpecially in the months of July and Auguſt, find in paſſing between the coaſt of Guinea and Braſil, notwithſtanding the width of this ſea is more than 500 leagues. This happens, becauſe the S. E. winds at that time of the year commonly extend ſome degrees beyond the ordinary limits of 4° N. latitude; and beſides come ſo much ſoutherly, as to be ſometimes ſouth, ſometimes a point or two to the weſt; it then only remains to ply to windward. And if, on the one ſide, they ſteer W. S. W. they get a wind more and more eaſterly; but then there is a danger of falling in with the Braſilian coaſt, or ſhoals; and if they ſteer E. S. E. they fall into the neighbourhood of the coaſt of Guinea, from whence they cannot depart without running eaſterly as far as the iſland of St. Thomas; and this is the conſtant practice of all the Guinea ſhips.

Secondly. All ſhips departing from Guinea for Europe, their direct courſe is northward; but on this courſe they cannot go, becauſe the coaſt bending nearly eaſt and weſt, the land is to the northward: Therefore as the winds on this coaſt are generally between the S. and W. S. W. they are obliged to ſteer S. S. E. or ſouth, and with theſe courſes they run off the ſhore; but in ſo doing they always find the winds more and more contrary; ſo that when near the ſhore, they can lie ſouth, at a greater diſtance they can make no better than S. E., and afterwards E. S. E.; with which courſes they commonly fetch the iſland of St. Thomas and Cape Lopez, where finding the winds to the eaſtward of the ſouth, they ſail weſterly with it, until they come to the latitude of 4 degrees ſouth, where they find the S. E. wind blowing perpetually.

On account of theſe general winds, all thoſe who uſe the Weſt India trade, even thoſe bound to Virginia, reckon it their beſt courſe to get as ſoon as they can to the ſouthward, that ſo they may be certain of a fair and freſh gale to run before to the weſtward. And for the ſame reaſon the homeward bound ſhips from America endeavour to gain the latitude of 30 degrees, where they firſt find the winds begin to be variable; though the moſt ordinary winds in the north Atlantic Ocean come from between the ſouth and weſt.

10th. Between the ſouthern latitudes of 10 and 30 degrees in the Indian Ocean, the general trade-wind about the S. E. by S. is found to blow all the year long in the ſame manner, as in the like latitude in the Ethiopic Ocean: and during the ſix months from May to December, theſe winds reach to within 2 degrees of the equator; but during the other ſix months, from November to June, a N. W. wind blows in the tract lying between the 3d and 10th degeees of ſouthern latitude, in the meridian of the north end of Madagaſcar: and between the 2d and 12th degree of ſouth latitude, near the longitude of Sumatra and Java.

11th. In the tract between Sumatra and the African coaſt, and from 3 degrees of ſouth latitude quite northward to the Aſiatic coaſts, including the Arabian Sea and the Gulf of Bengal, the Monſoons blow from September to April on the N. E.; and from March to October on the S. W. In the former half year the wind is more ſteady and gentle, and the weather clearer than in the latter ſix months; and the wind is more ſtrong and ſteady in the Arabian Sea than in the Gulf of Bengal.

12th. Between the Iſland of Madagaſcar and the coaſt of Africa, and thence northward as far as the equator, there is a tract, where from April to October there is a conſtant freſh S. S. W. wind; which to the northward changes into the W. S. W. wind, blowing at that time in the Arabian Sea.

13th. To the eaſtward of Sumatra and Malacca, on the north of the equator, and along the coaſts of Cambodia and China, quite through the Phillippines as far as Japan, the Monſoons blow northerly and ſoutherly, the northern ſetting in about October or November, and the ſouthern about May; the winds are not quite ſo certain as thoſe in the Arabian Seas.

14th. Between Sumatra and Java to the weſt, and New Guinea to the eaſt, the ſame northerly and ſoutherly winds are obſerved; but the firſt half year Monſoon inclines to the N. W. and the latter to the S. E. Theſe winds begin a month or ſix weeks after thoſe in the Chineſe Seas ſet in, and are quite as variable.

15th. Theſe contrary winds do not ſhift from one point to its oppoſite all at once; in ſome places the time of the change is attended with calms, in others by variable winds. And it often happens on the ſhores of Coromandel and China towards the end of the Monſoons, that there are moſt violent ſtorms, greatly reſembling the hurricanes in the Weſt Indies; when the wind is ſo vaſtly ſtrong, that hardly any thing can reſiſt its force.

All navigation in the Indian Ocean muſt neceſſarily be regulated by theſe winds; for if mariners ſhould delay their voyages till the contrary Monſoon begins, they muſt either ſail back, or go into harbour, and wait for the return of the trade-wind.

## SECTION VII.

## *Of the Tides.*

85. A Tide is that motion of the waters in the ſeas and rivers, by which they are found regularly to riſe and fall.

The general cauſe of the tides was diſcovered by Sir Iſaac Newton, and is deduced from the following conſiderations.

86. Daily experience ſhews, that all bodies thrown upwards from the Earth, fall down to its ſurface in perpendicular lines; and as lines perpendicular to the ſurface of a ſphere tend towards the center, therefore the lines, along which all heavy bodies fall, are directed towards the Earth's center.

As theſe bodies apparently fall by their weight, or gravity; therefore the law by which they fall, is called the Law of Gravitation.

87. A piece of glaſs, amber, or ſealing-wax, and ſome other things, being rubbed againſt the palm of a hand, or againſt a woollen cloth, until they are warmed, will draw bits of paper, or other light ſubſtances, towards them, when held ſufficiently near thoſe ſubſtances.

Alſo a magnet, or loadſtone, being held near the filings of iron or ſteel, or other ſmall pieces of theſe metals, will draw them to itſelf; and a piece of hammered iron or ſteel, that has been rubbed by a magnet, will have a like property of drawing iron or ſteel to itſelf. And this property is called Attraction.

88. Now as bodies by their gravity fall towards the Earth, it is not improper to ſay the Earth attracts thoſe bodies; and therefore in reſpect to the earth, the words gravitation and attraction may be uſed one for the other, as by them is meant no more than the power, or law, by which bodies tend towards its center.

And it is likely, that this is the cauſe, why the parts of the Earth adhere and keep cloſe to one another.

89. The incomparable Sir Iſaac Newton, by a ſagacity peculiar to himſelf, diſcovered from many obſervations, that this law of gravitation or attraction was univerſally diffuſed throughout the ſolar ſyſtem; and that the regular motions obſerved among the heavenly bodies were governed by this ſame principle; ſo that the Earth and Moon attracted each other, and both of them are attracted by the Sun. He diſcovered alſo that the force of attraction, exerted by theſe bodies one on the other, was leſs and leſs as the diſtance increaſed, in proportion to the ſquares of thoſe diſtances; that is, the power of attraction at double the diſtance was four times leſs, at triple the diſtance nine times leſs, at quadruple the diſtance ſixteen times leſs, and ſo on.

90. Now as the Earth is attracted by the Sun and Moon, therefore all the parts of the Earth will not gravitate towards its center in the ſame manner, as if thoſe parts were not affected by ſuch attractions. And it is very evident that, were the Earth entirely free from ſuch actions of the Sun and Moon, the ocean being equally attracted towards its center, on all ſides, by the force of gravity, would continue in a perfect ſtagnation without ever ebbing or flowing. But ſince the caſe is otherwiſe, the water in the ocean muſt needs riſe higher in thoſe places where the Sun and

Moon diminiſh its gravity, or where the Sun and Moon have the greateſt attraction.

As the force of gravity muſt be diminiſhed moſt in thoſe parts of the Earth to which the Moon is neareſt, that is, where ſhe is in the ZENITH, or vertical, and, conſequently, where her attraction is moſt powerful; therefore the waters in ſuch places will riſe higheſt, and it will be *full ſea* or *flood* in ſuch places.

91. *The parts of the Earth directly under the Moon, and alſo thoſe in her* NADIR, *viz. ſuch places as are diametrically oppoſite to thoſe where the Moon is in the Zenith, will have the flood, or high water, at the ſame time.*

FOR either half of the Earth would gravitate equally towards the other half, were they free from all external attraction.

But by the action of the moon, the gravitation of one half-earth towards its center is diminiſhed, and of the other is increaſed.

Now in the half-earth next the Moon, the parts in the zenith being moſt attracted, and thereby their gravitation towards the Earth's center diminiſhed, the waters in theſe parts muſt be higher than in any other part of this half-earth.

And in the half-earth fartheſt from the Moon, the parts in the nadir being leſs attracted by the Moon than the parts nearer to her, gravitate leſs towards the Earth's center, and conſequently the waters in theſe parts muſt be higher than they are in any other part of this half-earth.

92. *Thoſe parts of the Earth, where the Moon appears in the horizon, or is* 90 *degrees diſtant from the zenith and nadir, will have the ebbs or loweſt waters.*

For as the waters in the zenith and nadir riſe at the ſame time, the waters in their neighbourhood will preſs towards thoſe places to maintain the equilibrium; and to ſupply the places of theſe, others will move the ſame way, and ſo on to places of 90° diſtant from the ſaid zenith and nadir; conſequently, in thoſe places where the Moon appears in the horizon, the waters will have more liberty to deſcend towards the center; and therefore in thoſe places they will be the loweſt.

93. Hence it plainly follows, that the ocean, if it covered the ſurface of the Earth, muſt put on a ſpheroidal, or egg-like figure; in which the longeſt diameter paſſes through the place where the Moon is vertical; and the ſhorteſt diameter, will be, in the horizon of that place. And as the Moon apparently ſhifts her poſition from eaſt to weſt in going round the Earth every day, the longer diameter of the ſpheroid following her motion, will occaſion the two floods and ebbs obſervable in about every 25 hours, which is about the length of a lunar day, or the time ſpent between the Moon's leaving the meridian of any place, and coming to it again.

94. Hence the greater the Moon's meridian altitude is at any place, the greater theſe tides will be which happen when ſhe is above the horizon; and the greater her meridian depreſſion is, the greater will thoſe tides be which happen while ſhe is below the horizon.

Moreover the ſummer day, and the winter night-tides have a tendency to be higheſt, becauſe the Sun's ſummer altitude and his winter depreſſion are greateſt, but this is more eſpecially to be noted when the

Moon has north declination in ſummer, and ſouth declination in winter.

95. *The time of high water is not preciſely at the time of the Moon's coming to the meridian, but about an hour after.*

For the moon acts with ſome force after ſhe has paſt the meridian, and by that means adds to the libratory, or waving motion, which ſhe had put the water into, whilſt ſhe was in the meridian; in the ſame manner as a ſmall force applied upwards to a ball, already raiſed to ſome height, will raiſe it ſtill higher.

96. *The tides are greater than ordinary twice every month; that is, about the times of new and full Moon: theſe are called* SPRING-TIDES.

For at theſe times the actions of both Sun and Moon concur to draw in the ſame right line; and therefore the ſea muſt be more elevated. In *conjunction*, or when the Sun and moon are on the ſame ſide of the Earth, they both conſpire to raiſe the water in the zenith, and conſequently in the nadir. And when the Sun and Moon are in *oppoſition*, that is, when the Earth is between them, whilſt one makes high water in the zenith and nadir, the other does the ſame in the nadir and zenith.

97. *The tides are leſs than ordinary twice every month; that is, about the times of the firſt and laſt quarters of the Moon: and theſe are called* NEAP TIDES.

BECAUSE in the quarters of the Moon the Sun raiſes the water where the Moon depreſſes it; and depreſſes where the Moon raiſes the water; ſo that the tides are made only by the difference of their actions.

It muſt be obſerved, that the ſpring tides happen not directly on the new and full moons, but rather a day or two after, when the attractions of the Sun and moon have conſpired together for a conſiderable time. In like manner the neap-tides happen a day or two after the quarters, when the moon's attraction has been leſſened by the Sun for ſeveral days together.

98. *When the Moon is in her* PERIGÆUM, *or neareſt approach to the Earth, the tides increaſe more than in the ſame circumſtances at other times.*

FOR according to the laws of gravitation, the Moon muſt attract moſt when ſhe is neareſt to the Earth.

99. *The ſpring-tides are greater about the time of the* EQUINOXES, *that is, about the latter ends of March and September, than at other times of the year; and the neap-tides then are leſs.*

BECAUSE the longer diameter of the ſpheroid, or the two oppoſite floods, will at that time be in the Earth's equator; and conſequently will deſcribe a great circle of the Earth; by the diurnal rotation of which thoſe floods will move ſwifter, deſcribing a great circle in the ſame time they uſed to deſcribe a leſſer circle parallel to the equator, and conſequently the waters being thrown more forcibly againſt the ſhores, muſt riſe higher.

100. The following obſervations have been made on the riſe of the tides.

1ſt. The morning tides generally differ in their riſe from the evening-tides.

2d. The new and full Moon ſpring-tides riſe to different heights.

3d. In winter the morning-tides are higheſt.

4th. In ſummer the evening-tides are higheſt.

So that after a period of about ſix months the order of the tides are inverted; that is, the riſe of the morning and evening-tides will change places, the winter morning high-tides becoming the ſummer evening high-tides.

Some of theſe effects ariſe from the different diſtances of the Moon from the Earth after a period of ſix months, when ſhe is in the ſame ſituation with reſpect to the Sun; for, if ſhe is in perigee at the time of new moon, in about ſix months after ſhe will be in perigee about the time of full Moon.

Theſe particulars being known, a pilot may chuſe that time, which is moſt convenient for conducting a ſhip in or out of a port, where there is not ſufficient depth at low-water.

Small inland ſeas, ſuch as the Mediterranean and Baltic, are little ſubject to tides; becauſe the action of the Sun and Moon is always nearly equal at both extremities of ſuch ſeas. In very high latitudes the tides are alſo very inconſiderable. For the Sun and Moon acting towards the equator, and always raiſing the water towards the middle of the torrid zone, the neighbourhood of the poles muſt conſequently be deprived of thoſe waters, and the ſea muſt, within the frigid zones, be low, with relation to other parts.

101. All the things hitherto explained would exactly obtain, were the whole ſurface of the Earth covered with ſea. But ſince it is not ſo, and there being a multitude of iſlands, beſides continents, lying in the way of the tide, which interrupt its courſe; therefore in many places near the ſhores, there ariſes a great variety of other appearances beſide the foregoing ones which require particular ſolutions, in which the ſituations of the ſhores, ſtraits, ſhoals, winds, and other things, muſt neceſſarily be conſidered. For inſtance:

102. As the ſea * has no viſible paſſage between Europe and Africa, let them be ſuppoſed to be one continent, extending from 78 degrees north to 34 degrees ſouth, the middle between theſe two would be in latitude 19 degrees north, near Cape Blanco, on the weſt coaſt of Africa. But it is impoſſible that the flood-tide ſhould ſet to the weſtward upon the weſtern coaſt of Africa (for the general tide following the courſe of the Moon muſt ſet from eaſt to weſt), becauſe the continent, for above 50 degrees, both northward and ſouthward, bounds that ſea on the eaſt; therefore if any regular tide, proceeding from the motion of the ſea, from eaſt to weſt, ſhould reach this place, it muſt come either from the north of Europe ſouthward, or from the ſouth of Africa northward, to the ſaid latitude upon the weſt coaſt of Africa.

103. This opinion is further corroborated, or rather fully confirmed by common experience, which ſhews that the flood-tide ſets to the ſouthward along the weſt coaſt of *Norway*, from the north cape to the *Naze*, or entrance of the *Baltic Sea*, and ſo proceeds to the ſouthward along the eaſt coaſt of *Great Britain*, and in its paſſage ſupplies all thoſe ports with the tide one after another, the coaſt of *Scotland* having the tide firſt, be-

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* *Varenius's* Geogr. p. 815. Edit. 1734.

cauſe it proceeds from the northward to the ſouthward; and thus on the days of the full, or change, it is high-water at *Aberdeen* at 12 h. 45 m. but at *Tinmouth Bar*, the ſame day, not till 3 h. From thence rolling to the ſouthward, it makes high-water at the *Spurn* a little after 5 h.; but not till 6 h. at *Hull*, by reaſon of the time required for its paſſage up the river; from thence paſſing over the *Welbank* into *Yarmouth Road*, it makes high-water there a little after 8 h. but in the *Pier* not till 9 h., and it requires near an hour more to make high-water at *Yarmouth* town; in the mean time ſetting away to the ſouthward, it makes high-water at *Harwich* at 10 h. 30 m., at the *Nore* at 12, at *Graveſend* 1 h. 30 m., and at *London* at 3 h. all on the ſame day. And although this may ſeem to contradict that hypotheſis of the natural motion of the tide being from eaſt to weſt, yet as no tide can flow weſt from the main continent of *Norway* or *Holland*, or out of the *Baltic*, which is ſurrounded by the main continent, except at its entrance, it is evident that the tide we have been now tracing by its ſeveral ſtages from *Scotland* to *London*, is ſupplied by the tide, the original motion of which is from eaſt to weſt; yet as water always inclines to the level, it will in its paſſage fall towards any other point of the compaſs, to fill up vacancies where it finds them, and yet not contradict, but rather confirm, the firſt hypotheſis.

104. While the tide, or high-water, is thus gliding to the ſouthward along the eaſt coaſt of *England*, it alſo ſets to the ſouthward along the weſt coaſts of *Scotland* and *Ireland*, a branch of it falls into *St. George*'s channel, the flood running up north-eaſt, as may be naturally inferred from its being high-water at *Waterford* above three hours before it is high-water at *Dublin*, or thereabout, on that coaſt; and it is three quarters of an hour ebb at *Dublin* before it is high-water at the *Iſle of Man*, &c.

But not to proceed farther in particulars than to our own, or the *Britiſh* channel, we find the tides ſet to the ſouthward from the coaſt of *Ireland*, and in their paſſage a branch falls into the *Britiſh* channel between the *Lizard* and *Uſhant*; this progreſs to the ſouthward may be eaſily proved, by its being high-water on the day of the full and changes at *Cape Clear* a little after 4 h., and at *Uſhant* about 6 h., and at the *Lizard* after 7 h. The *Lizard* and *Uſhant* may properly be called the chaps of the *Britiſh* channel, between which the flood ſets to the eaſtward along the coaſts of *England* and *France*, till it comes to the *Goodwin* or *Galloper*, where it meets the tide before mentioned, which ſets to the ſouthward along the eaſtern coaſt of *England* to the *Downs*, where theſe two tides meeting, contribute very much towards ſending a powerful tide up the river *Thames* to *London*. And when the natural courſe of theſe two tides has been interrupted by a ſudden ſhift of the wind, by which means that tide was accelerated which had before been retarded, and that driven back which was before hurried in by the wind, it has been known to occaſion twice high-water in 3 or 4 hours, which, by thoſe who did not conſider this natural cauſe, was looked upon as a prodigy.

105. But now it may be objected, that this courſe of the flood-tide, eaſt, or eaſt-north-eaſt, up the channel, is quite contrary to the hypotheſis of the general motion of the tides being from eaſt to weſt, and conſequently of its being high-water where the moon is vertical, or any where elſe in the meridian.

In anſwer. This particular direction of any branch of the tide does not at all contradict the general direction of the whole; a river, with a weſtern courſe, may ſupply canals which wind north, ſouth, or even eaſt, and yet the river keep its natural courſe; and if the river ebbs and flows, the canals ſupplied by it would do the ſame, although they did not keep exact time with the river, becauſe it would be flood, and the river advanced to ſome height, before the flood reached the farther part of the canals; and the more remote the canals are, the longer time it would require; and it may be added, that if it was high-water in the river juſt when the moon was on the meridian, ſhe would be far paſt it before it could be high-water in the remoteſt part of thoſe canals, or ditches, and the flood would ſet according to the courſe of thoſe canals that received it, and could not ſet weſt up a canal of a different poſition; and as *St. George*'s channel, the *Britiſh* channel, &c. are no more in proportion to the vaſt ocean, than theſe canals are to a large navigable river; it will evidently follow, that among thoſe obſtructions and confinements the flood may ſet upon any other point of the compaſs as well as weſt, and may make high-water at any other time as well as when the Moon is upon the meridian, and yet no way contradict the general theory of the tides before aſſerted.

106. When the time of high-water at any place is, in general, mentioned, it is to be underſtood on the days of the ſyzygies, or days of new and full Moon, when the Sun and Moon paſs the meridian of that place at the ſame time. Among pilots it is cuſtomary to reckon the time of flood, or high-water, by the point of the compaſs the Moon is on at that time, allowing $\frac{3}{4}$ of an hour for each point: Thus on the full and change days, in places where it is flood at noon, the tide is ſaid to flow north and ſouth, or at 12 o'clock; in other places, on the ſame days, where the Moon bears 1, 2, 3, 4, or more points to the eaſt or weſt of the meridian, when it is high-water, the tide is ſaid to flow on ſuch point: Thus, if the Moon bears S. E. at flood, it is ſaid to flow S. E. and N.W., or 3 hours before the meridian, that is, at 9 o'clock; if it bears S. W., it flows S. W. and N. E., or at 3 hours after the meridian; and in like manner for other points of the Moon's bearing.

107. In ſome places it is high water on the ſhore, or by the ground, while the tide continues to flow in the ſtream, or offing; and according to the length of time it flows longer in the ſtream than on the ſhore, it is ſaid to flow tide, and ſuch part of tide; allowing 6 hours to a tide; Thus 3 hours longer in the offing than on the ſhore, make tide and half-tide; an hour and half longer makes tide and quarter-tide; three quartees of an hour longer make tide and half-quarter tide; &c.

108. Along an extent of coaſt next to the ocean, ſuch as the weſtern coaſt of Africa, and the eaſtern and weſtern coaſts of South America, it is generally high-water about the ſame hour. But ports on the coaſts of narrow ſeas, or within land, have the times of their high-water ſooner or later on the ſame day, according as thoſe ports are farther removed from the tide's way, or have their entrances more or leſs contracted.

109. The times of high-water, in any place, fall about the ſame hours after a period of 15 days nearly, which is the time between one ſpring-

tide and another: and during that period, the times of high-water fall each day later by about 48 minutes.

110. From the obſervations of different perſons, the times when it is high-water on the days of the new and full Moon, on moſt of the ſea-coaſts of Europe, and many other places, have been collected. Theſe times are uſually put in a table againſt the names of the places, digeſted in an alphabetical order; the like is followed in this work, only they are not given in a table by themſelves, but make a column in the table of the latitudes and longitudes of places; which column is not filled up againſt many of the names in that table, for want of a ſufficient collection of obſervations. This may be ſupplied by thoſe who have opportunity and inclination.

The uſe of ſuch a table is to find the time when it is high-water at any of the places mentioned in it. But as this depends upon knowing the time when the Moon comes to the meridian, and this on the Moon's age, and this on the knowledge how to find ſome of the common notes in the Calendar; therefore it was thought convenient to introduce in this place a compendium of Chronology, containing the ſeveral articles above mentioned.

## SECTION VIII.

## *Of Chronology.*

111. CHRONOLOGY is the art of eſtimating, and comparing together, the times when remarkable events have happened, ſuch as are related in hiſtory.

An ÆRA, or EPOCHA, is a time when ſome memorable tranſaction occurred; and from which ſome nations date and meaſure their computations of time.

| | Years of the Julian Period. | Years before Chriſt. |
|---|---|---|
| Some have dated their events from the creation of the world, and ſuppoſe it to have happened | 710 | 4004 |
| Others, from the deluge, or flood — — | 2366 | 2348 |
| The Greeks, from their Olympiads of 4 years each — — — — | 3938 | 776 |
| The Romans, from the building of Rome — | 3961 | 753 |
| The Aſtronomers, from Nabonaſſar king of Babylon — — — — | 3967 | 747 |
| Some Hiſtorians, from the death of Alexander the Great — — — — | 4390 | 324 |
| The Chriſtians, from the birth of Chriſt — | 4713 | A. D. |
| The Mahometans, from the flight of Mahomet, called the Hegira — — — | 5335 | 622 |

In order to aſſign the diſtance between theſe, and other events, the ancients found it neceſſary to have a large meaſure of time, the limits of which were naturally pointed out to them by the return of the ſeaſons and this interval they called a year.

112. The moſt natural diviſion of the year appeared to them to be the returns of the new Moon; and as they obſerved 12 new moons to happen within the time of the general return of the ſeaſons, they therefore firſt divided the year into 12 equal parts, which they called months; and as they reckoned about 30 returns of morning and evening between the times of the new moon and new moon, therefore they reckoned their month to conſiſt of 30 days, and their year, or 12 months, to contain 360 days; and this is what is generally underſtood by the lunar year of the ancients.

113. But in length of time it was found, that this year did not agree with the courſe of the ſun, the ſeaſons gradually falling later in the year than they had been formerly obſerved; this put them upon correcting the method of eſtimating their year, which they did from time to time, by taking a day or two from the month, as often as they found it too long for the courſe of the moon; and by adding a month, called an intercalary month, as often as they found 12 lunar months to be too ſhort for the return of the four ſeaſons and fruits of the Earth. This kind of year, ſo corrected from time to time by the prieſts, whoſe buſineſs it was, is what is to be underſtood by the *luni-ſolar* year; which was anciently uſed in moſt nations, and is ſtill among the *Arabs* and *Turks*.

As a great variety of methods was uſed in different countries to correct the length of the year, ſome by intercalating days in every year, and others by inſerting months and days in certain returns, or periods of years; and theſe different methods being obſerved by ſome eminent men to create a conſiderable difference in the accounts of time kept by neighbouring nations, introducing a confuſion in the chronological order of times, they therefore invented certain periods of years, called *cycles*, with which they compared the moſt memorable occurrences.

114. At length *Julius Cæſar* obſerving the confuſion which this variety of accounts occaſioned, and knowing that his order, as Emperor of the Romans, would be followed by a very conſiderable part of the world; he therefore, about 40 years before the birth of Chriſt, decreed that every fourth year ſhould conſiſt of 366 days, and the other three of 365 days each. This he did in conſequence of the information given him by *Soſigenes*, an eminent mathematician of Alexandria in Egypt; for at that time the philoſophers of the Alexandrinian ſchool knew from a length of experience, that the year conſiſted of about 365 days and a quarter; and this was the reaſon of ordering every fourth year to conſiſt of 366 days, thereby compenſating for the quarter day omitted in each of the preceding three years. This method, called the *Julian Account*, or *Old Stile*, continued to be uſed in moſt Chriſtian ſtates until the year 1582.

The Aſtronomers, ſince the time of *Julius Cæſar*, have found that the true length of the ſolar year, or common year, is 365 days, 5 hours, 48 minutes, 55 ſeconds, nearly; being leſs than the Julian, of 365 days, 6 hours, by about 11 minutes, 5 ſeconds, which is about the 130th part of 86400, the ſeconds contained in a day; ſo that in 130 Julian years there would be one day gained above 130 ſolar years; and in 400 Julian years there would be gained 3 days, 1 hour, 53 minutes, 20 ſeconds; conſequently one day omitted in every 130 common years, would bring the current account of time to agree very nearly with the motion of the Sun.

115. In the year of our Lord 325, when the *Council of Nice* ſettled the day for the celebration of Eaſter, the *Vernal Equinox* (that is, the day in the ſpring when the Sun roſe at ſix and ſet at ſix) happened on the 21ſt of March; but about the year 1580 the *Vernal Equinox* fell on the 11th of March, making a difference of about 10 days. Now *Gregory* the XIIIth, who was Pope at that time, obſerving that this difference of time in the falling out of the *Equinox* would affect the intention of the *Nicene Council*, concerning the time of the year appointed by them for the celebration of Eaſter; he therefore, in the year 1581, publiſhed a *Bull*, ordering that in the year 1582, the 5th of October ſhould be called the 15th, and ſo on; thus the 10 days taken off would cauſe the time of the *Vernal Equinox* to fall on the 21ſt of March, as at the time of the *Nicene Council*: and becauſe a little more than three days were gained in every 400 years by the *Julian* account; therefore to prevent any future difference, every century, or number of 100 years, not diviſible by 4, ſuch as 17 hundred, 18 hundred, 19 hundred, &c., ſhould contain only 365 days, which, by the *Julian* account, ſhould have contained 366; and the centuries diviſible by 4, ſuch as 16 hundred, 20 hundred, 24 hundred, &c. ſhould be leap-years of 366 days; and thus the three days would be omitted, which the anticipation of the equinoxes would gain in 400 years; the ſmall exceſs of 1 hour, 53 minutes, 20 ſeconds, not amounting to a whole day in leſs than 5082 years, being rejected as inconſiderable; the intermediate years to be reckoned as they uſed to be in the *Julian* or *Old Stile*. This Pope's alteration, called the *Gregorian* or *New Stile*, was received in moſt of the Chriſtian ſtates: But ſome at that time choſe to continue the *Julian*; among whom were the Engliſh; and they, in the year 1752, reformed their account, and introduced among themſelves a new one, that nearly correſponds with the *Gregorian*.

A certain length of the year being once ſettled, and a regular account of time in conſequence of it being ſo fitted, as to conform invariably to the ſeaſons; it was natural for the States who received ſuch account, to fit to it a regiſter of the days in each month, and to note the days when any remarkable occurrence was to be commemorated; and ſuch regiſter has obtained the name of *Calendar*.

116. The Calendar now in uſe among moſt of the Chriſtian ſtates conſiſts of 12 months, called *January*, *February*, *March*, *April*, *May*, *June*, *July*, *Auguſt*, *September*, *October*, *November*, *December*; theſe months are called *civil*, the number of days in each may be readily remembered by the following rule.

117. Thirty days has *September*, *April*, *June*, and *November*;
*February* has twenty-eight alone: All the reſt have thirty-one;
When the year conſiſts of 365 days: But in every fourth, which conſiſts of 366 days, *February* has 29. This additional day was interca-

lated after the 24th of *February*, which in the Old Roman Calendars was called the *ſixth of the calends of March*, and being this year reckoned twice over, the year was called *Biſſextilis*, or LEAP-YEAR.

Beſide the months, time is alſo divided into weeks, days, hours, minutes, &c. a year containing 52 weeks, a week 7 days, a day 24 hours, an hour 60 minutes, &c.

In the Calendars it has been uſual to mark the ſeven days of the week with the ſeven firſt letters of the alphabet, always calling the firſt of January A, the 2d B, the 3d C, the 4th D, the 5th E, the 6th F, and the 7th G, and ſo on, throughout the year: and that letter anſwering to all the Sundays for a year, is called the DOMINICAL LETTER.

According to this diſpoſition, the letters anſwering to the firſt day of every month in the year, will be known by the following rule:

118. At Dover Dwells George Brown, Eſquire,
Good Caleb Finch, and David Frier;

Where the firſt letter of each word anſwers to the letter belonging to the firſt day of the months in the order from January to December.

119. A year of 365 days contains 52 weeks and 1 day; and a leap-year has 52 weeks and 2 days; therefore the firſt and laſt days of a common year fall on the ſame week-day, ſuppoſe it *Monday*; then the next year begins on a *Tueſday*, the next year on *Wedneſday*, and ſo on to the eighth year, which would be on *Monday* again, did every year contain 365 days; alſo the Dominical letter would run backwards through all the ſeven letters. But this round of ſeven years is interrupted by the leap-years; for then *February* having a 29th day annexed, the firſt Dominical letter in *March* muſt fall a day ſooner than in the common year; ſo that leap-year has two Dominical letters, the one (ſuppoſing G) ſerving for *January* and *February*, and the other, which is the preceding letter (F) ſerves for the reſt of the Sundays in that year.

120. The SOLAR CYCLE, or cycle of the Sun, is a period of 28 years, in which all the varieties of the Dominical letters will have happened, and they will return in the ſame order as they did 28 years before. At the birth of Chriſt 9 years had paſt in this cycle.

For the changes, were all the years common ones, would be 7;
But the interruptions by leap-year being every fourth year;
Therefore the changes will be 4 times 7, or 28 years.

This return of the Dominical letter is conſtant in the *Julian* account. But in the *Gregorian*, where among the complete centuries, or hundredth years, only every fourth is leap-year; the other three hundred years, which according to the *Julian*, would be leap-years, are by the *Gregorian* only common years of 365 days: in theſe all the letters muſt be removed one place forwards in a direct order; and either year, inſtead of having two Dominical letters (as ſuppoſe D, C), will have only one (as D), the Dominical letters moving retrograde.

121. The LUNAR CYCLE, or cycle of the Moon (and ſometimes called the *Metonic Cycle*, from *Meton*, an Athenian who invented it about 432 years before the time of *Chriſt)*, is a period of nineteen years, containing all the variations of the days on which the new and full Moons happen; after which they fall on the ſame days they did 19 years before.

The PRIME, or GOLDEN NUMBER, is the number of years elapſed in this cycle.

At the birth of *Chriſt* the golden number was 2.

For many years after the *Nicene Council*, it was thought that 19 ſolar years, or 228 ſolar months, were exactly equal to 235 ſynodical, or lunar, months: and that the ſame yearly *golden number* ſet in their calendars againſt the days when the new Moons happened throughout one lunar cycle, would invariably ſerve for the new Moons of correſponding years throughout every ſucceſſive lunar cycle. But later obſervations ſhew, that this cycle is leſs than 19 years, by a little more than one hour, twenty-eight minutes; therefore, the new Moons will, in a little leſs than 311 years, happen a day earlier than by the Metonic account; and conſequently all the feſtivals depending on the new Moons, will in time be removed into other ſeaſons of the year than thoſe which they fell in at their firſt inſtitution: thus the new Moons in the year 1750 happened above 4½ days earlier than the times ſhewn by the calendar. But were the golden numbers, when once prefixed to the proper new Moon days in a Metonic period, to be ſet a day earlier at the end of every 310,7 years, a pretty regular correſpondence might be preſerved between the ſolar and lunar years.

122. The EPACT of any year is the Moon's age the beginning of that year; that is, the days paſt ſince the laſt new Moon.

The time between new Moon and new Moon is in the neareſt round numbers 29½ days; therefore the lunar year conſiſting of 12 lunations muſt be equal to 354 days, which is 11 days leſs than the ſolar year of 365 days. Now ſuppoſing the ſolar and lunar years to begin together, the epact is 0; the beginning of the next ſolar year, the epact is 11; the 3d year the epact is 22; the fourth 33, &c. But when the epact exceeds 30, an intercalary month of 30 days is added to the lunar year, making it conſiſt of 13 months; ſo that the epact at the beginning of the 4th year is only 3, the 5th 14, the 6th 25, the 7th 36, or only 6, on account of the intercalary month; and ſo on to the end of the cycle of 19 years; at the expiration of which the ſame epacts would run over again, were the cycle perfect; and the epact would always be 11 times the prime.

123. By the Nicene Council it was enacted,

1ſt. That Eaſter-day ſhould be celebrated after the vernal equinox, which at that time happened on the 21ſt of March.

2d. That it ſhould be kept after the full, or 14th day of that Moon which happened firſt after the 21ſt of March in common years, and firſt after the 20th of March in leap-years.

3d. That the Sunday next following the 14th, or day of full Moon, ſhould be Eaſter-Sunday: which muſt always fall between the 20th or 21ſt of March, and 25th of April.

124. The MOON'S SOUTHING at any place is the time when ſhe comes to the meridian of that place, which is every day later by about $\frac{4}{5}$ of an hour; becauſe 24, the hours in a day, being divided by 30, the number of times which ſhe paſſes the meridian between new Moon and new Moon, will give $\frac{4}{5} = 48'$ for the retardation of her paſſage over the meridian in one day.

The Sun and Moon come to the meridian at the ſame time on the day of the change, or at new Moon; alſo the Moon comes to the oppoſite part of the ſame meridian, when ſhe is in oppoſition, or at full Moon. Hence between new and full ſhe comes to the meridian in the afternoon; at full ſhe comes to the meridian at mid-night; and when paſt the full, after mid-night, or in the morning.

125. The ROMAN INDICTION is a cycle of 15 years, uſed by the ancient Romans for the times of taxing the provinces. Three years of this cycle were elapſed at the birth of Chriſt.

The DIONYSIAN PERIOD is a cycle of 532 years, ariſing by multiplying together 28 and 19, the ſolar and lunar cycles; it was contrived by *Dionyſius Exiguus*, a Roman abbot, about the year of Chriſt 527, as a period for comparing chronological events.

The JULIAN PERIOD contains 7980 years; it ariſes by multiplying together 28, 19, 15, the cycles of the Sun, Moon, and Indiction. This was alſo contrived as a period for chronological matters; and its beginning falls 710 years before the uſual date of the creation.

On the principles laid down in the preceding articles depend the ſolution of the following problems.

126. PROBLEM I. *To find whether any given year is leap-year.*

RULE. Divide the given year by 4; if 0 remains, it is leap-year; if 1, 2, or 3 remains, it is ſo many years after.

Obſerving that the years 1800, 1900, 2100, &c. are common years.

| EXAM. I. *Is 1788 leap-year?* | EXAM. II. *Is 1787 leap-year?* |
|---|---|
| 4)1788(447 | 4)1787(446 |
| 0 | Remains 3 years paſt leap-year. |
| Remains 0, ſo it is leap-year. | |

127. PROBLEM II. *To find the years of the ſolar, lunar, and indiction cycles.*

RULE. To the given year add 9 for the ſolar, 1 for the lunar, 3 for the indiction: Divide the ſums in order by 28, 19, 15; the remainder in each ſhews the year of its reſpective cycle.

EXAM. *Required the years of the ſolar, lunar, and indiction cycles for the year 1787?*

| 1787 | 1787 | 1787 |
|---|---|---|
| 9 | 1 | 3 |
| 28)1796(64 | 19)1788(94 | 15)1790(119 |
| Remains 4 = ſolar cycle. | 2 = lunar cyc. or golden No. | 5 = indict. cycle. |

Whereby it appears that the year 1787 is the
4th year of the 65th ſolar cycle
2d year of the 95th lunar cycle
5th year of the 120th indiction cycle
ſince the birth of Chriſt.

128. Problem III. *To find the Dominical letter till the year* 1800.

Rule. To the given year add its fourth part, divide the ſum by 7; the remainder taken from 7 leaves the index of the letter in common years, reckoning A 1, B 2, C 3, &c.

But in leap-year, this letter and its preceding one (in the retrograde order which theſe letters take), are the Dominical letters.

Exam. I. *For the year* 1787.

4)1787
446

7)2233(319

0

Remains 0. Then 7—0=7=G.
So G is the Dominical letter.

Exam. II. *For the year* 1788.

4)1788
447

7)2235(319

2

Remains 2. Then 7—2=5=E
So F and E are the Dominical letters.

And in this manner were the following numbers computed.

For the Dominical letters during the 18th century.

| Solar cycles | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dom. letters | DC | B | A | G | FE | D | C | B | AG | F | E | D | CB | A |
| Solar cycles | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| Dom. letters | G | F | ED | C | B | A | GF | E | D | C | BA | G | F | E |

The year 1800 being a common year, ſtops the above order, and the following are the Dominical letters for the 19th century.

| Solar cycles | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dom. letters | ED | C | B | A | GF | E | D | C | BA | G | F | E | DC | B |
| Solar cycles | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| Dom. letters | A | G | FE | D | C | B | AG | F | E | D | CB | A | G | F |

129. Problem IV. *To find the Epact till the year* 1900.

Rule. Multiply the golden number for the given year by 11, and divide the product by 30; from the remainder take 11, and it will leave the epact.

If the remainder is leſs than 11, add 19 to it, and it gives the epact

Ex. I. *To find the epact for* 1783.

The Golden number is 17. (127)
Multiply by 11

30)187(6
180

Remains 7
Add 19

Conſequently the Epact is 26

Ex. II. *To find the epact for* 1786.

The Golden number is 01 (127)
Multiply by 11

11
Subtract 11

Remains the Epact =00

And thus might the following numbers be found.

| Gold. N° | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Epacts | 29 | 11 | 22 | 3 | 14 | 25 | 6 | 17 | 28 | 9 | 20 | 1 | 12 | 23 | 4 | 15 | 26 | 7 | 18. |

The epacts here proceed by the difference 11, rejecting thirties.

### 130. PROBLEM V. *To find the Moon's age.*

RULE. To the epact add the number and day of the month; their ſum, if under 30, is the Moon's age; but if it be above 30, take 30 from it, and the remainder will be the Moon's age, or days ſince the laſt conjunction.

The numbers of the months, or monthly epacts are the Moon's age at the beginning of each month, when the ſolar and lunar years begin together;

| And are | 0 | 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Jan. | Feb. | Mar. | Apr. | May, | June, | July, | Aug. | Sept. | Oct. | Nov. | Dec. |

EXAM. I. *What is the Moon's age on the* 14*th of October*, 1787*?*

The epact is 2 (129)
The N° of month 8
The day of the month 14

The ſum is 24 the Moon's age.

EXAM. II. *What is the Moon's age on the* 29*th of March*, 1786*?*

The epact is 0. (129)
Then 0+1+29=30 is the ſum of the epact, number and day of the month.
And 30—30=0 is the Moon's age.

131. The day of next new Moon is readily found by taking her age from 30.

The day of new Moon in any month is equal to the difference between the ſum of the year's and month's epacts, and 30. Thus;

On March 29, the Moon is 0 days old.
So that new Moon is on the 29th.
Now 0+1=1, is the ſum of the epacts.
Then 30—1=29, the day of new Moon, as it ſhould be.

### 132. PROBLEM VI. *The day of the month in any year being given, to know on what week-day it will fall.*

RULE. Find the Dominical letter (128): alſo the week day on which the firſt of the propoſed month falls (118); and hence the name of the propoſed day of the month will be known; obſerving that the 1ſt, 8th, 15th, 22d, and 29th days of any month fall on the ſame week-days.

EX. I. *On what day of the week does the* 14*th of Oct. fall, in* 1787*?*

The Dominical letter is G. (128)
The 1ſt of October is A, (118)
Therefore October 7th is Sunday.
Conſequently 14th is alſo Sunday.

EX. II. *In* 1788, *on what week-day does the* 20*th of March fall?*

The Dominical letter is E. (128)
The 1ſt of March is D. (118)
Then March 2d is Sunday.
And ſo March 20th is Thurſday.

133. Problem VII. *To find when Eaſter-day will fall in any year between 1700 and 1899.*

Rule. Find what day that new Moon falls on which is neareſt to the 21ſt of March in common years, or to the 20th in leap-years; then the Sunday next after the full, or 15th day of that new Moon, will be Eaſter-day.

If the 15th day fall on a Sunday, the next Sunday is Eaſter-day.

Ex. I. *When does Eaſter-day fall in the year* 1787?
The Dominical letter is G. (128)
March 21, Moon's age is 3. (130)
New Moon on March 18.
The 15th day is April 2.
April the 1ſt is G, on Sunday.
Then Eaſter-Sunday is April 8th.

Ex. II. *Required the time of Eaſter-day in the year* 1788?
The Dominical letter is E. (128)
March 20, Moon's age 13. (130)
New Moon on March 7th.
Full Moon on March 22.
March 1ſt is D, on Saturday.
Then Eaſter-Sunday is March 23d.

134. Eaſter-day is always found by the Paſchal full Moons, and theſe are readily found in the following curious table, which was communicated to the Royal Society in the year 1750, by the Earl of Macclesfield, and publiſhed in the Philoſophical Tranſactions for the ſame year; and its uſe ſhewn in the following precepts.

“ To find the day, on which the Paſchal limit, or full Moon, falls in
“ any given year; look, in the column of golden numbers belonging to
“ that period of time wherein the given year is contained, for the golden
“ number of that year; over-againſt which, in the ſame line continued
“ to the column intitled Paſchal full Moons, you will find the day of the
“ month, on which the Paſchal limit, or full Moon, happens in that
“ year. And the Sunday next after that day is Eaſter-day in that year,
“ according to the Gregorian account.”

His Lordſhip alſo gave with the following table an account of the principles upon which he conſtructed it; and which the more inquiſitive readers may conſult, if they pleaſe.

A TABLE, ſhewing, by means of the Golden Numbers, the ſeveral days on which the Paſchal limits, or full Moons, according to the Gregorian account, have already happened, or will hereafter happen; from the Reformation of the Calendar in the year 1582, to the year 4199, incluſive.

| Golden Numbers from the year 1583 to 1699, and ſo on to 4199, all incluſive. | | | | | | | | | | | | | | | | Paſchal full Moons. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1583 to 1699 | 1700 to 1899 | 1900 to 2199 | 2200 to 2299 | 2300 to 2399 | 2400 to 2499 | 2500 to 2599 | 2600 to 2899 | 2900 to 3099 | 3100 to 3399 | 3400 to 3499 | 3500 to 3599 | 3600 to 3699 | 3700 to 3799 | 3800 to 4099 | 4100 to 4199 | Days of the month and Sun. letters. |
| 3 | 14 | — | 6 | 17 | 6 | 17 | — | 9 | — | 1 | 12 | 1 | 12 | — | 4 | March 21 C |
| — | 3 | 14 | — | 6 | — | 6 | 17 | — | 9 | — | 1 | — | 1 | 12 | — | 22 D |
| 11 | — | 3 | 14 | — | 14 | — | 6 | 17 | — | 9 | — | 9 | — | 1 | 12 | 23 F |
| — | 11 | — | 3 | 14 | 3 | 14 | — | 6 | 17 | — | 9 | — | 9 | — | 1 | 24 F |
| 19 | — | 11 | — | 3 | — | 3 | 14 | — | 6 | 17 | — | 17 | — | 9 | — | 25 G |
| 8 | 19 | — | 11 | — | 11 | — | 3 | 14 | — | 6 | 17 | 6 | 17 | — | 9 | 26 A |
| — | 8 | 19 | — | 11 | — | 11 | — | 3 | 14 | — | 6 | — | 6 | 17 | — | 27 B |
| 16 | — | 8 | 19 | — | 19 | — | 11 | — | 3 | 14 | — | 14 | — | 6 | 17 | 28 C |
| 5 | 16 | — | 8 | 19 | 8 | 19 | — | 11 | — | 3 | 14 | 3 | 14 | — | 6 | 29 D |
| — | 5 | 16 | — | 8 | — | 8 | 19 | — | 11 | — | 3 | — | 3 | 14 | — | 30 E |
| 13 | — | 5 | 16 | — | 16 | — | 8 | 19 | — | 11 | — | 11 | — | 3 | 14 | 31 F |
| 2 | 13 | — | 5 | 16 | 5 | 16 | — | 8 | 19 | — | 11 | — | 11 | — | 3 | April 1 G |
| — | 2 | 13 | — | 5 | — | 5 | 16 | — | 8 | 19 | — | 19 | — | 11 | — | 2 A |
| 10 | — | 2 | 13 | — | 13 | — | 5 | 16 | — | 8 | 19 | 8 | 19 | — | 11 | 3 B |
| — | 10 | — | 2 | 13 | 2 | 13 | — | 5 | 16 | — | 8 | — | 8 | 19 | — | 4 C |
| 18 | — | 10 | — | 2 | — | 2 | 13 | — | 5 | 16 | — | 16 | — | 8 | 19 | 5 D |
| 7 | 18 | — | 10 | — | 10 | — | 2 | 13 | — | 5 | 16 | 5 | 16 | — | 8 | 6 E |
| — | 7 | 18 | — | 10 | — | 10 | — | 2 | 13 | — | 5 | — | 5 | 16 | — | 7 F |
| 15 | — | 7 | 18 | — | 18 | — | 10 | — | 2 | 13 | — | 13 | — | 5 | 16 | 8 G |
| 4 | 15 | — | 7 | 18 | 7 | 18 | — | 10 | — | 2 | 13 | 2 | 13 | — | 5 | 9 A |
| — | 4 | 15 | — | 7 | — | 7 | 18 | — | 10 | — | 2 | — | 2 | 13 | — | 10 B |
| 12 | — | 4 | 15 | — | 15 | — | 7 | 18 | — | 10 | — | 10 | — | 2 | 13 | 11 C |
| 1 | 12 | — | 4 | 15 | 4 | 15 | — | 7 | 18 | — | 10 | — | 10 | — | 2 | 12 D |
| — | 1 | 12 | — | 4 | — | 4 | 15 | — | 7 | 18 | — | 18 | — | 10 | — | 13 E |
| 9 | — | 1 | 12 | — | 12 | — | 4 | 15 | — | 7 | 18 | 7 | 18 | — | 10 | 14 F |
| — | 9 | — | 1 | 12 | 1 | 12 | — | 4 | 15 | — | 7 | — | 7 | 18 | — | 15 G |
| 17 | — | 9 | — | 1 | — | 1 | 12 | — | 4 | 15 | — | 15 | — | 7 | 18 | 16 A |
| 6 | 17 | 17 | 9 | — | 9 | — | 1 | 12 | 12 | 4 | 15 | 4 | 15 | 15 | 7 | 17 B |
| 14 | 6 | 6 | 17 | 9 | 17 | 9 | 9 | 1 | 1 | 12 | 4 | 12 | 4 | 4 | 15 | 18 C |

135. PROBLEM VIII. *To find the time of the Moon's ſouthing on a given day.*

RULE. The Moon's age in days, multiplied by 0,8, gives the time of her ſouthing, nearly, in hours and tenth parts.

That time, if leſs than 12 hours, is the time after mid-day.

But if greater, the exceſs is the time after laſt midnight.

EX. I. *At what time does the Moon come to the meridian of London, on the 14th of October, 1787?*

The Moon's age is 3 days.
Which multiplied by 0,8

Moon So. $2^{h}\ 24^{m} = 2,4$

EX. II. *Required the time of the Moon's ſouthing on the 20th of March, 1788?*

The Moon's age 13 days. (130)
Which multiplied by 0,8

Moon So. $10^{h}\ 24^{m} = 10,4$

Each tenth part of an hour being 6 minutes, any number of ſuch tenth parts multiplied by 6, produces minutes.

136. **Problem** IX. *To find the time of high-water at any place.*

**Rule.** To the time of the Moon's ſouthing add the time the Moon has paſſed the meridian on the full and change days to make high-water at that place; the ſum ſhews the time of high-water on the given day.

The time of high-water, on the full and change days, is found in the right-hand column of the geographical table, art. 137, againſt the name of the place.

Ex. I. *On the 14th of October 1787, at what time will it be high-water at London?*

Moon ſouths at 2h. 24m. (135).
H.W. at Lond. 3 0 on ſyzygies

Sum 5 24

H. W. at 5h 24m. P. M. on the day propoſed.

Ex. II. *Required the time when it will be high-water at Uſhant on March 20th,* 1788.

Moon ſouths at 10h. 24m. (135)
High-water at Uſhant 4 30 P. M.

14 54
Subtract 12 00

High-water at 2 54 A. M. on the day propoſed.

The v. viii. ix. problems preceding have ſolutions, ſuch as are common in books of pilotage, and which in ſome caſes will produce concluſions conſiderably wide of the truth; it has therefore been judged neceſſary to conſider theſe articles in a more accurate manner in Book IX. of Days works.

## SECTION IX.

## 137. *A Geographical Table.*

Containing the latitudes and longitudes of the chief towns, iſlands, bays, capes, and other parts of the ſea-coaſts in the known world, collected from the moſt authentic obſervations and charts extant; with the times of high-water on the days of the new and full Moon.

The longitudes are reckoned from the meridian of London. By the latitude and longitude of an iſland, or harbour, is meant the middle of that place.

*Note.* B. ſtands for bay; C. for cape; R. for river; P. for port; Pt. for point; I. for Iſle; St. for ſaint; G. for gulf; M. for mount; Eu. for Europe; Am. for America; Atl. for the Atlantic; Ind. for Indian; Med. Sea for Mediterranean Sea; Wh. Sea for White Sea; Archip. for Archipelago; Nov. Sco. for Nova Scotia; Phil. I. for Philippine Iſles; Adriat. for Adriatic; Eng. for England; D. Neth. for the Dutch Netherlands: Beſides other contractions which will be eaſily underſtood.

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| A | | | | ° ′ | ° ′ | |
| I. Abacco, N. point<br>or Lucayos S. point | Am. | Bahama I. | Atl. Ocean | 27 12 N.<br>26 15 N. | 77 05 W.<br>77 01 W. | |
| Abbreviak | Eu. | France | Eng. Channel | 48 32 N. | 4 15 W. | 4h. 30m. |
| St. Abbſhead | Eu. | Scotland | Germ. Ocean | 55 55 N. | 1 56 W. | |
| I. Abdeleur | Africa | Anian | Indian Sea | 11 55 N. | 51 45 E. | |
| Aberdeen | Eu. | Scotland | Germ. Ocean | 57 06 N. | 01 44 W. | 0 45 |
| Abo | Eu. | Finland | Baltic Sea | 60 27 N. | 22 15 E. | |
| Abrolhos Bank | Am. | Braſil | Atl. Ocean | 18 22 S. | 38 45 W. | |
| Abrollo Bank, N. part | Am. | Bahama | Atl. Ocean | 21 33 N. | 69 50 W. | |
| Achen | Aſia | I. Sumatra | Indian Ocean | 5 22 N. | 95 40 E. | |
| Aden | Aſia | Arabia | Indian Sea | 12 55 N. | 45 35 E. | |
| I. Admiralties | Eu. | Nova Zem. | North Ocean | 75 05 N. | 52 50 E. | |
| Adventure Iſland | Aſia | Soc. Iſles | Pacif. Ocean | 17 6 S. | 144 18 W. | |
| I. Agalega, or Gallega | Africa | Madagaſcar | Indian Ocean | 10 15 S. | 54 46 E. | |
| C. St. Agnes | Am. | Patagonia | S. Atl. Ocean | 53 55 S. | 66 29 W. | |
| Agra | Aſia | India | Mogul's | 26 43 N. | 76 49 E. | |
| I. St. Aguſta | Eu. | Dalmatia | Adriatic Sea | 42 40 N. | 18 57 E. | |
| C. Ajuga | Am. | Peru | Pacif. Ocean | 6 38 S. | 80 50 W. | |
| B. Alagoa | Africa | Caffers | Indian Ocean | 25 30 S. | 33 33 E. | |
| Iſ. Aland | Eu. | Sweden | Baltic Sea | 60 20 N. | 21 30 E. | |
| R. Albany | Am. | NewS.Wales | Hud. Bay | 52 35 N. | 85 18 W. | |
| I. Alboran | Africa | Algiers | Medit. Sea | 36 00 N. | 2 27 W. | |
| Aldborough | Eu. | England | Germ. Ocean | 52 20 N. | 1 25 E. | 9 45 |
| I. Alderney | Eu. | England | Eng. Channel | 49 48 N. | 2 11 W. | 12 00 |
| Aleppo | Aſia | Syria | Medit. Sea | 35 45 N. | 37 25 E. | |
| Alexandretta | Aſia | Syria | Medit. Sea | 36 35 N. | 36 20 E. | |
| Alexandria | Africa | Egypt | Medit. Sea | 31 11 N. | 30 17 E. | |
| I. Algeranca | Africa | Canaries | Atl. Ocean | 29 23 N. | 15 53 W. | |
| Algiers | Africa | Algiers | Medit. Sea | 36 49 N. | 2 18 E. | |
| Alicant | Eu. | Spain | Medit. Sea | 38 34 N. | 0 07 W. | |
| I. Alicur, Lipari Iſ. | Eu. | Italy | Medit. Sea | 38 31 N. | 14 37 E. | |
| Alkoſir | Africa | Egypt | Red Sea | 26 20 N. | 34 41 E. | |
| B. All Saints, or Todos Sanctos | Am. | Braſil | Atl. Ocean | 13 05 S. | 38 45 W. | |
| Almeria | Eu. | Spain | Medit. Sea | 36 51 N. | 2 15 W. | |
| Iſ. Almirante, limits | Africa | Zanguebar | Indian Sea | 5 45 S.<br>4 30 S. | 52 30 E.<br>55 40 E. | |
| St. Alphonſo's Iſ. | Am. | T. del Fuego | Pacif. Ocean | 55 51 S. | 69 28 W. | |
| Altur | Aſia | Arabia | Red Sea | 28 20 N. | 34 19 E. | |
| R. Amazons, mouths | Am. | Terra Firma | Atl. Ocean | 0 30 S. | 47 35 W.<br>49 20 W. | 6 00 |
| I. Amboyna | Aſia | Molucca I. | Indian Ocean | 4 25 N. | 127 25 E. | |
| Ambrym | Aſia | N. Hebrides | Pacif. Ocean | 16 10 S. | 168 12 E. | |
| Iſ. Ambroſa | Am. | Chili | Pacif. Ocean | 26 40 S. | 82 30 W. | |
| I. Ameyland | Eu. | D. Neth. | Germ. Ocean | 53 30 N. | 6 20 E. | 7 30 |
| I. Amoy | Aſia | China | Pacif. Ocean | 24 30 N. | 118 45 E. | |
| Amſterdam | Eu. | D. Neth. | Germ. Ocean | 52 23 N. | 04 52 E. | 3 00 |
| I. Amſterdam | Aſia | Madagaſcar | Indian Ocean | 37 55 S. | 75 15 E. | |
| I. Amſterdam, or Tonga-Tabu | Aſia | Friendly Iſ. | Pacif. Ocean | 21 09 S. | 174 41 W. | 8 30 |
| I. Anabona | Africa | Eth. Coaſt | Atl. Ocean | 2 36 S. | 5 35 E. | |
| Ancona | Eu. | Italy | Mediterran. | 43 38 N. | 13 31 E. | |
| Iſ. Andaman, limits | Aſia | India | B. Bengal | 14 00 N.<br>10 08 N. | 93 03 E.<br>93 35 E. | |
| I. Andaro | Aſia | India | Indian Ocean | 10 00 N. | 73 40 E. | |
| I. St. Andero, Sotovento | Am. | Mexico | Atl. Ocean | 12 30 N. | 81 35 W. | |
| C. St. Andrea | Africa | Madagaſcar | Indian Ocean | 15 46 S. | 45 22 E. | |
| St. Andrew's | Eu. | Scotland | Germ. Ocean | 56 18 N. | 2 37 W. | 2 15 |
| Iſ. Androſs N. point<br>S. point | Am. | Bahama I. | Atl. Ocean | 25 00 N.<br>23 30 N. | 77 58 W.<br>77 00 W. | |
| Iſ. Angaſay | Africa | Madagaſcar | Indian Ocean | 17 00 S. | 58 40 E. | |
| C. St. Angelo | Eu. | Turkey | Archipelago | 36 27 N. | 33 38 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Mount St. Angelo | Eu. | Italy | Medit. Sea | 41 42 N. | 16 16 E. | |
| R. d'Angra | Africa | Ethiopia | N. Atl. Ocean | 01 00 N. | 9 35 E. | |
| C. d'Anguilhas | Africa | Caffers | Indian Ocean | 34 50 S. | 20 06 E. | |
| I. Anguilla | Am. | Antilles Iſ. | Atl. Ocean | 18 15 N. | 62 57 W. | |
| C. Anguille | Am. | Newfoundl. | Atl. Ocean | 47 55 N. | 59 11 W. | |
| I. Anholt | Eu. | Denmark | Sound | 56 40 N. | 12 00 E. | oh.oom. |
| C. Anne | Am. | New Eng. | Weſt. Ocean | 42 50 N. | 70 27 W. | |
| C. Queen Anne | Am. | Greenland | North Ocean | 64 15 N. | 50 30 W. | |
| Q. Anne's Foreland | Am. | N. Main | Hudſon's Str. | 64 08 N. | 74 36 W. | |
| Annamocka, or Rotterdam | Aſia | Friendly Iſ. | Pacif. Ocean | 20 16 S. | 174 30 W. | |
| Annapolis Royal | Am. | Nova Scotia | B. Fundy | 44 52 N. | 64 00 W. | |
| I. Antego | Am. | Caribbee Iſ. | Atl. Ocean | 16 57 N. | 61 56 W. | |
| Antibes | Eu. | France | Medit. Sea | 43 35 N. | 7 09 E. | |
| I. Antecoſt { W. point | Am. | Canada | B. St. Laurence | 49 52 N. | 64 04 W. | |
| E. point | | | | 49 10 N. | 61 42 W. | |
| Antiochetta | Aſia | Syria | Medit. Sea | 36 08 N. | 36 17 E. | |
| C. d'Antiſer | Eu. | France | Eng. Channel | 49 47 N. | 0 34 E. | |
| C. Antonio | Am. | Iſle Cuba | Atl. Ocean | 21 45 N. | 84 05 W. | |
| I. St. Antonio | Africa | Cape Verd | Atl. Ocean | 17 00 N. | 25 02 W. | |
| C. St. Antony | Am. | Magellan | Atl. Ocean | 54 46 S. | 63 42 W. | |
| Antwerp | Eu. | Flanders | R. Scheld | 51 13 N. | 4 24 E. | 6 00 |
| B. Apalaxy | Am. | Florida | G. Mexico | 30 00 N. | 83 53 W. | |
| I. Apalioria | Aſia | India | Indian Ocean | 9 08 S. | 79 40 E. | |
| Aquapulco | Am. | Mexico | Pacif. Ocean | 17 10 N. | 101 40 W. | |
| Aquatulco | Am. | Mexico | Pacif. Ocean | 15 27 N. | 96 03 W. | |
| Archangel | Eu. | Ruſſia | White Sea | 64 34 N. | 38 59 E. | 6 00 |
| I. d'Areas | Am. | Mexico | G. Mexico | 20 45 N. | 92 35 W. | |
| Arica | Am. | Peru | Pacif. Ocean | 18 27 S. | 71 05 W. | |
| I. Arran | Eu. | Ireland | St. Geo. Ch. | 54 48 N. | 8 59 W. | 11 00 |
| I. Aſcenſion | Am. | Braſil | Atl. Ocean | 7 56 S. | 14 18 W. | |
| I. Aſſinaria, Sardinia | Eu. | Italy | Medit. Sea | 41 06 N. | 8 36 E. | |
| R. Aſhley | Am. | Carolina | Atl. Ocean | 33 22 N. | 79 50 W. | 0 45 |
| R. Aſſene | Africa | Guinea | Atl. Ocean | 5 30 N. | 2 20 W. | |
| I. Aſtores | Africa | Madagaſcar | Indian Ocean | 10 22 S. | 53 25 E. | |
| Athens | Eu. | Turkey | Archipelago | 38 5 N. | 23 52 E. | |
| Atkin's Key | Am. | Bahama Iſles | Atl. Ocean | 22 07 N. | 74 26 W. | |
| Atwood's Keys | Am. | Bahama Iſles | Atl. Ocean | 21 22 N. | 72 04 W. | |
| C. Ava | Aſia | Japan | Pacif. Ocean | 34 45 N. | 141 00 E. | |
| Iſ. Aves, Sotovento | Am. | Terra Firma | Atl. Ocean | 15 26 N. | 66 15 W. | |
| C. St. Auguſtine | Am. | Braſil | Atl. Ocean | 8 48 S. | 35 00 W. | |
| C. St. Auguſtine | Aſia | Mindanao | Pacif. Ocean | 6 40 N. | 126 25 E. | |
| St. Auguſtine | Am. | Florida | Atl. Ocean | 30 10 N. | 81 29 W. | 7 30 |
| Aurora | Aſia | N. Hebrides | Pacif. Ocean | 15 8 S. | 168 17 E. | |
| Aydhab | Africa | Egypt | Red Sea | 21 53 N. | 36 26 E. | |
| Aylah | Aſia | Arabia | Red Sea | 29 08 N. | 35 41 E. | |
| B | | | | | | |
| Babelmondel Straits | Africa | Abyſſinia | Red Sea | 12 50 N. | 43 50 E. | |
| C. Baba | Aſia | Natolia | Archipelago | 39 33 N. | 26 22 E. | |
| I. Bachian | Aſia | Molucca Iſles | Pacif. Ocean | 00 40 N. | 123 00 E. | |
| I. Bahama | Am. | Bahama Iſles | Atl. Ocean | 26 45 N. | 78 35 W. | |
| Bahama Bank, N. pt. | Am. | Bahama Iſles | Atl. Ocean | 27 50 N. | 78 43 W. | |
| C. Bajador | Africa | Negroland | Atl. Ocean | 26 29 N. | 14 36 W. | 0 00 |
| Baker's Dozen | Am. | Labrador | Hudſon's Bay | 57 0 N. | | |
| Balaſor | Aſia | India | B. Bengal | 21 20 N. | 86 00 E. | |
| Baldivia | Am. | Chili | Pacif. Ocean | 39 38 S. | 73 20 W. | |
| I. Bali | Aſia | Sunda Iſles | Indian Ocean | 8 05 S. | 114 30 E. | |
| Baltimore | Eu. | Ireland | Weſt. Ocean | 51 16 N. | 9 26 W. | 4 30 |
| I. Banca { S. end | Aſia | Sunda Iſles | Indian Ocean | 3 15 S. | 107 10 E. | |
| N.W. end | | | | 1 50 S. | 105 30 E. | |
| I. Banda | Aſia | Molucca Iſles | Indian Ocean | 4 30 N. | 127 25 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Banjar | Aſia | I. Borneo | Indian Ocean | 2 27 S. | 113 50 E. | |
| Banks's Iſle | Aſia | N. Zealand | Pacif. Ocean | 43 45 S. | 172 40 E. | |
| Bantam | Aſia | I. Java | Indian Ocean | 6 15 S. | 106 25 E. | |
| B. Bantry | Eu. | Ireland | Atl. Ocean | 51 45 N. | 10 46 W. | |
| I. Barbadoes<br>Bridge-town | Am. | CaribbeeIſles | Atl. Ocean | 13 05 N. | 59 36 W. | |
| C. Barbas | Africa | Sanaga | N. Atl. Ocean | 21 50 N. | 16 26 W. | |
| I. Barbuda | Am. | CaribbeeIſles | Atl. Ocean | 17 46 N. | 61 47 W. | |
| C. Barcam | Eu. | Greenland | North Ocean | 78 18 N. | 20 06 E. | |
| Barcelona | Eu. | Spain | Medit. Sea | 41 26 N. | 2 18 E. | |
| C. Barfleur | Eu. | France | Eng. Channel | 49 38 N. | 1 16 W. | 7h. 30m. |
| Bargazar Point | Eu. | Iceland | North Ocean | 66 30 N. | 17 12 W. | |
| I. Bardſey | Eu. | Wales | St. Geo. Cha. | 52 44 N. | 5 00 W. | |
| C. Barſo | Eu. | Ruſſia | White Sea | 66 30 N. | 38 00 E. | |
| I. Bartholomew | Am. | CaribbeeIſles | Atl. Ocean | 17 56 N. | 63 11 W. | |
| I. de Bas | Eu. | France | Eng. Channel | 48 50 N. | 4 00 W. | 3 45 |
| Baſſora | Aſia | Arabia | Perſian Gulf | 29 45 N. | 47 40 E. | |
| C. Baſſos, or Baxos | Africa | Anian | Indian Sea | 4 12 N. | 47 07 E. | |
| Baſſos de Banhos | Africa | Zanguebar | Indian Ocean | 5 00 S. | 48 08 E. | |
| Baſſos de Chagos | Aſia | India | Indian Ocean | 6 42 S. | 68 20 E. | |
| I. Baſſus des Indes | Africa | Zanguebar | Indian Ocean | 21 19 S. | 41 43 E. | |
| Batavia | Aſia | I. Java | Indian Ocean | 6 12 S. | 106 45 E. | |
| Bayonne | Eu. | France | B. Biſcay | 43 30 N. | 1 30 W. | 3 30 |
| Bayona Iſles | Eu. | Spain | Atl. Ocean | 41 45 N. | 9 01 W. | |
| Beachy Head | Eu. | England | Eng. Channel | 50 44 N. | 0 25 E. | 0 00 |
| Bear-bay | Eu. | Greenland | North Ocean | 79 10 N. | 24 15 E. | |
| N. Bear<br>S. Bear | Am. | Labradore | Hudſon's Bay | 54 40 N.<br>54 25 N. | 80 0 W. | 12 00 |
| I. Beerenberg | Eu. | ——— | North Ocean | 71 45 N. | 4 30 E. | |
| Belcher's Iſles | Am. | Labrādore | Hudſon's Bay | 56 0 N. | 83 4 W. | |
| Belfaſt | Eu. | Ireland | Iriſh Sea | 54 43 N. | 5 52 W. | 10 0 |
| Belliſle | Eu. | France | B. Biſcay | 47 21 N. | 3 13 W. | 3 30 |
| Belliſle | Am. | Newfound. | Atl. Ocean | 51 55 N. | 55 25 W. | |
| Bembridge Point | Eu. | Iſle Wight | Eng. Channel | 50 41 N. | 1 5 W. | |
| Straits of Belliſle | Am. | Newfoundl. | Atl. Ocean | 51 48 N. | 56 00 W. | |
| Bell Sound | Eu. | Greenland | North Ocean | 77 15 N. | 12 40 E. | |
| Bencolin | Aſia | I. Sumatra | Indian Ocean | 3 49 S. | 102 5 E. | |
| Bengal | Aſia | India | B. Bengal | 22 00 N. | 92 45 E. | |
| Bergen | Eu. | Norway | Weſtern Oc. | 60 10 N. | 6 14 E. | |
| Berlin | Eu. | Germany | R. Elbe | 52 33 N. | 13 26 E. | |
| I. Bermudas | Am. | Bahama Iſles | Atl. Ocean | 32 35 N. | 63 23 W. | 7 00 |
| I. Bermaja | Am. | Mexico | G. Mexico | 21 40 N. | 92 53 W. | |
| Berwick | Eu. | England | Germ. Ocean | 55 45 N. | 1 50 W. | 2 30 |
| Berry Point | Eu. | England | Eng. Channel | 50 37 N. | 3 49 W. | |
| Bird Iſland | Am. | Acadia | G. St. Lawr. | 47 44 N. | 60 24 W. | |
| Bilboa | Eu. | Spain | B. Biſcay | 43 26 N. | 3 18 W. | |
| I. du Bic | Am. | Acadia | R. St. Lawr. | 48 30 N. | 68 36 W. | 2 0 |
| Blackney | Eu. | England | Germ. Ocean | 53 20 N. | 0 55 E. | 6 00 |
| Black Point | Eu. | Greenland | North Ocean | 78 00 N. | 10 50 E. | |
| Black Iſle | Eu. | Nova Zem. | North Ocean | 72 52 N. | 52 35 E. | |
| C. Blanco | Africa | Negroland | Atl. Ocean | 20 55 N. | 17 5 W. | 9 45 |
| C. Blanco | Am. | Patagonia | Atl. Ocean | 47 20 S. | 64 37 W. | |
| C. Blanco | Eu. | Greenland | North Ocean | 77 58 N. | 20 04 E. | |
| C. Blanco | Am. | Mexico | Pacif. Ocean | 9 42 N. | 85 55 W. | |
| I. Blanco, Sotovento | Am. | Terra Firma | Atl. Ocean | 11 42 N. | 64 20 W. | |
| Blanch. de Race | Eu. | France | Eng. Channel | 49 42 N. | 2 08 W. | 0 00 |
| Blaſques | Eu. | Ireland | Atl. Ocean | 52 00 N. | 10 56 W. | |
| Blavet, or Port Louis | Eu. | France | B. Biſcay | 47 45 N. | 3 13 W. | 3 0 |
| Bocachica | Am. | Terra Firma | Carib. Sea | 10 20 N. | 75 30 W. | |
| Bolabola | Aſia | Society Iſ. | Pacif. Ocean | 16 33 S. | 151 52 W. | |
| R. Bolſhaya | Aſia | Siberia | Pacif. Ocean | 52 48 N. | 157 00 E. | |
| I. Bombay | Aſia | India | Indian Ocean | 18 57 N. | 72 43 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Bona | Africa | Tunis | Mediterran. | 37 08 N. | 7 10 E. | |
| C Bona | Africa | Tunis | Mediterran. | 37 10 N. | 10 00 E. | |
| C Bona viſta | Am. | Newfoundl. | Atl. Ocean | 48 54 N. | 52 33 W. | |
| I. Bona viſta | Africa | C. Verd Iſles | Atl. Ocean | 16 05 N. | 22 42 W. | |
| C. Bona fortuna | Eu. | Ruſſia | White Sea | 65 35 N. | 38 25 E. | |
| I. Bonayre, Sotovento | Am. | Terra Firma | Atl. Ocean | 11 52 N. | 67 20 W. | |
| B. Bonaventura | Am. | Terra Firma | Pacif. Ocean | 3 18 N. | 76 50 W. | |
| C. Bon Eſperance | Africa | Caffers | Indian Ocean | 34 29 S. | 18 23 E. | 3h. 00m. |
| Bourdeaux | Eu. | France | B. Biſcay | 44 50 N. | 0 30 W. | 3 00 |
| I. Borneo Eaſt point | Aſia | | Indian Ocean | 1 12 N. | 117 10 E. | |
| Weſt point | | | | 3 15 N. | 108 57 E. | |
| North point | | | | 7 05 N. | 113 40 E. | |
| South point | | | | 3 32 S. | 112 05 E. | |
| I. Borneo Borneo | Aſia | | Indian Ocean | 5 00 N. | 112 15 E. | |
| Succadano | | | | 0 50 S. | 108 35 E. | |
| I. Bornholm | Eu. | Sweden | Baltic Sea | 55 12 N. | 15 50 E. | |
| Boſton | Eu. | England | Germ. Ocean | 53 10 N. | 0 25 E. | |
| Boſton | Am. | New Eng. | Atl. Ocean | 42 25 N. | 70 32 W. | |
| Botany Iſle | Aſia | N. Caledonia | Pacif. Ocean | 22 27 S. | 167 12 E. | |
| Botany Bay | Aſia | N. Holland | Pacif. Ocean | 34 00 S. | 151 28 E. | |
| Boulogne | Eu. | France | Eng. Channel | 50 44 N. | 1 40 E. | 10 30 |
| I. Bourbon, St. Den. | Africa | Madagaſcar | Indian Ocean | 20 52 S. | 55 35 E. | |
| I. St. Brandon | Africa | Madagaſcar | Indian Ocean | 16 45 S. | 64 48 E. | |
| B. Brandwyns | Eu. | Greenland | North Ocean | 79 50 N. | 26 20 E. | |
| I. Bravas | Africa | C. Verd | Atl. Ocean | 14 54 N. | 24 45 W. | |
| Bremen | Eu. | Germany | R. Weſer | 53 30 N. | 9 00 E. | 6 00 |
| Breefound, a ſand | Eu. | D. Neth. | Germ. Ocean | 53 12 N. | 5 15 E. | 4 30 |
| Breſlau | Eu. | Sileſia | R. Oder | 51 03 N. | 17 13 E. | |
| Breſt | Eu. | France | B. Biſcay | 48 23 N. | 4 26 W. | 3 45 |
| P. Breſt | Am. | New Britain | Weſt. Ocean | 52 10 N. | 52 30 W. | |
| Cape Bret | Aſia | N. Zealand | Pacif. Ocean | 35 07 S. | 173 52 E. | |
| Bridge Town | Am. | I. Barbadoes | Atl. Ocean | 13 05 N. | 59 36 W. | |
| Bridlington Bay | Eu. | England | Germ. Ocean | 54 07 N. | 00 04 E. | 3 45 |
| Brill | Eu. | D. Neth. | Germ. Ocean | 51 56 N. | 4 10 E. | 1 30 |
| Brion Iſle | Am. | Acadia | G. St. Lawr. | 47 50 N. | 60 47 W. | |
| Briſtol | Eu. | England | St. Geo. Ch. | 51 28 N. | 2 30 W. | 6 45 |
| C. Briſtol | Am. | Sandwich L. | Atl. Ocean | 59 2 S. | 26 46 W. | |
| I. Cape Britain Louiſbourgh | Am. | Acadia | Atl. Ocean | 45 54 N. | 59 55 W. | |
| I. Scateri | | | | 46 01 N. | 61 57 W. | |
| North Cape | | | | 47 05 N. | 60 8 W. | |
| New Britain I. Mathias | Aſia | New Guinea | Pacif. Ocean | 2 00 S. | 147 50 E. | |
| North point | | | | 2 30 S. | 148 40 E. | |
| S. W. point | | | | 6 00 S. | 146 37 E. | |
| Strait Dampier | | | | 6 15 S. | 146 15 E. | |
| C. St. George | | | | 5 30 S. | 150 55 E. | |
| I. St. John | | | | 4 20 S. | 152 40 E. | |
| Buchaneſs | Eu. | Scotland | Germ. Ocean | 57 29 N. | 1 23 W. | 3 00 |
| Buenos Ayres | Am. | Braſil | Atl. Ocean | 34 35 S. | 58 26 W. | |
| C. Buller | Am. | S. Georgia | Atl. Ocean | 53 58 S. | 37 40 W. | |
| Burgaford point | Eu. | Iceland | North Ocean | 66 03 N. | 16 34 W. | |
| Burgeo Iſles | Am. | Newfoundl. | Atl. Ocean | 47 36 N. | 57 31 W. | |
| Burlings, rocks | Eu. | Portugal | Atl. Ocean | 39 20 N. | 9 32 W. | |
| Burlington | Eu. | England | Germ. Ocean | 54 00 N. | 0 08 E. | |
| Button's Iſles | Am. | New Britain | Hudſ. Straits | 60 35 N. | 65 20 W. | 6 50 |
| Cape Byron | Aſia | N. Zealand | Pacif. Ocean | 28 39 S. | 153 31 E. | |
| Byron's Iſle | Aſia | | Pacif. Ocean | 1 18 S. | 170 6 W. | |
| C | | | | | | |
| I. Cabrera | Eu. | Italy | Mediterran. | 43 10 N. | 9 11 E. | |
| Cadiz | Eu. | Spain | Atl. Ocean | 36 31 N. | 6 07 W. | 4 30 |
| Caen | Eu. | France | Eng. Channel | 49 11 N. | 0 17 W. | 9 00 |
| Cagliari, I. Sardinia | Eu. | Italy | Medit. Sea | 39 25 N. | 9 38 E. | |
| Cajaneburg | Eu. | Finland | Baltic Sea | 64 13 N. | 27 51 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Iſles Caicos, or Cankroſs, from | Am. | Bahama Iſles | Atl. Ocean | 21 27 N.<br>22 5 N. | 71 24 W.<br>72 15 W. | |
| Calabar { Old<br>New | Africa | Guinea | Eth. Ocean | 4 30 N.<br>5 00 N. | 8 10 E.<br>7 00 E. | |
| C. Calaberno | Aſia | Natolia | Archipelago | 38 42 N. | 26 44 E. | |
| Calais | Eu. | France | Eng. Channel | 50 58 N. | 01 51 E. | 11h. 30m. |
| C. Calamadon | Aſia | India | B. Bengal | 10 22 N. | 80 40 E. | |
| Calcutta | Aſia | India | B. Bengal | 22 35 N. | 88 34 E. | |
| Caldera | Aſia | I. Mindano | Pacif. Ocean | 7 00 N. | 121 25 E. | |
| I. Caldy | Eu. | England | St. Geo. Ch. | 51 33 N. | 5 14 W. | 5 15 |
| Calecut | Aſia | India | Indian Ocean | 11 15 N. | 75 39 E. | |
| Cairo | Africa | Egypt | R. Nile | 30 02 N. | 31 26 E. | |
| Callao | Am. | Peru | Pacif. Ocean | 12 2 S. | 76 53 W. | |
| I. Great Camanis | Am. | Weſt Indies | Atl. Ocean | 19 18 N. | 80 29 W. | |
| I. Little Camanis | Am | Weſt Indies | Atl. Ocean | 19 42 N. | 79 20 W. | |
| Camboida | Aſia | India | Indian Ocean | 10 35 N. | 104 45 E. | |
| Cambridge | Eu. | England | ——— | 52 13 N. | 0 9 E. | |
| Cambridge | Am. | N. England | ——— | 42 25 N. | 71 5 W. | |
| C. Cambron, or Carbon | Africa | Algiers | Medit. Sea | 37 18 N. | 4 58 E. | |
| C. Cameron | Am. | New Spain | Atl. Ocean | 15 35 N | 83 29 W. | |
| R. Camerones | Africa | Guinea | Atl. Ocean | 3 30 N. | 9 10 E. | |
| B. Camerones | Am. | Magellan | Atl. Ocean | 44 50 S. | 67 10 W. | |
| Camfer, a ſand | Eu. | D. Neth. | Germ. Ocean | 53 33 N. | 5 30 E. | 1 30 |
| Camin | Eu. | Germany | Baltic | 54 04 N. | 15 40 E. | |
| C. Campbell | Aſia | N. Zealand | Pacif. Ocean | 41 51 S. | 174 41 E. | |
| Compeachy | Am. | Yucatin | Atl. Ocean | 19 36 N. | 90 53 W. | |
| I. Canaria | Africa | Canaries | Atl. Ocean | 28 01 N. | 15 0 W. | 3 0 |
| C. Candenoſe | Eu. | Ruſſia | North Ocean | 69 25 N. | 45 30 E. | |
| I. Candia { C. St. John, W. end<br>Candia<br>C. Salomon, E. end | Eu. | Turkey | Medit. Sea | 35 12 N.<br>35 19 N.<br>34 57 N. | 23 54 E.<br>25 23 E.<br>27 06 E. | |
| Candia | Aſia | I. Ceylon | Indian Ocean | 7 54 N. | 81 53 E. | |
| Candlemas Iſles | Am. | Sandwich L. | Atl. Ocean | 57 10 S. | 27 13 W. | |
| I. Candu | Aſia | India | Indian Ocean | 7 30 S. | 77 55 E. | |
| C. Canſo | Am. | Nova Scotia | Atl. Ocean | 45 18 N. | 60 48 W. | |
| Canſo Paſſage | Am. | Nova Scotia | Atl. Ocean | 45 30 N. | 61 00 W. | |
| C. Cantin | Africa | Barbary | Atl. Ocean | 32 41 N. | 9 01 W. | 0 00 |
| Cantire, Mul | Eu. | Scotland | Weſt Ocean | 55 22 N. | 5 45 W. | |
| Canton | Aſia | China | Pacif. Ocean | 23 08 N. | 113 07 E. | |
| Cape Town | Africa | Caffers | Atl. Ocean | 33 55 S. | 18 23 E. | 2 30 |
| I. Capri | Eu. | Italy | Medit. Sea | 40 34 N. | 14 11 E. | |
| I. Capraya | Eu. | Italy | Medit. Sea | 43 03 N. | 10 15 E. | |
| C. C[illegible] | Aſia | India | B. Bengal | 19 22 N. | 86 05 E. | |
| Caracca | Am. | Terra Firma | Atl. Ocean | 10 06 N. | 66 45 W. | |
| Carys, or Hermits Iſles | Am. | Greenland | Baffin's Bay | 77 15 N. | 62 00 W. | |
| Carmel Point | Am. | California | Pacif. Ocean | 38 24 N. | 124 25 W. | |
| Carleſcroon | Eu. | Sweden | Baltic | 56 20 N. | 15 31 E. | |
| Carliſle | Eu. | England | Iriſh Sea | 54 47 N. | 2 35 W. | |
| C. Carmel | Aſia | Syria | Levant | 33 08 N | 35 35 E. | |
| Caroline Iſles, Limits | Aſia | ——— | Pacif. Ocean | 7 10 N.<br>12 00 N. | 137 25 E.<br>127 25 E. | |
| C. Carthage | Africa | Barbary | Medit. Sea | 36 52 N. | 10 31 E. | |
| Carthagena | Am. | Terra Firma | Caribbean Sea | 10 27 N. | 75 22 W. | |
| Carthagena | Eu. | Spain | Medit. Sea | 37 37 N. | 1 03 W. | |
| Carteret's Iſle | Aſia | New Britain | Pacif. Ocean | 8 26 S. | 159 14 E. | |
| Cary's Swan's Neſt | Am. | ——— | Hudſon's Bay | 62 20 N. | 83 30 W. | |
| Caſket. | Eu. | I. Guernſey | Eng. Channel | 49 50 N | 2 26 W. | 8 15 |
| C. Caſſander | Eu. | Turkey | Archipelago | 40 02 N. | 23 41 E. | |
| I. St Catherine's | Am. | Braſil | Atl. Ocean | 27 35 N | 49 12 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. | |
|---|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | | |
| Cat Iſle { N. point | Am. | Bahama | Atl. Ocean | 24 50 N. | 75 38 W. | | |
| S. point | | | | 23 48 N. | 75 35 W. | | |
| Cathneſs Point<br>Dinnet Head | Eu. | Scotland | Weſt. Ocean | 58 42 N. | 3 17 W. | 9h. com. | |
| Catenea | Eu. | I. Sicily | Medit. Sea | 42 40 N. | 20 30 E. | | |
| C. Catocha | Am. | New Spain | Caribbean Sea | 20 48 N. | 86 35 W. | | |
| I. Cayenne | Am. | Terra Firma | Atl. Ocean | 4 56 N. | 52 10 W. | | |
| I. Celebes { N. E. point | Aſia | Spice Iſles | Indian Ocean | 1 43 N. | 124 37 E. | | |
| W. point | | | | 3 00 S. | 117 55 E. | | |
| S. W. point Macaſſer | | | | 5 11 S. | 117 50 E. | | |
| S. point | | | | 5 40 S. | 119 55 E. | | |
| S. E. point | | | | 5 20 S. | 121 58 E. | | |
| I. Cephalonia | Eu. | Turkey | Medit. Sea | 38 20 N. | 20 11 E. | | |
| Ceuta | Africa | Barbary | Medit. Sea | 35 49 N. | 5 25 W. | | |
| I. Ceylon { Infanapatam, N. point | Aſia | India | Indian Ocean | 9 47 N. | 80 55 E. | | |
| Trinquemale, S. E. end | | | | 8 40 N. | 81 40 E. | | |
| C. Gallo, S. Weſt end | | | | 6 27 N. | 82 10 E. | | |
| | | | | 6 15 N. | 80 20 E. | | |
| Chain Iſland | Aſia | Society Iſles | Pacif. Ocean | 17 25 S. | 145 30 W. | | |
| Chandernagar | Aſia | Bengal | River Ganges | 22 51 N. | 88 34 E. | | |
| Charles Town | Am. | Carolina | Aſhley River | 33 22 N. | 79 50 W. | 3 | 0 |
| C. Charles | Am. | Virginia | Atl. Ocean | 37 11 N. | 76 07 W. | | |
| I. of Charles { Eaſt end | Am. | Labradore | Hudſon's Str. | 62 46½ N. | 74 15 W. | | |
| Weſt end | | | | 62 48 N. | 75 30 W. | 10 | 15 |
| C. Charles | Am. | New Britain | Weſt Ocean | 51 50 N. | 51 10 W. | | |
| Charlotte's Iſles | Aſia | Guadalcanal | Pacif. Ocean | 11 0 S. | 164 0 E. | | |
| C. Charlotte | Am. | S. Georgia | Atl. Ocean | 54 32 S. | 36 12 W. | | |
| Q. Charlotte's Sound | Aſia | N. Zealand | Pacif. Ocean | 41 6 S. | 174 19 E. | 9 | 00 |
| Q. Charlotte's Foreld. | Aſia | N. Caledonia | Pacif. Ocean | 22 15 S. | 167 18 E. | | |
| I. Charlton | Am. | New Wales | Hudſon's Bay | 52 03 N. | 79 00 W. | | |
| Chatteaux Bay | Am. | Labradore | Atl. Ocean | 52 1 N. | 55 50 W. | | |
| B. Chebucto | Am. | Nova Scotia | Atl. Ocean | 44 45 N. | 63 18 W. | | |
| Cheignecto | Am. | Nova Scotia | B. Fundy | 46 15 N. | 63 11 W. | 0 | 45 |
| Cherbourgh | Eu. | France | Eng. Channel | 49 38 N. | 01 33 W. | 7 | 30 |
| Cherry Iſle | Eu. | Greenland | North Ocean | 74 35 N. | 18 05 E. | | |
| Cheſter | Eu. | England | Iriſh Sea | 53 10 N. | 2 25 W. | | |
| Chiddock | Eu. | England | Eng. Channel | 50 47 N. | 3 00 W. | | |
| C. Chidley | Am. | New Britain | Hudſ. Straits | 60 22 N. | 65 00 W. | | |
| I. Chiloe { N. point | Am. | Patagonia | Pacif. Ocean | 41 45 S. | 73 05 W. | | |
| S. point | | | | 43 50 S. | | | |
| C. Chickotſkago | Aſia | Siberia | North Ocean | 64 00 N. | 174 45 W. | | |
| Chriſtiana | Eu. | Norway | Sound | 59 25 N. | 10 30 E. | | |
| Chriſtianople | Eu. | Sweden | Baltic Sea | 55 55 N. | 15 10 E. | | |
| Chriſtianſtadt | Eu. | Sweden | G. Bothnia | 62 47 N. | 22 50 E. | | |
| Chriſtmas Sound | Am. | T. del Fuego | Pacif. Ocean | 55 22 S. | 70 01 W. | 2 | 30 |
| I. St. Chriſtopher's | Am. | Carib. Iſles | Atl. Ocean | 17 15 N. | 62 38 W. | | |
| R. St. Chriſtopher's | Africa | Caffers | Indian Ocean | 32 47 S. | 30 00 E. | | |
| C. Chukcherſe | Aſia | Siberia | North Ocean | 66 30 N. | 171 10 W. | | |
| C. Churchill | Am. | New Wales | Hudſon's Bay | 58 48 N. | 93 10 W. | | |
| R. Churchill | | | | 58 47½ N. | 94 03 W. | 7 | 20 |
| I. Chuſan | Aſia | China | Chineſe Sea | 30 00 N. | 121 50 E. | | |
| Civita Vecchia | Eu. | Italy | Medit. Sea | 42 5 N. | 11 51 E. | | |
| C. Clear | Eu. | Ireland | Weſt. Ocean | 51 18 N. | 9 50 W. | 4 | 30 |
| rk's Iſles | Am. | S. Georgia | Atl. Ocean | 55 6 S. | 34 37 W. | | |
| I. C ate | Aſia | India | Indian Ocean | 22 00 S. | 95 40 E. | | |
| Cochin | Aſia | India | Indian Ocean | 9 50 N. | 76 05 E. | | |
| I. Cocos | Aſia | India | Indian Ocean | 12 20 S. | 98 10 E. | | |
| I. Cocos | Am. | Mexico | Pacif. Ocean | 5 00 N. | 88 45 W. | | |
| C. Cod | Am. | New Eng. | Atl. Ocean | 42 15 N. | 69 27 W. | | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Colcheſter | Eu. | England | Germ. Ocean | 52 00 N. | 0 58 E. | |
| C. Cold | Eu. | Greenland | North Ocean | 79 00 N. | 10 00 E. | |
| C. Colnet | Aſia | N.Caledonia | Pacif. Ocean | 20 30 S. | 164 56 E. | |
| R. Colerado | Am. | New Spain | G. California | 31 40 N. | 115 25 W. | |
| I. Colgau | Eu. | Ruſſia | North Ocean | 69 20 N. | 45 00 E. | |
| Collioure | Eu. | Spain | Medit. Sea | 42 31 N. | 3 10 E. | |
| C. Colone | Eu. | Turkey | Archipelago | 37 43 N. | 24 41 E. | |
| C. Colone | Aſia | Natolia | Archipelago | 39 10 N. | 27 04 E. | |
| C. Colonni | Eu. | Italy | Medit. Sea | 38 56 N. | 18 05 E. | |
| C. Colville | Aſia | N. Zealand | Pacif. Ocean | 36 27 S. | 174 48 E. | |
| Comana | Am. | Terra Firma | Atl. Ocean | 10 00 N. | 65 07 W. | |
| C. Comarin | Aſia | India | Indian Ocean | 7 55 N. | 78 7 E. | |
| C. Comfort | Am. | New Wales | Hudſon's Bay | 64 45 N. | 82 30 W. | |
| Concarneau | Eu. | France | B. Biſcay | 47 54 N. | 3 50 W. | 3h. 00m. |
| C. Conception | Am. | Calefornia | Pacif. Ocean | 35 40 N. | 120 01 W. | |
| B. Conception Entra | Am. | Newfoundl. | Atl. Ocean | 48 25 N. | 50 07 W. | |
| Conception | Am. | Chili | Pacif. Ocean | 36 43 S. | 73 13 W. | |
| R. Congo | Africa | Congo | Eth. Ocean | 5 45 S. | 11 53 E. | |
| I. Coningen | Aſia | N. Zealand | Pacif. Ocean | 34 30 S. | 164 25 E. | |
| Coningſburgh | Eu. | Poland | Baltic Sea | 54 44 N. | 21 53 E. | |
| Conquet | Eu. | France | Eng. Channel | 48 30 N. | 4 35 W. | 2 15 |
| C. Conquibaco | Am. | Terra Firma | Atl. Ocean | 12 15 N. | 69 57 W. | |
| Conſtantinople | Eu. | Turkey | Archipelago | 41 00 N. | 28 53 E. | |
| Cook's Straits | Aſia | N. Zealand | Pacif. Ocean | 41 6 S. | 174 30 E. | |
| Cooper's Iſle | Am. | S. Georgia | Atl. Ocean | 54 57 S. | 36 0 W. | |
| Copenhagen | Eu. | Denmark | Baltic Sea | 55 41 N. | 12 40 E. | |
| Coperwic | Eu. | Norway | Sound | 59 20 N. | 10 10 E. | |
| I. Copland | Eu. | Ireland | Iriſh Sea | 54 40 N. | 6 40 W. | |
| I. Coquet | Eu. | England | Germ. Ocean | 55 20 N. | 1 25 W. | 3 00 |
| R. Coquimbo | Am. | Chili | Pacif. Ocean | 29 54 S. | 71 10 W. | |
| C. Corbau | Aſia | Natolia | Archipelago | 38 03 N. | 26 58 E. | |
| Cordoue | Eu. | France | B. Biſcay | 45 30 N. | 01 10 W. | |
| Corea, South limit | Aſia | China | Pacif. Ocean | 34 50 N. | 124 25 E.<br>127 25 E. | |
| I. Corfu | Eu. | Turkey | Mediterran. | 39 50 N. | 19 48 E. | |
| C. Corientes | Africa | Caffres | Indian Ocean | 24 08 S. | 36 49 E. | |
| C. Corientes | Am. | Mexico | Pacif. Ocean | 20 18 N. | 108 00 W. | |
| Corinth | Eu. | Turkey | Archipelago | 37 30 N. | 23 00 E. | |
| Corke | Eu. | Ireland | St. Geo. Ch. | 51 54 N. | 8 30 W. | 6 30 |
| C. Coronation | Aſia | N.Caledonia | Pacif. Ocean | 22 5 S. | 167 8 E. | |
| C. Corſe | Africa | Guinea | Eth. Sea | 5 12 N. | 0 23 W. | 3 30 |
| Corſica { C. Corſe, North point | Eu. | Italy | Mediterran. | 42 53 N. | 9 40 E. | |
| Bonifacio, South point | | | | 41 22 N. | 9 26 L. | |
| I. Corvo | Fu. | Azores | Atl. Ocean | 39 42 N. | 31 02 W. | |
| I. Coſmoledo | Africa | Madagaſcar | Indian Ocean | 10 28 S. | 51 40 E. | |
| I. Coudre | Am. | Canada | R. St. Lawr. | 47 30 N. | 69 2 W. | |
| Cow and Calf | Eu. | Ireland | Weſt. Ocean | 51 22 N. | 10 30 W. | |
| I. Cozumel | Am. | Yucatan | Atl. Ocean | 19 36 N. | 86 35 W. | |
| R. Cocei | Aſia | China | B. Nankin | 34 06 N. | 120 10 E. | |
| Cromer | Eu. | England | Germ. Ocean | 53 05 N. | 0 56 E. | 7 00 |
| Crooked I. N. point | Am. | Bahama | Atl. Ocean | 22 47 N. | 73 50 W. | |
| Croſs Iſle | Eu. | Ruſſia | White Sea | 66 31 N. | 36 33 E. | |
| Croſs Point | Eu. | Nova Zem. | North Ocean | 72 00 N. | 53 12 E. | |
| S. Cruz | Africa | Barbary | Atl. Ocean | 30 36 N. | 9 35 W. | |
| I. St. Cruz | Am. | Antilles I. | Atl. Ocean | 17 53 N. | 64 55 W. | |
| Iſl. Cuba { C. Antonio, W. point | Am. | Antilles I. | Atl. Ocean | 21 45 N. | 84 05 W. | |
| P. de Mais, E. point | | | | 20 03 N. | 74 52 W. | |
| [illegible] Cape | | | | 19 42 N. | 77 25 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Ifl. Cuba | | | | | | |
| St. Jago | Am. | Antilles I. | Atl. Ocean | 20 03 N. | 75 51 W. | |
| St. Mary | | | | 21 26 N. | 78 10 W. | |
| Le St. Eſprit | | | | 21 56 N. | 79 50 W. | |
| Havannah | | | | 23 12 N. | 81 45 W. | |
| B. Hondy | | | | 22 54 N. | 82 40 W. | |
| Cubbs Iſles | Am. | New Wales | Hudſon's Bay | 54 15 N. | 82 34 W | |
| Cubello | Aſia | Ind. Malab. | Indian Ocean | 7 50 N. | 71 55 E. | |
| C. Cumberland | Aſia | N. Hebrides | Pacif. Ocean | 14 40 S. | 166 47 E. | |
| Cumberland Iſles | Aſia | Society Iſles | Pacif. Ocean | 19 18 S. | 140 36 E. | |
| B. Cumberland | Am. | North Main | Davis's Str. | 66 40 N. | 65 20 W. | |
| Curaſſoa | Am. | Terra Firma | Atl. Ocean | 11 56 N. | 68 20 W. | |
| I. Cuzzola | Eu. | Turkey | Medit. Sea | 42 50 N. | 16 55 E. | |
| Cuſco | Am. | Peru | Inland | 12 25 S. | 73 35 W. | |
| I. Cypreſs | | | | | | |
| C. Baſſa, W. end | Aſia | Syria | Medit. Sea | 35 04 N. | 33 04 E. | |
| C. St. Andr. E. end | | | | 35 40 N. | 35 08 E. | |
| C. de Gaſſe, S. point | | | | 34 35 N. | 33 41 E. | |
| C. Grego, S. E. point | | | | 34 57 N. | 34 36 E. | |
| D. | | | | | | |
| Dabul | Aſia | India | Arabian Sea | 18 24 N. | 73 33 E. | |
| Dahlak | Aſia | Arabia | Red Sea | 15 50 N. | 41 44 E. | |
| Iſles of Danger | Aſia | Society Iſles | Pacif. Ocean | 10 15 S. | 165 50 W. | |
| I. Dageroort Light-houſe | Eu. | Livonia | Baltic Sea | 58 55 N. | 22 32 E. | |
| Dantzic | Eu. | Poland | Baltic Sea | 54 22 N. | 18 36 E. | |
| Str. Dardanels | Eu. | Turkey | Archipelago | 40 10 N. | 26 26 E. | |
| Gulph Darien | Am. | Terra Firma | Caribbean Sea | 8 45 N. | 76 35 W. | |
| Dartmouth | Eu. | England | Eng. Channel | 50 27 N. | 3 36 W. | 6h. 30m. |
| I. Dauphin | Am. | Louiſiana | G. Mexico | 29 40 N. | 87 53 W. | |
| St. David's Head | Eu. | Wales | St. Geo. Ch. | 51 55 N. | 5 22 W. | 6 00 |
| Fort St. David's | Aſia | India | Corom. Coaſt | 12 05 N. | 80 55 E. | |
| I. Deſeada | Am. | Carib. Iſles | Atl. Ocean | 16 36 N. | 61 10 W. | |
| C. Deſeada | Am. | T. del Fuego | Pacif. Ocean | 53 4 S. | 74 13 W. | |
| C. Deſire | Eu. | Nova Zem. | North Ocean | 77 45 N. | 79 20 E. | |
| C. Deſolation | Am. | Greenland | North Ocean | 61 45 N. | 47 00 W. | |
| Devil's Iſles | Eu. | Greenland | North Ocean | 80 00 N. | 11 43 E. | |
| Dewpoint | Aſia | India | B. Bengal | 16 07 N. | 81 47 E. | |
| I. Diego Rayes | Aſia | Ind. Malab. | Indian Ocean | 0 45 S.<br>0 30 N. | 70 25 E. | |
| I. Diego Garcia | Africa | India | Indian Ocean | 8 45 S. | 68 10 E. | |
| Str. Diemen | Aſia | Japan Iſles | Pacif. Ocean | 31 12 N. | 130 55 E. | |
| I. Dieu | Eu. | France | Bay of Biſcay | 46 26 N. | 2 20 W. | |
| Dieppe | Eu. | France | Eng. Channel | 49 55 N. | 1 12 E. | 10 30 |
| Digge's, or Dudley's Cape | Am. | Greenland | Baffin's Bay | 76 48 N. | 59 07 W. | |
| C. Diggs | Am. | Labradore | Hudſon's Bay | 62 45 N. | 78 50 W. | |
| Po. Diu | Aſia | India | Indian Ocean | 21 37 N. | 70 28 E. | |
| C. Dobbs | Am. | North Wales | Hudſon's Bay | 65 10 N. | 86 25 W. | |
| Dofar | Aſia | Arabia | Indian Ocean | 16 24 N. | 53 40 E. | |
| St. Domingo Hiſpaniola | Am. | Antilles | Atl. Ocean | 18 25 N. | 69 30 W. | |
| I. Dominica | Am. | Caribbee | Atl. Ocean | 15 18 N. | 61 28 W. | |
| Dordrecht | Eu. | D. Neth. | R. Maes | 52 00 N. | 4 26 E. | |
| C. Dorfui | Africa | Ajan | Indian Ocean | 10 15 N. | 50 44 E. | |
| C. Doro | Eu. | Turkey | Archipelago | 38 02 N. | 25 12 E. | |
| Dort | Eu. | D. Neth. | Germ. Ocean | 51 47 N. | 4 40 E. | 3 00 |
| I. Doſel | Eu. | Livonia | Baltic Sea | 58 20 N. | 23 00 E. | |
| Is. Do. Banhos | Africa | Zanguebar | Indian Ocean | 5 15 N. | 49 24 E. | |
| Dover | Eu. | England | Eng. Channel | 51 07 N. | 1 19 E. | 11 30 |
| Downs | Eu. | England | Germ. Ocean | 51 25 N. | 1 27 E. | 1 15 |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H.Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Po. Dradate | Africa | Egypt | Red Sea | 19 56 N. | 37 40 E. | |
| Po. Drake, ſir Francis | Am. | California | Pacif. Ocean | 38 45 N. | 128 35 W. | |
| Drontheim | Eu. | Norway | North Ocean | 63 26 N. | 11 08 E. | |
| Dublin | Eu. | Ireland | Iriſh Sea | 53 21 N. | 6 5 W. | 9h. 15m |
| Dunbar | Eu. | Scotland | Germ. Ocean | 55 58 N. | 2 22 W. | 2 30 |
| Dundalk | Eu. | Ireland | Iriſh Sea | 53 57 N. | 6 28 W. | |
| Dundee | Eu. | Scotland | Germ. Ocean | 56 26 N. | 2 48 W. | 2 15 |
| Dungarven | Eu. | Ireland | Atl. Ocean | 51 57 N. | 7 55 W. | 4 30 |
| Dungeneſs | Eu. | England | Eng. Channel | 50 53 N. | 0 59 E. | 9 45 |
| Duncanſby Head | Eu. | Scotland | Germ. Ocean | 58 40 N. | 2 57 W. | |
| Dunkirk | Eu. | France | Germ. Ocean | 51 02 N. | 2 27 E. | 0 00 |
| Dunnoſe | Eu. | I. White | Eng. Channel | 50 34 N. | 1 15 W. | 9 45 |
| Durazzo | Eu. | Turkey | Medit. Sea | 41 58 N. | 25 00 E. | |
| Duſky Bay | Aſia | N. Zealand | Pacif. Ocean | 45 47 S. | 166 23 E. | 10 57 |
| E | | | | | | |
| C. Eaſt | Am. | Statenland | Stra. le Maire | 54 54 S. | 64 47 W. | |
| Eaſter Iſl. | Am. | Chili | Pacif. Ocean | 27 7 S. | 109 42 W. | 2 00 |
| Edinburgh | Eu. | Scotland | Germ. Ocean | 55 58 N. | 3 7 W. | 4 30 |
| Edyſtone | Eu. | England | Eng. Channel | 50 8 N. | 4 20 W. | 5 30 |
| Egmont Iſle | Aſia | Society Iſles | Pacif. Ocean | 19 20 S. | 138 30 W. | |
| C. Egmont | Aſia | N. Zealand | Pacif. Ocean | 39 20 S. | 173 45 E. | |
| I. Elba | Eu. | Italy | Mediterran. | 42 52 N. | 10 38 E. | |
| R. Elbe mouth | Eu. | Germany | Germ. Ocean | 54 18 N. | 7 10 E. | 0 00 |
| Elbing | Eu. | Poland | Baltic Sea | 54 12 N. | 20 35 E. | |
| Elſingburgh | Eu. | Sweden | Baltic Sea | 56 00 N. | 13 35 E. | |
| Elſinore | Eu. | Denmark | Baltic Sea | 56 00 N. | 13 23 E. | |
| I. Elutheria { N.point<br>S.point | Am. | Bahama | Atl. Ocean | 25 45 N.<br>24 57 N. | 76 42 W.<br>75 53 W. | |
| Emblen | Eu. | Germany | Germ. Ocean | 53 05 N. | 7 26 E. | 0 00 |
| R. Emes mouth | Eu. | Germany | Germ. Ocean | 53 10 N. | 7 20 E. | 7 30 |
| Enchuyſen | Eu. | D. Neth. | Zuyder Sea | 52 43 N. | 5 06 E. | 0 00 |
| Endeavour R. | Aſia | N. Holland | Pacif. Ocean | 15 26 S. | 145 12 E. | |
| I. Engano, or Trompouſe | Am. | Sumatra | Indian Ocean | 6 00 S. | 102 35 E. | |
| B. Enhora | Eu. | Greenland | North Sea | 78 45 N. | 26 05 E. | |
| Epheſus | Aſia | Natolia | Archipelago | 38 00 N. | 27 53 E. | |
| Erramanga | Aſia | N. Hebrides | Pacif. Ocean | 18 44 S. | 169 20 E. | |
| Eſtaples | Eu. | France | Eng. Channel | 50 34 N. | 1 42 E. | 11 00 |
| Euſtatia | Am. | Caribbee | Atl. Ocean | 17 30 N. | 63 14 W. | |
| I. Exuma | Am. | Bahama | Atl. Ocean | 23 25 N. | 75 35 W. | |
| F | | | | | | |
| Fairhead | Eu. | Ireland | Weſt. Ocean | 55 19 N. | 6 20 W. | |
| C. Falcon | Africa | Barbary | Medit. Sea | 36 03 N. | 0 14 W. | |
| I. Falkland { E. end<br>Iſl. Anifant | Am. | Patagonia | Atl. Ocean | 51 05 S.<br>52 27 S. | 56 40 W.<br>61 53 W. | |
| Falmouth | Eu. | England | Eng. Channel | 50 8 N. | 5 0 W. | 5 30 |
| C. Falo | Eu. | Turkey | Archipelago | 40 12 N. | 24 27 E. | |
| C. Falſo | Africa | Caffers | Indian Ocean | 34 16 S. | 18 44 E. | |
| C. Faiſo | Africa | Zanguebar | Indian Ocean | 8 52 S. | 59 55 E. | |
| Falſterbom | Eu. | Sweden | Baltic Sea | 55 20 N. | 13 36 E. | |
| I. Fana | Eu. | Turkey | Medit. Sea | 40 14 N. | 19 32 E. | |
| R. Farate | Africa | Egypt | Red Sea | 21 40 N. | 36 29 E. | |
| Faro Head | Eu. | Scotland | Weſt. Ocean | 58 40 N. | 4 50 W. | |
| C. Farewell | Aſia | N. Zealand | Pacif. Ocean | 40 35 S. | 172 47 E. | |
| C. Farewell | Am. | Greenland | North Ocean | 59 37 N. | 42 37 W. | |
| C. Fartack | Aſia | Arabia | Indian Ocean | 15 41 N. | 51 31 E. | |
| C. Fear | Am. | Carolina | Atl. Ocean | 34 04 N. | 78 09 W. | |
| I. Fernand Noronha | Am. | Braſil | Atl. Ocean | 3 56 S. | 32 23 W. | |
| I. Felicudi, Lipari Iſle | Eu. | Italy | Medit. Sea | 38 33 N. | 14 51 E. | |
| I. Fermina | Eu. | Turkey | Archipelago | 37 24 N. | 25 05 E. | |
| I. Fernando Po | Africa | Guinea | Atl. Ocean | 3 02 N. | 3 33 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. Ferro | Africa | Canaries | Atl. Ocean | 27 48 N. | 17 40 W. | |
| C. Finiſterre | Eu. | Spain | Atl. Ocean | 42 52 N. | 9 16 W. | |
| I. Fironda | Aſia | Corea | Pacif. Ocean | 33 30 N. | 127 25 E. | |
| Flamborough Head | Eu. | England | Germ. Ocean | 54 08 N. | 0 11 E. | 4h. 00m. |
| I. Flores | Eu. | Azores | Atl. Ocean | 39 34 N. | 30 59 W. | |
| C. Florida | Am. | Florida | G. Mexico | 25 50 N. | 80 20 W. | 7 30 |
| Fluſhing | Eu. | D. Neth. | Germ. Ocean | 51 33 N. | 3 20 W. | 0 45 |
| I. Fly | Eu. | D. Neth. | Germ. Ocean | 53 16 N. | 5 35 E. | 7 30 |
| Forbiſher's Straits | Am. | Greenland | Atl. Ocean | 62 05 N. | 47 18 W. | |
| North Foreland | Eu. | England | Germ. Ocean | 51 28 N. | 1 25 E. | 9 45 |
| South Foreland | Eu. | England | Eng. Channel | 51 12 N. | 1 24 E. | 9 45 |
| Foreland Fair | Eu. | Ireland | North Ocean | 55 05 N. | 6 30 W. | |
| Foreland Fair | Eu. | Greenland | North Ocean | 79 18 N. | 10 50 E. | |
| Foreland Merchants | Am. | Greenland | North Ocean | 63 20 N. | 17 05 W. | |
| I. Formentaria | Eu. | Spain | Medit. Sea | 38 33 N. | 1 15 E. | |
| I. Formigas | Eu. | Azores | Atl. Ocean | 37 17 N. | 24 43 W. | |
| C. Formoſa | Africa | Guinea | Eth. Sea | 4 22 N. | 5 43 E. | |
| R. Formoſa | Africa | Guinea | Eth. Sea | 6 10 N. | 4 49 E. | |
| I. Formoſa { N. point<br>S. point | Aſia | China | Indian Ocean | { 21 25 N.<br>22 00 N. | 121 25 E.<br>120 40 E. | |
| I. Forteventura, S. W. end | Africa | Canaries | Atl. Ocean | 28 35 N. | 14 04 W. | |
| Foulneſs | Eu. | England | Germ. Ocean | 52 57 N. | 0 58 E. | 6 45 |
| Foulſound | Eu. | Greenland | North Ocean | 77 30 N. | 12 50 E. | |
| Fowey | Eu. | England | Eng. Channel | 50 25 N. | 4 30 W. | 5 15 |
| I. France, P. Louis | Africa | Madagaſcar | Indian Ocean | 20 10 S. | 57 33 E. | |
| C. St. Francis | Am. | Peru | Pacif. Ocean | 0 30 N. | 80 35 W. | |
| I. St. Franciſco | Africa | Zanguebar | Indian Ocean | 6 23 S. | 53 22 E. | |
| R. St. Franciſco | Am. | Braſil | Atl. Ocean | 10 55 S. | 36 30 W. | |
| C. Francois | Am. | Domingo | Atl. Ocean | 19 47 N. | 72 15 W. | |
| Frederickſtadt | Eu. | Norway | Sound | 59 00 N. | 11 10 E. | |
| French Keys | Am. | Bahama | Atl. Ocean | 21 30 N. | 72 10 W. | |
| Fretum Borough | Eu. | Ruſſia | North Ocean | 70 00 N. | 61 20 E. | |
| C. Frio | Am. | Braſil | Atl. Ocean | 23 00 S. | 40 11 W. | |
| R. Fugor | Africa | Zanguebar | Indian Ocean | 00 10 N. | 42 05 E. | |
| I. Fuego | Africa | De Verd | Atl. Ocean | 14 55 N. | 24 28 W. | |
| Furneaux Iſland | Aſia | Soc. Iſles | Pacif. Ocean | 17 11 S. | 143 07 W. | |
| B. Fuſhan | Aſia | China | Pacif. Ocean | 23 00 N. | 112 35 E. | |
| I. Fyal | Eu. | Azores | Atl. Ocean | 38 32 N. | 28 36 W. | 2 20 |
| G | | | | | | |
| I. Galla | Am. | Terra Firma | Pacif. Ocean | 2 40 N. | 79 35 W. | |
| R. Gallega | Am. | Patagonia | Atl. Ocean | 51 37 S. | 65 35 W. | |
| I. Gallego | Am. | Terra Firma | Pacif. Ocean | 1 40 N. | 104 35 W. | |
| Gallipoli | Eu. | Italy | Medit. Sea | 40 19 N. | 18 08 E. | |
| Gallipoly | Eu. | Turkey | Archipelago | 40 36 N. | 27 02 E. | |
| I. Gallita | Africa | Barbary | Medit. Sea | 37 42 N. | 9 03 E. | |
| C. Gallo | Aſia | I. Ceylon | Indian Ocean | 6 15 N. | 80 20 E. | |
| Is. Gallepago | Am. | Peru | Pacif. Ocean | { 2 00 N.<br>2 00 S. | 89 00 W. | |
| Gally Head | Eu. | Ireland | Weſt. Ocean | 52 40 N. | 9 30 W. | |
| Galway | Eu. | Ireland | Weſt. Ocean | 53 10 N. | 10 03 W. | |
| R. Gambia | Africa | Negroland | Atl. Ocean | 13 00 N. | 14 58 W. | |
| I. Gamo | Aſia | India | Indian Ocean | 3 05 S. | 77 25 E. | |
| C. Gardafui | Africa | Anian | Indian Ocean | 11 48 N. | 50 25 E. | |
| R. Garronne | Eu. | France | B. Biſcay | 45 30 N. | 1 05 W. | 3 00 |
| Gaſpey Bay | Am. | Acadia | G. St. Lawr. | 48 49 N. | 63 34 W. | 1 30 |
| C. de Gatt | Eu. | Spain | Medit. Sea | 36 32 N. | 2 05 W. | |
| C. Gear | Africa | Barbary | Atl. Ocean | 30 35 N. | 10 01 W. | |
| Genoa | Eu. | Italy | Medit. Sea | 44 25 N. | 8 41 E. | |
| C. St. George | Am. | Newfoundl. | Atl. Ocean | 48 28 N. | 57 43 W. | |
| C. George | Am. | S. Georgia | Atl. Ocean | 54 17 S. | 36 33 W. | |
| B. St. George | Am. | Newfoundl. | Atl. Ocean | 48 19 N. | 57 30 W. | |

I. St. George

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H.Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. St. George | Aſia | Natolia | Archipelago | 38 47 N. | 25 07 E. | |
| I. St. George | Eu. | Azores | Atl. Ocean | 38 39 N. | 28 00 W. | |
| St. George's Fort | Aſia | India | B. Bengal | 13 05 N. | 80 34 E. | |
| Gibraltar | Eu. | Spain | Medit. Sea | 36 05 N. | 5 17 W. | oh. oom. |
| Gilbert's Iſland | Am. | T. del Fuego | Pacif. Ocean | 55 13 S. | 71 4 W. | |
| I. Gilolo { N. point | Aſia | Spice Iſlands | Indian Ocean | { 2 30 N. | 128 00 E. | |
| S. point | | | | 1 30 S. | 129 25 E. | |
| Glaſgow | Eu. | Scotland | R. Clyde | 55 52 N. | 4 10 W. | |
| Glouceſter Iſles | Aſia | Society Iſles | Pacif. Ocean | 19 11 S. | 140 4 W. | |
| Glouceſter Iſles | Aſia | Society Iſles | Indian Ocean | 20 36 S. | 146 7 W. | |
| Goa | Aſia | India | Malabar | 15 31 N. | 73 50 E. | |
| Goes | Eu. | D. Neth. | Germ. Ocean | 51 39 N. | 4 05 E. | |
| Golfe triſte | Am. | Terra Firma | Carrib. Sea | 10 20 N. | 67 40 W. | |
| Gombroon | Aſia | Perſia | Perſian Gulf | 27 40 N. | 55 20 E. | |
| I. Gomero | Africa | Canaries | Atl. Ocean | 28 06 N. | 17 03 W. | |
| C. Gondewar | Aſia | India | B. Bengal | 16 55 N. | 82 55 E. | |
| C. Good Hope | Africa | Caffers | Indian Ocean | 34 29 S. | 18 28 E. | 3 00 |
| I. Gorea | Africa | Negroland | Atl. Ocean | 14 40 N. | 17 20 W. | |
| I. Gorgona | Eu. | Italy | Medit. Sea | 43 21 N. | 9 11 E. | |
| I. Gothland { N. end | Eu. | Sweden | Baltic Sea | { 58 00 N. | 20 15 E. | |
| S. end | | | | 56 58 N. | 19 37 E. | |
| Wiſby | | | | 57 40 N. | 19 50 E. | |
| I. Goto | Aſia | Corea | Pacif. Ocean | 34 25 N. | 125 50 E. | |
| Gottenberg | Eu. | Sweden | Sound | 57 42 N. | 11 44 E. | |
| Gottingen | Eu. | Germany | Inland | 51 32 N. | 9 58 E. | |
| Gower's Iſle | Aſia | N. Britain | Pacif. Ocean | 7 56 S. | 158 56 E. | |
| R. Grand | Am. | Paraguay | Atl. Ocean | 31 58 S. | 50 35 W. | |
| Granville | Eu. | France | Eng. Channel | 48 50 N. | 1 32 W. | 7 00 |
| C. De Grat | Am. | Newfoundl. | Atl. Ocean | 51 36 N. | 55 33 W. | |
| I. Gratioſa | Africa | Canaries | Atl. Ocean | 29 15 N. | 13 07 W. | |
| I. Gratioſa | Eu. | Azores | Atl. Ocean | 39 02 N. | 27 53 W. | |
| C. Gratios a Dios | Am. | New Spain | Carribbe. Sea | 14 48 N. | 82 15 W. | |
| Graveline | Eu. | France | Eng. Channel | 50 59 N. | 2 13 E. | 0 00 |
| Graveſend | Eu. | England | R. Thames | 51 35 N. | 0 20 E. | 1 30 |
| I. Grenada | Am. | Caribbee | Atl. Ocean | 11 52 N. | 61 39 W. | |
| Greenwich | Eu. | England | R. Thames | 51 29 N. | 0 05 E. | |
| C. Gremia | Eu. | Turkey | Archipelago | 40 33 N. | 26 20 E. | |
| Gripſwald | Eu. | Germany | Baltic Sea | 54 04 N. | 13 43 E. | |
| Grimeſby | Eu. | England | Germ. Ocean | 53 30 N. | 0 56 E. | |
| Groin, or C. Corunna | Eu. | Spain | B. Biſcay | 43 28 N. | 9 20 W. | 3 00 |
| I. Groy | Am. | Newfoundl. | Atl. Ocean | 50 56 N. | 55 35 W. | |
| I. Guadaloupe | Am. | Carribbee | Atl. Ocean | 16 00 N. | 61 55 W. | |
| Guayaquil | Am. | Peru | Pacif. Ocean | 2 10 S. | 81 05 W. | |
| I. Guernſey | Eu. | England | Eng. Channel | 49 30 N. | 2 47 W. | 1 30 |
| Gulf | Eu. | England | St. Geo. Ch. | 50 06 N. | 6 00 W. | |
| Gurjef | Aſia | Aſtracan | Caſpian Sea | 47 7 N. | 52 02 E. | |
| H | | | | | | |
| Hacluit's Headland | Eu. | Greenland | North Ocean | 79 55 N. | 12 00 E. | |
| I. Hainam { N. E. point | Aſia | China | Indian Ocean | { 19 45 N. | 110 13 E. | |
| S. W. point | | | | 18 22 N. | 108 13 E. | |
| Halifax | Am. | Nova Scotia | Weſtern Oc. | 44 46 N. | 63 20 W. | [illegible] 3[illegible] |
| I. Hall | Am. | Greenland | Atl. Ocean | 63 56 N. | 44 26 W. | |
| Hallitord | Eu. | Iceland | North Ocean | 64 30 N. | 27 15 W. | |
| Hamburgh | Eu. | Germany | R. Elbe | 53 34 N. | 9 55 E. | |
| Hare Iſle | Am. | Canada | R. St. Lawr. | 48 00 N. | 63 26 W. | |
| Harlem | Eu. | D. Neth. | Germ. Ocean | 52 24 N. | 4 [illegible] E. | |
| Hartland Point | Eu. | England | Briſtol Chan. | 51 06 N. | 4 3[illegible] W. | |
| Hartlepool | Eu. | England | Germ. Ocean | 54 4[illegible] N. | 0 5[illegible] | |
| Harwich | Eu. | England | Germ. Ocean | [illegible]2 11 N. | 1 18 | |
| C. Hatteras | Am. | Carolina | Atl. Ocean | 3[illegible] 24 N. | [illegible]6 2[illegible] | |
| Havannah | Am. | I. Cuba | Atl. Ocean | 23 1[illegible] N. | [illegible] | |
| Havre de Grace | Eu. | France | Eng. Channel | 49 [illegible] N. | [illegible] | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Hawke's Bay | Aſia | N. Zealand | Pacif. Ocean | 39 30 S. | 177 6 E. | |
| I. St. Helena | Africa | Caffers | Atl. Ocean | 15 55 S. | 5 44 W. | |
| Helie's Sound | Eu. | Greenland | North Ocean | 79 15 N. | 12 50 E. | |
| Is. Heligh's Land | Eu. | Norway | North Ocean | 65 15 N. | 9 30 E. | |
| C. Henlopen | Am. | Maryland | Atl. Ocean | 38 48 N. | 75 08 W. | |
| C. Henrietta Maria | Am. | New Wales | Hudſon's Bay | 55 10 N. | 84 00 W. | 12h. 00m. |
| C. Henry | Am. | Virginia | Atl. Ocean | 37 00 N. | 76 23 W. | 11 15 |
| Hervey's Iſle | Aſia | Society Iſles | Pacif. Ocean | 19 17 S. | 158 43 W. | |
| I. Heys | Eu. | France | B. Biſcay | 46 24 N. | 2 14 W. | |
| High Mount | Eu. | Greenland | North Ocean | 83 23 N. | 26 40 E. | |
| Hinchingbrook I. | Aſia | N. Hebrides | Pacif. Ocean | 17 25 S. | 168 38 E. | |
| Iſl. Hiſpaniola: C. Tiberoon, W. pt. | Am. | Antilles | Atl. Ocean | 18 17 N. | 74 24 W. | |
| S. Louis | | | | 18 19 N. | 73 11 W. | |
| C. St. Nichol. N. W. pt. | | | | 19 50 N. | 73 18 W. | |
| Po. Grave | | | | 18 28 N. | 72 42 W. | |
| St. Domingo | | | | 18 25 N. | 69 30 W. | |
| C. Raphael N. E. pt. | | | | 19 05 N. | 68 30 W. | |
| Hogſties | Am. | Bahama | Atl. Ocean | 21 41 N. | 73 25 W. | |
| New Holland: W. limit | Aſia | ——— | Indian Ocean | 25 30 S. | 111 10 E. | |
| N. Ditto | | | | 12 35 S. | 141 31 E. | |
| S. Ditto | | | | 43 38 S. | 146 00 E. | |
| E. Ditto | | | | 27 10 S. | 153 39 E. | |
| Holy Cape | Aſia | Siberia | North Ocean | 72 32 N. | 179 45 E. | |
| Holy Head | Eu. | Wales | Iriſh Sea | 53 23 N. | 4 40 W. | 1 30 |
| C. Honduras | Am. | New Spain | Caribbean Sea | 16 18 N. | 85 23 W. | |
| B. Hondy, I. Cuba | Am. | Antilles | Atl. Ocean | 22 54 N. | 82 40 W. | |
| Honfleur | Eu. | France | R. Seine | 49 24 N. | 0 20 E. | 9 00 |
| Hood's Iſle | Aſia | Marqueſas | Pacif. Ocean | 9 26 S. | 138 47 W. | |
| Hope Iſle | Eu. | Greenland | North Ocean | 76 22 N. | 23 40 E. | |
| C. Horn | Am. | T. del Fuego | Pacif. Ocean | 55 59 S. | 67 21 W. | |
| Hornſound | Eu. | Greenland | North Ocean | 76 41 N. | 13 36 E. | |
| La Hogue | Eu. | France | Eng. Channel | 49 45 N. | 1 52 W. | |
| Howe's Iſle | Aſia | Society's Iſ. | Pacif. Ocean | 16 46 S. | 154 2 W. | |
| C. How | Aſia | N. Holland | Pacif. Ocean | 37 24 S. | 150 00 E. | |
| R. Hughly | Aſia | India | B. Bengal | 21 45 N. | 89 15 E. | |
| Hull | Eu. | England | R. Humber | 53 50 N. | 0 28 W. | 6 00 |
| R. Humber, Ent. | Eu. | England | Germ. Ocean | 53 55 N. | 0 24 E. | 5 13 |
| I. Hyneago | Am. | Bahama | Atl. Ocean | 21 27 N. | 73 29 W. | |
| I | | | | | | |
| Jado | Aſia | Japan | Pacif. Ocean | 36 00 N. | 139 40 E. | |
| C. Jaffanapatan | Aſia | I. Ceylon | Indian Ocean | 9 47 N. | 80 55 E. | |
| I. Jago | Africa | C. Verd | Atl. Ocean | 15 07 N. | 23 30 W. | |
| Jakutſkoi | Aſia | Siberia | Pacif. Ocean | 62 2 N. | 129 52 E. | |
| I. Jam.: Weſt end | Am. | Weſt Indies | Atl. Ocean | 18 45 N. | 78 00 W. | |
| Port Royal | | | | 18 00 N. | 76 40 W. | |
| Eaſt End | | | | 17 58 N. | 76 05 W. | |
| James Town | Am. | Virginia | B. Cheſapeak | 37 30 N. | 76 00 W. | |
| R. Janeiro | Am. | Braſil | Atl. Ocean | 22 54 S. | 42 40 W. | |
| Japan Iſles | Aſia | ——— | Pacif. Ocean | 40 40 N.<br>31 45 N. | 141 25 E.<br>126 10 E. | |
| Java I.: C. Delo, W. pt. | Aſia | Siam | Indian Ocean | 6 50 S. | 105 15 E. | |
| | | | | 7 00 S. | | |
| Eaſt limit | | | | 8 30 S. | 115 55 E. | |
| Ice Cove | Am. | N. Main | Hudſ. Straits | 62 20 N. | 69 00 W. | 10 00 |
| Ice Point | Eu. | Nova Zem. | North. Ocean | 77 40 N | 69 10 E. | |
| Ice Sound | Eu. | Greenland | North Ocean | 78 13 N. | 12 00 E. | |
| I. Jerſey | Eu. | England | Eng. Channel | 49 07 N. | 2 26 W. | |
| Jeruſalem | Aſia | Paleſtine | Inland | 31 55 N. | 35 25 E. | |
| I. Hay, S. pt. | Eu. | Scotland | Weſt. Ocean | 55 39 N. | 6 20 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| R. Indus | Aſia | India | Indian Ocean | 25 50 N. | 66 33 E. | |
| Inverneſs | Eu. | Scotland | Germ. Ocean | 57 33 N. | 4 02 W. | |
| I. Joanna | Africa | Zanguebar | Indian Ocean | 12 05 S. | 45 45 E. | |
| Juddah | Aſia | Arabia | Red Sea | 22 00 N. | 39 27 E. | |
| C. St. John | Am. | Newfoundl. | Atl. Ocean | 50 08 N. | 55 32 W. | |
| C. St. John | Africa | Maiumbo | Eth. Ocean | 1 17 N. | 9 34 E. | |
| St. John's | Am. | Newfoundl. | Atl. Ocean | 47 34 N. | 52 18 W. | 6h. om. |
| I. St. John { E. pt. | Am. | Canada | Bay St. Laurence | 46 30 N. | 62 03 W. | |
| N. pt. | | | | 47 07 N. | 64 05 W. | |
| I. St. John de Nova | Africa | Madagaſcar | Indian Ocean | 17 00 S. | 44 02 E. | |
| St. John de Luz | Eu. | France | B. Biſcay | 43 10 N. | 1 38 W. | 3 30 |
| Cape Jones | Am. | New Britain | Hudſon's Bay | 58 50 N. | 79 00 W. | |
| Joppa | Aſia | Syria | Levant | 32 45 N. | 36 00 E. | |
| Jones Sound | Am. | Greenland | Baffin's Bay | 71 07 N. | 91 30 W. | |
| St. Joſeph | Am. | California | Pacif. Ocean | 23 3 S. | 109 35 W. | |
| Ipſwich | Eu. | England | Germ. Ocean | 52 14 N. | 1 00 E. | |
| Iſpahan | Aſia | Perſia | R. Zenduro | 32 25 N. | 52 55 E. | |
| C. St. Juan | Am. | Statenland | Atl. Ocean | 54 47 S. | 63 42 W. | |
| I. Juan Fernandez | Am. | Chili | Pacif. Ocean | 33 45 S. | 78 37 W. | |
| Port. St. Julian | Am. | Patagonia | S. Atl. Ocean | 49 10 S. | 66 10 W. | 4 45 |
| I. Ivica | Eu. | Spain | Medit. Sea | 38 54 N. | 1 15 E. | |
| K | | | | | | |
| Kalmer | Eu. | Sweden | Baltic Sea | 56 40 N | 17 25 E. | |
| Kambaya | Aſia | India | Indian Ocean | 23 36 N. | 72 50 E. | |
| Kamtſchatka { Lower | Aſia | Siberia | Pacif. Ocean | 56 11 N. | 159 25 E. | |
| Upper | | | | 54 48 N. | 157 25 E. | |
| I. Karaghinſkoy | Aſia | Siberia | Pacif. Ocean | 58 00 N. | 162 10 E. | |
| I. St. Katharine's | Am. | Braſil | Atl. Ocean | 27 35 S. | 49 12 W. | |
| Keſo | Aſia | Tonquin | Indian Ocean | 21 55 N. | 106 10 E. | |
| Kegor | Eu. | Muſcovy | North Ocean | 70 18 N. | 34 00 E. | |
| R. Kennebeck | Am. | N. England | Atl. Ocean | 44 00 N. | 69 45 W. | |
| Kentiſh Knock, a ſand | Eu. | England | Germ. Ocean | 51 42 N. | 1 45 E. | 0 00 |
| I. St. Kilda | Eu. | Scotland | Weſt. Ocean | 57 44 N. | 8 18 W. | |
| I. Kilduin | Eu. | Lapland | North Ocean | 69 30 N. | 31 20 E. | 7 30 |
| Kinſale | Eu. | Ireland | Atl. Ocean | 51 32 N. | 9 01 W. | 5 15 |
| Klip | Eu. | Greenland | North Ocean | 80 10 N. | 12 22 E. | |
| R. Kola | Eu. | Lapland | North Ocean | 68 53 N. | 33 08 E. | |
| C. Kol | Eu. | Sweden | Sound | 56 50 N. | 13 13 E. | |
| Port Komol | Africa | Abyſſinia | Red Sea | 22 30 N. | 36 17 E. | |
| Komero Iſles | Africa | Zanguebar | Indian Ocean | { 10 48 S. | 44 45 E. | |
| | | | | 13 10 S. | 46 22 E. | |
| R. Kowimia | Aſia | Siberia | North Ocean | 70 40 N. | 159 00 E. | |
| L | | | | | | |
| Ladrone, or Marian Iſles | Aſia. | ———— | Pacif. Ocean | { 21 00 N. | 144 00 E. | |
| | | | | 13 15 N. | 142 55 E. | |
| C. L'Aigulle | Africa | Caffraria | Indian Ocean | 34 50 S. | 20 06 E. | |
| Lancaſter | Eu. | England | St. Geo. Ch. | 54 42 N. | 4 36 W. | |
| I. Lancerota | Africa | Canaries | Atl. Ocean | 29 10 N. | 13 20 W. | |
| Land's End | Eu. | England | St. Geo. Ch. | 50 06 N. | 5 50 W. | 7 30 |
| Langeneſs | Eu. | Nova Zem. | North Ocean | 74 40 N | 53 36 E. | |
| I. Lambay | Eu. | Ireland | Iriſh Sea | 53 22 N | 7 30 W. | 8 15 |
| I. Lampedoſa | Africa | Tunis | Medit. Sea | 35 32 N. | 12 45 E. | |
| B. St. Lazarus | Am. | Patagonia | Pacif. Ocean | 48 42 S. | 73 35 W. | |
| C. Ledo | Africa | Angola | Atl. Ocean | 9 24 S. | 12 [illegible] E. | |
| Leith | Eu. | Scotland | Germ. Ocean | 55 58 N. | 2 59 W. | 4 30 |
| Leghorn | Eu. | Italy | Medit. Sea | 43 33 N. | 10 25 E. | |
| I. Lemnos | Aſia | Natolia | Archipelago | 40 02 N. | 25 36 E. | |
| C. Lengua | Eu. | Turkey | Medit. Sea | 40 44 N. | 19 36 E. | |
| Leoſtoff | Eu. | England | Germ. Ocean | 52 38 N | 1 54 E. | 9 45 |
| Lepanto | Eu. | Turkey | Medit. Sea | 38 20 N. | 22 03 E. | |
| Leper's Iſle | Aſia | N. Hebrides | Pacif. Ocean | 15 23 S | 167 [illegible] E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. Leſow | Eu. | Denmark | Sound | 57 05 N. | 11 06 E. | |
| Liverpool | Eu. | England | Iriſh Sea | 53 22 N. | 3 10 W. | 11h. 15m. |
| I. Lewes, N. point | Eu. | Scotland | Weſt. Ocean | 58 35 N. | 6 37 W. | 6 30 |
| Liampo, or Ningpo | Aſia | China | Pacif. Ocean | 29 58 N. | 120 23 E. | |
| Lima | Am. | Peru | Pacif. Ocean | 12 01 S. | 76 44 W. | |
| Lime | Eu. | England | Eng. Channel | 50 45 N. | 3 15 W. | 7 00 |
| Limerick | Eu. | Ireland | R. Shannon | 52 22 N. | 10 00 W. | |
| I. Limoſa | Eu. | Italy | Medit. Sea | 36 08 N. | 13 01 E. | |
| I. Lipari | Eu. | Italy | Medit. Sea | 38 35 N. | 15 31 E. | |
| I. Liqueo | Aſia | Japan | Pacif. Ocean | 28 00 N. | 127 30 E. | |
| Liſbon | Eu. | Portugal | R. Tagus | 38 42 N. | 9 4 W. | 2 15 |
| Liſbon Rock | Eu. | Portugal | Weſt. Ocean | 38 42 N. | 9 25 W. | |
| C. Liſburne | Aſia | N. Hebrides | Pacif. Ocean | 15 41 S. | 166 57 W. | |
| I. Liſſa | Eu. | Dalmatia | Adriatic Sea | 42 56 N. | 18 32 E. | |
| Lizard | Eu. | England | Eng. Channel | 49 57 N. | 5 10 W. | 7 30 |
| iſles Loffout { S. W. end | Eu. | Norway | North Ocean | 68 15 N. | 10 20 E. | |
| iſles Loffout { N. E. end | | | | 69 00 N. | 12 00 E. | |
| R. Loire, Ent. | Eu. | France | B. Biſcay | 47 07 N. | 2 05 W. | 3 00 |
| London | Eu. | England | R. Thames | 51 32 N. | 0 00 | 3 00 |
| New London | Am. | N. England | Weſt. Ocean | 41 50 N. | 72 14 W. | 1 30 |
| Londonderry | Eu. | Ireland | Weſt. Ocean | 55 01 N. | 7 31 W. | |
| Long Iſle | Am. | N. England | Weſt. Ocean | 41 00 N. | { 71 59 W.<br>74 20 W. | 3 00 |
| I. Longo | Eu. | Dalmatia | Adriat. Sea | 43 45 N. | 17 58 E. | |
| Longſand Head | Eu. | England | Germ. Ocean | 51 47 N. | 1 41 E. | 10 30 |
| Lookout Point | Eu. | Greenland | North Ocean | 76 40 N. | 16 25 E. | |
| C. Lopas | Africa | Loango | Atl. Ocean | 0 47 S. | 8 30 E. | |
| B. St. Louis | Am. | Louiſiana | G. Mexico | 28 50 N. | 97 08 W. | |
| Louiſbourg | Am. | C. Breton | B. St. Law. | 45 54 N. | 59 50 W. | |
| Lubec | Eu. | Germany | Baltic Sea | 54 00 N. | 11 40 E. | |
| C. St. Lucar | Am. | California | Pacif. Ocean | 23 15 N. | 109 40 W. | |
| R. Lucia | Africa | Caffers | Indian Ocean | 27 52 S. | 33 28 E. | |
| I. St. Lucia | Africa | C. de Verd | Atl. Ocean | 16 43 N. | 24 33 W. | |
| I. St. Lucia | Am. | Caribbee | Atl. Ocean | 13 25 N. | 60 46 W. | |
| I. Luconia { N. E. point | Aſia | Phil. Iſles | Pacif. Ocean | 19 25 N. | 121 45 E. | |
| I. Luconia { C. Bajador | | | | 18 50 N. | 120 25 E. | |
| I. Luconia { Manilla | | | | 14 36 N. | 120 58 E. | |
| I. Luconia { S. W. point | | | | 13 30 N. | 119 35 E. | |
| I. Luconia { E. point | | | | 14 00 N. | 124 00 E. | |
| Lunden | Eu. | Sweden | Baltic Sea | 55 42 N. | 13 26 E. | |
| I. Lundy | Eu. | England | St. Geo. Ch. | 51 20 N. | 4 04 W. | 5 15 |
| Lupis's Head | Eu. | Ireland | Weſt. Ocean | 52 24 N. | 10 15 W. | |
| Lynn | Eu. | England | Germ. Ocean | 52 46 N. | 0 32 E. | 6 45 |
| M | | | | | | |
| C. Mabo | Aſia | New Guinea | Pacif. Ocean | 0 40 S. | 130 05 E. | |
| Macao, or Makau | Aſia | China | Pacif. Ocean | 22 12 N. | 113 46 E. | |
| Macaſſar | Aſia | I. Celebes | Pacif. Ocean | 5 09 S. | 119 50 E. | |
| C. Machian | Eu. | Spain | B. Biſcay | 43 44 N. | 3 05 W. | |
| I. Madagaſcar { C. St. Mary, S. point | Africa | —— | Indian Ocean | 25 24 S. | 45 53 E. | |
| I. Madagaſcar { B. St. Auguſtine | | | | 25 35 S. | 43 13 E. | |
| I. Madagaſcar { Terra de Gada | | | | 19 36 S. | 43 46 E. | |
| I. Madagaſcar { C. St. Andrew | | | | 15 46 S. | 45 22 E. | |
| I. Madagaſcar { C. St. Sebaſtian | | | | 12 30 S. | 46 10 E. | |
| I. Madagaſcar { C. de Ambre N. point | | | | 12 15 S. | 50 15 E. | |
| I. Madagaſcar { B. d'Antongil | | | | 16 00 S. | 49 40 E. | |
| I. Madagaſcar { Antavare | | | | 20 57 S. | 47 48 E. | |
| I. Madagaſcar { Po. Dauphin | | | | 24 48 S. | 47 06 E. | |
| I. Madeira { Funchal | Africa | Canaries | Atl. Ocean | 32 38 N. | 17 [illegible] W. | 12 4 |
| I. Madeira { W. end | | | | 32 25 N. | 17 21 W. | 12 0 |
| Madraſs | Aſia | India | Indian Ocean | 13 5 N. | 80 34 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Madrid | Eu. | Spain | R. Manzana | 40 25 N. | 03 21 W. | |
| Madura | Aſia | India | Indian Ocean | 10 15 N. | 78 35 E. | |
| R. Maes, Mouth | Eu. | D. Neth. | Germ. Ocean | 52 06 N. | 3 50 E. | 1h. 30m. |
| Str. Le Maire | Am. | Patagonia | Atl. Ocean | 54 51 S. | 65 00 W. | |
| Magadoxa | Africa | Zanguebar | Indian Ocean | 2 53 N. | 45 25 E. | |
| Str. Magellan E. ent. | Am. | Patagonia | Atl. Ocean | 52 30 S. | 67 50 W. | |
| Str. Magellan W. ent. | | | Pacif. Ocean | 52 55 S. | 74 18 W. | |
| Magiſiland | Aſia | India | Malabar Coa. | 12 10 N. | 74 14 E. | |
| I. Maguana | Am. | Bahama I. | Atl. Ocean | 22 36 N. | 72 25 W. | |
| P. Mahon, Iſle Minorca | Eu. | Spain | Medit. Sea | 39 51 N. | 3 53 E. | |
| Majorca, Iſl. Majorca | Eu. | Spain | Medit. Sea | 39 35 N. | 2 35 E. | |
| C. Mala | Eu. | Turkey | Archipelago | 37 20 N. | 24 07 E. | |
| Malacca | Aſia | India | Str. Malacca | 2 12 N. | 102 10 E. | |
| Malaga | Eu. | Spain | Medit. Sea | 36 43 N. | 4 02 W. | |
| Iſles Maldive N. end | Aſia | India | Indian Ocean | 7 20 N. | 73 03 E. | |
| Iſles Maldive S. end | | | | 0 20 S. | 76 10 E. | |
| Maleſtroom Whirlpool | Eu. | Norway | Weſt. Ocean | 68 08 N. | 10 40 E. | |
| I. Malique | Aſia | Maldive I. | Indian Ocean | 7 45 N. | 72 40 E. | |
| St. Maloes | Eu. | France | Eng. Channel | 48 39 N. | 1 57 W. | 6 00 |
| I. Malta | Eu. | Italy | Medit. Sea | 35 54 N. | 14 28 E. | |
| I. Man, W. end | Eu. | England | Iriſh Sea | 53 45 N. | 5 00 W. | 9 00 |
| Mangalore | Aſia | India | Indian Ocean | 13 02 N. | 75 10 E. | |
| Manilla | Aſia | I. Luconia | Pacif. Ocean | 14 36 N. | 120 58 E. | |
| I. Mansfield, N. pt. | Am. | New Britain | Hudſon's Bay | 62 38 N. | 80 33 W. | |
| I. Manſia | Africa | Zanguebar | Indian Ocean | 8 36 S. | 40 40 E. | |
| I. Mardou | Eu. | Norway | Sound | 58 14 N. | 8 55 E. | |
| I. Margarita | Am. | Terra Firma | Atl. Ocean | 11 15 N. | 63 35 W. | |
| R. Maragnon | Am. | Braſil | Atl. Ocean | 1 48 S. | 44 17 W. | |
| Margate | Eu. | England | Eng. Channel | 51 29 N. | 1 10 E. | 11 15 |
| C. St. Maria | Eu. | Portugal | Atl. Ocean | 36 45 N. | 7 45 W. | |
| C. St. Maria, or Lucia | Eu. | Italy | Medit. Sea | 40 04 N. | 18 31 E | |
| Marian or Ladrone Iſles N. lim. | Aſia | ——— | Pacif. Ocean | 21 00 N. | 144 00 E. | |
| Marian or Ladrone Iſles S. lim. | | | | 13 15 N. | 142 55 E. | |
| I. St. Maries | Eu. | Azores | Atl. Ocean | 37 00 N. | 25 00 W. | |
| St. Maries | Eu. | I. Scilly | Eng. Channel | 49 57 N. | 6 38 W. | |
| I. Marigallante | Am. | Weſt Indies | Atl. Ocean | 16 00 N. | 61 10 W. | |
| I. Maritimo, Sicily | Eu. | India | Medit. Sea | 38 04 N. | 12 33 E. | |
| Marqueſa Iſ. | Aſia | ——— | Pacif. Ocean | 9 56 N. | 139 00 W. | 2 30 |
| C. Martelo | Eu. | Turkey | Medit. Sea | 38 00 N. | 26 00 E. | |
| St. Martha | Am. | Terra Firma | Atl. Ocean | 11 26 N. | 74 00 W. | |
| I. St. Martin | Am. | Weſt Indies | Atl. Ocean | 18 06 N. | 63 06 W. | |
| C. St. Martin | Africa | Caffers | Atl. Ocean | 32 08 S. | 18 58 E. | |
| C. St. Martin | Eu. | Spain | Medit. Sea | 38 44 N. | 0 25 E. | |
| I. Martinique, Port Royal | Am. | Weſt Indies | Atl. Ocean | 14 36 N. | 61 04 W. | |
| Marſeilles | Eu. | France | Medit. Sea | 43 18 N. | 5 27 E. | |
| C. St. Mary | Am. | Newfoundl. | Atl. Ocean | 46 52 N. | 54 01 W. | |
| C. St. Mary | Am. | Braſil | Atl. Ocean | 34 52 S. | 52 55 W. | |
| C. St. Mary | Aſia | Natolia | Archipelago | 37 46 N. | 27 21 E. | |
| C. St. Mary | Eu. | Spain | N. Atl. Ocean | 36 46 N. | 7 49 W. | |
| C. Virgin Mary | Am. | Patagonia | S. Atl. Ocean | 52 23 S. | 68 10 W. | |
| [illegible] | Am. | Chili | Pacif. Ocean | 33 45 S. | 80 34 W. | |
| I. Mauritius | Africa | Zanguebar | Indian Ocean | 20 52 S. | 55 35 E. | |
| I. [illegible] | Am. | Peru | Pacif. Ocean | 1 20 S. | 88 50 W. | |
| [illegible] | Aſia | Arabia | Indian Ocean | 23 10 N. | 57 40 E. | |
| Mallicollo Iſl. | Aſia | N. Hebrides | Pacif. Ocean | 16 32 S. | 167 59 E. | |
| Marſtrand | Eu. | Sweden | Sound | 57 53 N. | 12 00 E. | |
| Maſulipatam | Aſia | India | B. Bengal | 15 28 N. | 81 40 E. | |

C. Matapan

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| C. Matapan | Eu. | Turkey | Archipelago | 36 25 N. | 22 40 E. | |
| I. Matbare | Aſia | Japan | Pacif. Ocean | 26 30 N. | 137 00 E. | |
| I. St. Mathew's | Africa | Guinea | Eth. Ocean | 1 23 S. | 6 11 W. | |
| I. Mauritius | Africa | Madagaſcar | Indian Ocean | 20 10 S. | 57 33 E. | |
| Maurua | Aſia | Society Iſles | Pacif. Ocean | 16 26 S. | 152 33 W. | |
| I. May | Africa | C. Verd | Atl. Ocean | 15 10 N. | 23 00 W. | |
| C. May | Am. | Penſilvania | Atl. Ocean | 39 15 N. | 74 43 W. | |
| I. Mayette | Africa | Madagaſcar | Indian Ocean | 12 53 S. | 46 10 E. | |
| Mecca | Aſia | Arabia | Red Sea | 21 40 N. | 41 00 E. | |
| Medina | Aſia | Arabia | Red Sea | 24 58 N. | 39 53 E. | |
| I. Melada | Eu. | Dalmatia | Adriat. Sea | 42 40 N. | 19 34 E. | |
| Melinde | Africa | Zanguebar | Indian Ocean | 3 07 S. | 39 40 E. | |
| I. Melo | Eu. | Turkey | Archipelago | 36 41 N. | 25 05 E. | |
| Memel | Eu. | Courland | Baltic Sea | 55 48 N. | 22 23 E. | |
| Memiſſan | Eu. | France | B. Biſcay | 44 20 N. | 1 23 W. | 3h. 30m. |
| I. Menado | Aſia | I. Celebes | Pacif. Ocean | 1 36 N. | 122 25 E. | |
| C. Mendozin | Am. | California | Pacif. Ocean | 41 20 N. | 130 15 W. | |
| Mercury Bay | Aſia | N. Zealand | Pacif. Ocean | 36 50 S. | 175 12 E. | |
| R. Metaparvous | Am. | Bahama | Atl. Ocean | 21 58 N. | 74 13 W. | |
| Meſſina | Eu. | I. Sicily | Medit. Sea | 38 21 N. | 16 21 E. | |
| C. Meſurato | Africa | Tripoli | Medit. Sea | 32 18 N. | 16 36 E. | |
| I. Metylene — C. Sigre | Aſia | Natolia | Archipelago | 39 21 N. | 26 08 E. | |
| Metylene | | | | 39 11 N. | 26 47 E. | |
| Po. Olivica | | | | 39 00 N. | 26 50 E. | |
| I. Meun | Eu. | Denmark | Baltic Sea | 55 00 N. | 13 15 E. | |
| Mexico | Am. | Mexico | Inland | 19 54 N. | 100 01 W. | |
| Miatea | Aſia | Society Iſles | Pacif. Ocean | 17 52 S. | 148 1 W. | |
| I. St. Michael | Eu. | Azores | Atl. Ocean | 37 45 N. | 25 38 W. | |
| Middleburgh | Eu. | D. Neth. | Germ. Ocean | 51 37 N. | 3 58 E. | |
| Middleburgh, or Eaoowe | Aſia | Friendly Iſl. | Pacif. Ocean | 21 21 S. | 174 34 W. | |
| Milford | Eu. | Wales | St. Geo. Ch. | 51 45 N. | 5 15 W. | 5 15 |
| Milo, I. Milo | Aſia | Turkey | Archipelago | 36 41 N. | 25 05 E. | |
| Mill Iſles | Am. | North Main | Hudſon's Bay | 64 36 N. | 80 30 W. | |
| I. Mindanao — N. point | Aſia | Spice Iſlands | Pacif. Ocean | 9 40 N. | 124 25 E. | |
| S. E. pt. C. St. Auguſtine | | | | 6 40 N. | 126 25 E. | |
| S. W. pt. Caldera | | | | 7 00 N. | 121 25 E. | |
| S. point | | | | 3 50 N. | 124 43 E. | |
| I. Mindora | Aſia | Philip. Iſles | Pacif. Ocean | 13 00 N. | 119 37 E. | |
| I. Minorca — N. W. pt. | Eu. | Spain | Mediterran. | 39 58 N. | 3 54 E. | |
| S. E. pt. | | | | 40 24 N. | 4 18 E. | |
| G. Miquelon | Am. | Newfoundl. | Atl. Ocean | 47 3 N. | 56 13 W. | |
| L. Miquelon | Am. | Newfound. | Atl. Ocean | 46 50 N. | 56 13 W. | |
| I. Miſco | Am. | Nova Scotia | G. St. Lawr. | 48 04 N. | 64 19 W. | |
| C. Miſerata | Africa | Guinea | Atl. Ocean | 6 25 N. | 9 35 W. | |
| R. Miſſiſſippi, mouth | Am. | Louiſiana | G. Mexico | 29 00 N. | 89 17 W. | |
| Mizen Head | Eu. | Ireland | Atl. Ocean | 51 16 N. | 10 20 W. | |
| Mocha | Aſia | Arabia | Red Sea | 13 45 N. | 44 04 E. | |
| Modon | Eu. | Turkey | Medit. Sea | 36 55 N. | 21 03 E. | |
| I. Mohilla | Africa | Zanguebar | Indian Ocean | 11 55 S. | 45 00 E. | |
| I. Monſerat | Am. | Weſt Indies | Atl. Ocean | 16 48 N. | 62 12 W. | |
| Montagu Iſle | Aſia | N. Hebrides | Pacif. Ocean | 17 26 S. | 168 36 E. | |
| Montreal | Am. | Canada | R. St. Lawr. | 45 52 N. | 73 11 W. | |
| I. Monte Chriſto | Eu. | Italy | Medit. Sea | 42 17 N. | 10 28 E. | |
| C. Monte Sancto | Eu. | Turkey | Archipelago | 40 27 N. | 24 39 E. | |
| Monument | Aſia | N. Hebrides | Pacif. Ocean | 17 14 S. | 168 38 E. | |
| Mount St. Michael | Eu. | France | Eng. Channel | 48 39 N. | 1 35 W. | |
| I. Morgo | Aſia | Natolia | Archipelago | 36 55 N. | 26 30 E | |
| Morlaix | Eu. | France | Eng. Channel | 48 30 N. | 3 50 W. | |
| Mort Point | Eu. | England | St. Geo. Ch. | 51 12 N. | 4 40 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Moſambique | Africa | Zanguebar | Indian Ocean | 15 00 S. | 41 40 E. | |
| Moſcow | Eu. | Ruſſia | R. Moſcow | 55 45 N. | 37 51 E. | |
| Moſquitos Bank | Am. | Mexico | Atl. Ocean | 14 45 N. | 80 05 W. | |
| C. Mount | Africa | Guinea | Atl. Ocean | 7 12 N. | 10 44 W. | |
| Mount's Bay | Eu. | England | Eng. Channel | 50 05 N. | 5 45 W. | 4h. 30m. |
| Mouſe River | Am. | New Wales | Hudſon's Bay | 51 25 N. | 83 15 W. | |
| C. Muſaldon | Aſia | Arabia | Perſian Gulf | 26 04 N. | 55 22 E. | |
| N | | | | | | |
| C. Nabo | Aſia | Japan | Pacif. Ocean | 40 35 N. | 141 25 E. | |
| Nangaſack | Aſia | Japan | Pacif. Ocean | 32 32 N. | 128 50 E. | |
| Nankin | Aſia | China | Pacif. Ocean | 32 07 N. | 118 35 E. | |
| Nantes | Eu. | France | B. Biſcay | 47 13 N. | 1 29 W. | 3 00 |
| Nantucket Iſle | Am. | New Eng. | Weſt. Ocean | 41 34 N. | 69 40 W. | |
| Naples | Eu. | Italy | Medit. Sea | 40 51 N. | 14 19 E. | |
| Narbonne | Eu. | France | Medit. Sea | 43 11 N. | 3 05 E. | |
| Narſinga | Aſia | India | B. Bengal | 18 05 N. | 85 20 E. | |
| Narva | Eu. | Livonia | G. Finland | 59 08 N. | 29 18 E. | |
| I. Naſſau | Aſia | Sumatra | Indian Ocean | 3 00 S. | 100 25 E. | |
| C. Naſſau | Am. | Terra Firma | Atl. Ocean | 7 53 N. | 58 07 W. | |
| Naſſau Str. | Eu. | Ruſſia | North Ocean | 69 55 N. | 57 30 E. | |
| B. Natal | Africa | Caffers | Indian Ocean | 29 25 S. | 33 10 E. | |
| I. Naxos | Eu. | Turkey | Archipelago | 37 06 N. | 25 58 E. | |
| Naze | Eu. | Norway | Weſt. Ocean | 57 50 N. | 7 32 E. | 11 15 |
| Needles | Eu. | England | Eng. Channel | 50 41 N. | 1 28 W. | 10 15 |
| C. Negrailles | Aſia | Pegu | B. Bengal | 16 20 N. | 94 15 E. | |
| C. Negro | Africa | Caffers | Atl. Ocean | 16 30 S. | 11 30 E. | |
| C. Negro | Africa | Barbary | Medit. Sea | 37 17 N. | 9 09 E. | |
| Negropont | Eu. | Turkey | Archipelago | 38 30 N. | 24 05 E. | |
| Port Nelſon | Am. | New Wales | Hudſon's Bay | 57 07 N. | 92 37 W. | |
| Port Nelſon's Shoals | Am. | New Wales | Hudſon's Bay | 57 35 N. | 92 07 W. | 8 20 |
| I. Nevis | Am. | Caribbee Iſles | Atl. Ocean | 17 11 N. | 62 52 W. | |
| Newcaſtle | Eu. | England | Germ. Ocean | 55 03 N. | 1 28 W. | 3 15 |
| R. Nicaragua | Am. | New Spain | Atl. Ocean | 11 40 N. | 82 47 W. | |
| Nice | Eu. | Italy | Medit. Sea | 43 42 N. | 7 22 E. | |
| Iſ. Nicobar | Aſia | Siam | B. Bengal | 7 22 N. | 94 40 E. | |
| I. St. Nicholas | Africa | C. Verd I. | Atl. Ocean | 16 35 N. | 24 06 W. | |
| Nicotera | Eu. | Italy | Medit. Sea | 38 33 N. | 16 30 E. | |
| Nieuport | Eu. | Flanders | Germ. Ocean | 51 08 N. | 2 50 E. | 12 00 |
| Ninhay | Aſia | China | Pacif. Ocean | 37 10 N. | 122 25 E. | |
| Ningpo, or Liampo | Aſia | China | Pacif. Ocean | 29 58 N. | 120 23 E. | |
| I. Nio | Eu. | Turkey | Archipelago | 36 48 N. | 26 02 E. | |
| I. Noel | Aſia | India | Indian Ocean | 10 30 S. | 105 25 E. | |
| C. Noir | Am. | T. del Fuego | Pacif. Ocean | 54 32 S. | 73 3 W. | |
| Norfolk Iſle | Aſia | N. Holland | Pacif. Ocean | 29 2 S. | 168 15 E. | |
| C. de Non | Africa | Barbary | Atl. Ocean | 28 04 N. | 10 32 W. | |
| Nombre de Dios | Am. | Terra Firma | Carribbe. Sea | 9 43 N. | 78 35 W. | |
| Nore | Eu. | England | R. Thames | 51 28 N. | 0 48 E. | 0 00 |
| Noriton | Am. | Penſylvania | Inland | 40 10 N. | 75 17 W. | |
| C. North | Am. | Terra Firma | Atl. Ocean | 1 45 N. | 49 00 W. | |
| C. North | Am. | C. Breton | Atl. Ocean | 47 5 N. | 60 8 W. | |
| C. North | Am. | S. Georgia | Atl. Ocean | 54 5 S. | 38 10 W. | |
| N. Cape, I. Maggoroe | Eu. | Lapland | North Ocean | 71 10 N. | 26 02 E. | 3 00 |
| North Point | Eu. | Norway | North Ocean | 62 15 N. | 6 15 E. | |
| North Bluff | Am. | North Main | Hudſon's Str. | 62 30 N. | 70 59 W. | |
| I. Nottingham, E. pt. | Am. | New Britain | Hudſon's Str. | 63 35 N. | 77 48 W. | 10 00 |
| O | | | | | | |
| Oaite Peha Bay | Aſia | Otaheite | Pacif. Ocean | 17 46 S. | 149 9 W. | |
| Oczakow | Eu. | Turkey | Black Sea | 45 12 N. | 34 40 E. | |
| I. Oeland { S. end<br>N. end | Eu. | Sweden | Baltic Sea | 56 15 N.<br>57 23 N. | 16 35 E.<br>17 36 E. | |
| O[illegible] Bay | Aſia | Society Iſles | Ulietea | 16 46 S. | 151 33 W. | 11 20 |
| Old Head of Kinſale | Eu. | Ireland | Atl. Ocean | 51 30 N. | 8 [illegible] W. | |

I. Oleron

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. Oleron | Eu. | France | B. Biſcay | 46 03 N. | 1 20 W. | |
| Olinde | Am. | Braſil | S. Atl. Ocean | 8 13 S. | 35 00 W. | |
| Oliva | Eu. | Germany | Baltic Sea | 54 20 N. | 18 30 E. | |
| Ollone | Eu. | France | B. Biſcay | 46 32 N. | 1 36 W. | 3h. 45m. |
| Onegka | Eu. | Italy | Medit. Sea | 43 57 N. | 7 52 E. | |
| Oporto | Eu. | Portugal | Atl. Ocean | 41 10 N. | 8 22 W. | |
| Oran | Africa | Barbary | Medit. Sea | 35 45 N. | 0 00 | |
| C. Orange | Am. | Terra Firma | Atl. Ocean | 4 27 N. | 50 50 W. | |
| Orbitello | Eu. | Italy | Medit. Sea | 42 30 N. | 12 00 E. | |
| I. Orchillo | Am. | Terra Firma | Carribean Sea | 11 32 N. | 65 25 W. | |
| Orenburg | Aſia | Aſtracan | Inland | 51 46 N. | 55 14 E. | |
| Orfordneſs | Eu. | England | Germ. Ocean | 52 17 N. | 1 11 E. | 9 45 |
| Orkney Iſles, limits | Eu. | Scotland | Weſt. Ocean | 59 24 N.<br>58 44 N. | 3 23 W.<br>2 11 W. | 3 00 |
| New Orleans | Am. | Louiſiana | R. Miſſiſſipi | 30 00 N. | 89 54 W. | |
| I. Ormus | Aſia | Perſia | G. Perſia | 27 30 N. | 55 17 E. | |
| C. del Oro, or Olerada | Africa | Negroland | Atl. Ocean | 23 30 N. | 14 31 W. | |
| R. Oronoque | Am. | Terra Firma | Atl. Ocean | 8 08 N. | 59 50 W. | |
| C. Oropeſo | Eu. | Spain | Medit. Sea | 40 20 N. | 0 49 E. | |
| Orſk | Aſia | Aſtracan | Inland | 51 12 N. | 58 37 E. | |
| C. Ortegal | Eu. | Spain | B. Biſcay | 43 47 N. | 8 32 W. | |
| Ortona | Eu. | Italy | Medit. Sea | 42 19 N. | 14 37 E. | |
| I. Oruba | Am. | Terra Firma | Caribbean Sea | 12 03 N. | 69 03 W. | |
| Oſnaburg Iſle | Aſia | Society Iſles | Pacif. Ocean | 22 00 S. | 141 34 W. | |
| Oſtend | Eu. | Flanders | Germ. Ocean | 51 14 N. | 3 00 E. | 12 00 |
| C. Otranto | Eu. | Italy | Medit. Sea | 40 23 N. | 17 41 E. | |
| Owharre Bay | Aſia | Huaheine | Pacif. Ocean | 16 44 S. | 151 3 W. | |
| Ozaca | Aſia | Japan | Pacif. Ocean | 35 10 N. | 134 05 E. | |
| P | | | | | | |
| C. Padron | Africa | Congo | Atl. Ocean | 6 00 S. | 11 40 E. | |
| Paita | Am. | Peru | Pacif. Ocean | 5 20 S. | 80 35 W. | |
| C. Faillouri | Eu. | Turkey | Archipelago | 39 59 N. | 24 03 E. | |
| Palermo, I. Sicily | Eu. | Italy | Medit. Sea | 38 10 N. | 13 43 E. | |
| Palliakate | Aſia | India | B. Bengal | 13 40 N. | 80 50 E. | |
| Palliſer's Iſles | Aſia | Society Iſles | Pacif. Ocean | 15 38 S. | 146 25 W. | |
| C. Palliſer | Aſia | N. Zealand | Pacif. Ocean | 41 40 S. | 175 28 E. | |
| I. Palma | Africa | Canaries | Atl. Ocean | 28 36 N. | 17 45 W. | |
| I. Palmaria | Eu. | Italy | Medit. Sea | 41 00 N. | 13 03 E. | |
| Palmerſton's Iſle | Aſia | Society Iſles | Pacif. Ocean | 18 00 S. | 162 52 W. | |
| C. Palmiras | Aſia | India | B. Bengal | 20 40 N. | 87 35 E. | |
| C. Palmas | Africa | Guinea | Atl. Ocean | 4 26 N. | 5 56 W. | |
| Panama | Am. | Mexico | Pacif. Ocean | 8 45 N. | 80 16 W. | |
| I. Panaria | Eu. | Italy | Medit. Sea | 38 40 N. | 15 41 E. | |
| Panorma | Eu. | Turkey | Medit. Sea | 40 05 N. | 21 40 E. | |
| I. Pantalaria | Eu. | Italy | Medit. Sea | 36 55 N. | 12 31 E. | |
| R. Panuco | Am. | Mexico | G. Mexico | 24 02 N. | 100 13 W. | |
| R. Paraiba | Am. | Braſil | Atl. Ocean | 21 26 S. | 39 50 W. | |
| Paris | Eu. | France | R. Seine | 48 50 N. | 2 25 E. | |
| C. Paſſero | Eu. | I. Sicily | Medit. Sea | 36 35 N. | 15 22 E. | |
| C. Patam | Aſia | Malacca | Indian Ocean | 7 27 N. | 101 20 E. | |
| I. Patmos | Aſia | Natolia | Archipelago | 37 22 N. | 26 48 E. | |
| R. Pattahan | Aſia | I. Sumatra | Str. Malacca | 0 28 N. | 102 25 E. | |
| C. Paul | Eu. | Spain | Medit. Sea | 37 50 N. | 0 15 W. | |
| I. St. Paul | Am. | Newfoundl. | B. St. Lawr. | 47 12 N. | 59 59 W. | |
| I. St. Paul | Aſia | Madagaſcar | Indian Ocean | 37 51 S. | 77 53 E. | |
| St. Paul de Leon | Eu. | France | Eng. Channel | 48 41 N. | 3 55 W. | 4 00 |
| I. Paxeros | Am. | California | Pacif. Ocean | 30 18 N. | 120 45 W. | |
| I. Pearl, or Serana | Am. | Weſt Indies | Atl. Ocean | 14 55 N. | 79 00 W. | |
| Pegu | Aſia | India | B. Bengal | 17 00 N. | 96 58 E. | |
| Pekin | Aſia | China | Inland | 39 55 N. | 116 29 E. | |
| I. Pelugoſa | Eu. | Italy | Adriatic Sea | 42 20 N. | 18 32 E. | |
| I. Pemba | Africa | Zanguebar | Indian Ocean | 5 38 S. | 40 09 E. | |

C. Pembroke

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H.Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| C. Pembroke | Am. | New Wales | Hudſon's Bay | 63 12 N. | 82 54 W. | |
| I. Pengwin | Am. | Newfoundl. | Atl. Ocean | 47 23 N. | 56 56 W. | |
| Penmark | Eu. | France | B. Biſcay | 47 50 N. | 4 20 W. | |
| R. Penobſcot | Am. | New Eng. | Atl. Ocean | 44 40 N. | 68 52 W. | |
| Pernambuco | Am. | Braſil | S. Atl. Ocean | 8 30 S. | 35 07 W. | |
| Petapoli | Aſia | India | B. Bengal | 16 14 N. | 81 10 E. | |
| I. St. Peter | Am. | Newfoundl. | Atl. Ocean | 46 46 N. | 56 5 W. | |
| Peterſburg | Eu. | Ruſſia | Baltic Sea | 59 56 N. | 30 24 E. | |
| C. Petra | Aſia | Natolia | Archipelago | 37 02 N. | 27 38 E. | |
| Peverel Point | Eu. | England | Eng. Channel | 50 34 N. | 1 22 W. | |
| Philadelphia | Am. | Penſilvania | R. Delawar | 39 57 N. | 75 8 W. | |
| St. Philip | Africa | Benguela | Atl. Ocean | 12 22 S. | 13 20 E. | |
| I. Pianoſa | Eu. | Italy | Medit. Sea | 42 46 N. | 10 34 E. | |
| Iſle of Pines | Aſia | N.Caledonia | Pacif. Ocean | 22 38 S. | 167 43 E. | |
| I. Pico (Pike) | Eu. | Azores | Atl. Ocean | 38 29 N. | 28 19 W. | |
| C. Pinas | Eu. | Spain | B. Biſcay | 43 51 N. | 6 14 W. | |
| Pickerſgill's I. | Am. | S. Georgia | Atl. Ocean | 54 42 S. | 36 53 W. | |
| Mo. Pintados, or St. Martin | Am. | California | Pacif. Ocean | 27 30 N. | 117 15 W. | |
| Piſcadore Iſles | Aſia | China | Pacif. Ocean | 23 30 N. | 119 25 E. | |
| Pitcairn's Iſles | Am. | Chili | Pacif. Ocean | 25 2 S. | 133 21 W. | |
| Placentia | Am. | Newfoundl. | Atl. Ocean | 47 15 N. | 53 43 W. | 9h. 00m. |
| R. Plata | Am. | La Plata | Atl. Ocean | 36 00 S. | 57 40 W. | |
| R. Platewrack | Am. | Bahama Iſles | Atl. Ocean | 20 04 N. | 68 37 W. | |
| Plymouth | Eu. | England | Eng. Channel | 50 22 N. | 4 10 W. | 6 00 |
| Policaſtro | Eu. | Italy | Medit. Sea | 40 18 N. | 15 45 E. | |
| Is. Polſapate | Aſia | Cambaya | Indian Ocean | 9 45 N. | 109 55 E. | |
| I. Poma | Eu. | Dalmatia | Adriat. Sea | 42 57 N. | 18 14 E. | |
| Pondicherry | Aſia | India | B. Bengal | 11 42 N. | 79 58 E. | |
| Pontorſon | Eu. | France | Eng. Channel | 48 33 N. | 1 27 W. | |
| Ponoi | Eu. | Lapland | North Sea | 67 5 N. | 38 48 E. | |
| I. Ponza | Eu. | Italy | Medit. Sea | 40 53 N. | 13 09 E. | |
| Pool | Eu. | England | Eng. Channel | 51 00 N. | 1 50 W. | |
| Porto Port | Eu. | Portugal | Atl. Ocean | 41 10 N. | 8 22 W. | |
| Port Mahon | Eu. | Spain | Medit. Sea | 39 51 N. | 3 53 E. | |
| Portland | Eu. | England | Eng. Channel | 50 30 N. | 2 48 W. | 8 15 |
| Port l'Orient | Eu. | France | B. Biſcay | 47 47 N. | 3 13 W. | |
| Porto Bello | Am. | New Spain | Carrib. Sea | 9 33 N. | 79 45 W. | |
| Porto Praya | Africa | Cape Verd | Atl. Ocean | 14 54 N. | 23 24 W. | 11 00 |
| I. Porto Rico, E. point | Am. | Antilles | Atl. Ocean | 18 35 N. | 65 58 W. | |
| Port Rico | | | | 18 29 N. | 66 35 W. | |
| W. point | | | | 18 34 N. | 67 51 W. | |
| I. Porto Sancto | Africa | Canaries | Atl. Ocean | 32 58 N. | 16 20 W. | |
| Portſall | Eu. | France | Eng. Channel | 48 36 N. | 4 43 W. | |
| Portſmouth.R.Aca. | Eu. | England | Eng. Channel | 50 48 N. | 1 01 W. | 11 15 |
| Praken | Aſia | Coch. Chi. | Indian Ocean | 17 15 N. | 106 15 E. | |
| Prenau | Eu. | Livonia | Baltic Sea | 58 26 N. | 24 58 E. | |
| I. Princes | Africa | Guinea | Atl. Ocean | 1 47 N. | 6 39 E. | |
| C. Prior | Eu. | Spain | Atl. Ocean | 43 29 N. | 8 15 W. | |
| I. Providence | Am. | Bahama | Atl. Ocean | 24 51 N. | 77 01 W. | |
| I. Providence, or St. Catherine | Am. | Mexico | Atl. Ocean | 13 26 N. | 80 42 W. | |
| Padyoua | Aſia | N.Caledonia | Pacif. Ocean | 20 18 S. | 164 46 E. | 6 00 |
| I. Pulo condor | Aſia | Cambaya | Indian Ocean | 8 40 N. | 107 25 E. | |
| Q | | | | | | |
| Quaqua, or Ivory Coaſt | Africa | Guinea | Eth. Sea | 5 00 N. | 4 00 W. | |
| Quebec | Am. | Canada | R. St. Lawr. | 46 55 N. | 69 48 W. | 7 30 |
| Queda | Aſia | Malaya | B. Bengal | 6 15 N. | 100 12 E. | |
| I. Quelpert | Aſia | Korea | Pacif. Ocean | 33 32 N. | 128 04 E. | |
| Quiloa | Africa | Zanguebar | Indian Ocean | 9 30 S. | 39 09 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Quimper | Eu. | France | B. Biſcay | 47 58 N. | 4 02 W. | |
| Quinam | Aſia | Coch. Chi. | Indian Ocean | 12 52 N. | 109 10 E. | |
| Quiraba Iſles | Africa | Zanguebar | Indian Ocean | 11 00 N. | 41 39 E. | |
| C. Quiros | Aſia | N. Hebrides | Pacif. Ocean | 14 56 S. | 167 15 W. | |
| Quito | Am. | Peru | Inland | 0 13 S. | 77 50 W. | |
| R | | | | | | |
| C. Race | Am. | Newfoundl. | Atl. Ocean | 46 40 N. | 52 38 W. | |
| Raguſa | Eu. | Dalmatia | Medit. Sea | 42 45 N. | 20 00 E. | |
| Rajapour | Aſia | India | Indian Ocean | 17 19 N. | 73 50 E. | |
| Ramſgate | Eu. | England | Downs | 51 20 N. | 1 22 E. | |
| Ramhead | Eu. | England | Eng. Channel | 50 19 N. | 4 15 W. | |
| C. Raſalgate | Aſia | Arabia | Indian Ocean | 22 46 N. | 58 48 E. | |
| Ravenna | Eu. | Italy | Medit. Sea | 44 26 N. | 12 21 E. | |
| C. Ray | Am. | Newfoundl. | Atl. Ocean | 47 37 N. | 59 8 W. | |
| I. Rhee | Eu. | France | B. Biſcay | 46 15 N. | 1 28 W. | 3h. 00m. |
| Regio | Eu. | Italy | Mediterran. | 38 22 N. | 16 37 E. | |
| Cape Reſolution | Am. | N. Main | Hudſon's Str. | 61 29 N. | 65 10 W. | |
| Reſolution Bay | Aſia | Marqueſas | Pacif. Ocean | 9 55 S. | 139 4 W. | |
| Reſolution Iſland | Aſia | Society Iſles | Pacif. Ocean | 17 23 S. | 141 40 W. | |
| Revel | Eu. | Livonia | Baltic Sea | 59 22 N. | 25 33 E. | |
| Rhodes Iſle: Rhodes, N. end | Aſia | Natolia | Archipelago | 36 27 N. | 28 36 E. | |
| Rhodes Iſle: C. Tranquil, S. end | | | | 35 55 N. | 28 23 E. | |
| Riga | Eu. | Livonia | Baltic Sea | 56 55 N. | 24 51 E. | |
| Ripraps, a ſand | Eu. | England | Straits Dover | 51 53 N. | 1 25 E. | |
| Robin Hood's Bay | Eu. | England | Germ. Ocean | 54 25 N. | 0 08 W. | 3 00 |
| I. Rocca | Am. | Terra Firma | Atl. Ocean | 11 21 N. | 66 17 W. | |
| Rochefort | Eu. | France | B. Biſcay | 46 03 N. | 0 54 W. | 4 15 |
| Rochel | Eu. | France | Bay Biſcay | 46 10 N. | 1 5 W. | 3 45 |
| Rocheſter | Eu. | England | R. Medway | 51 26 N. | 0 30 E. | 0 45 |
| I. Rodrigue | Aſia | Madagaſcar | Indian Ocean | 19 41 S. | 62 45 E. | |
| C. Romain | Am. | Terra Firma | Atl. Ocean | 11 40 N. | 69 05 W. | |
| Rome | Eu. | Italy | Medit. Sea | 41 54 N. | 12 34 E. | |
| I. Roncadore | Am. | Mexico | Atl. Ocean | 13 30 N. | 78 53 W. | |
| Rood Bay | Eu. | Greenland | North Ocean | 79 53 N. | 14 00 E. | |
| C. Roque | Am. | Braſil | Atl. Ocean | 5 00 S. | 35 43 W. | |
| I. Roquepiz | Africa | Madagaſcar | Indian Ocean | 9 51 S. | 64 30 E. | |
| G. Roſes | Eu. | Spain | Medit. Sea | 42 10 N. | 3 18 E. | |
| Roſtock | Eu. | Germany | Baltic Sea | 54 10 N. | 12 50 E. | |
| I. Rotterdam | Aſia | Friendly Is. | Pacif. Ocean | 20 16 S. | 174 25 W. | |
| Rotterdam | Eu. | D. Neth. | Germ. Ocean | 51 56 N. | 4 33 E. | 3 00 |
| Rouen | Eu. | France | R. Seine | 49 27 N. | 1 10 E. | 1 15 |
| C. Roxant | Eu. | Portugal | Atl. Ocean | 38 45 N. | 9 30 W. | |
| C. Roxo | Africa | Negroland | Atl. Ocean | 11 42 N. | 14 33 W. | |
| Po. Royal | Am. | I. Jamaica | Caribbean Sea | 17 40 N. | 76 37 W. | |
| C. Rozier | Am. | Nova Scotia | G. St. Law. | 48 55 N. | 63 36 W. | |
| I. Rugen | Eu. | Germany | Baltic Sea | 54 32 N. | 14 30 E. | |
| I. Rum Key, or Samana | Am. | Bahama | Atl. Ocean | 23 00 N. | 74 20 W. | |
| R. Rupert | Am. | New Britain | Hudſon's Bay | 51 45 N. | 78 40 W. | |
| C. Ruſeto | Africa | Barca | Medit. Sea | 32 53 N. | 20 41 E. | |
| Ruſt Iſles | Eu. | Norway | North Sea | 67 40 N. | 10 25 E. | |
| Rye | Eu. | England | Eng. Channel | 51 03 N. | 0 45 E. | 11 15 |
| S | | | | | | |
| C. Sable | Am. | Nova Scotia | Atl. Ocean | 43 24 N. | 65 35 W. | |
| I. Sable, W. end | Am. | Nova Scotia | Atl. Ocean | 44 09 N. | 60 29 W. | |
| I. Saddle back | Am. | North Main | Hudſ. Straits | 62 07 N. | 68 13 W. | 10 00 |
| Saffia | Africa | Barbary | Atl. Ocean | 32 30 N. | 8 50 W. | |
| I. Safenjal bahr. | Africa | Egypt | Red Sea | 27 05 N. | 34 40 E. | |
| B. Saldanna | Africa | Caffers | Atl. Ocean | 32 35 S. | 19 30 E. | |

I. Sal

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. Sal | Africa | C. Verd | Atl. Ocean | 16 38 N. | 22 51 W. | |
| Salerno | Eu. | Italy | Medit. Sea | 40 39 N. | 14 48 E. | |
| I. Salini, Lipari Iſl. | Eu. | Italy | Medit. Sea | 38 39 N. | 15 24 E. | |
| I. Saliſbury | Am. | N—Main | Hudſon's Bay | 63 29 N. | 76 47 W. | |
| Sallee | Africa | Barbary | Atl. Ocean | 33 58 N. | 6 20 W. | |
| Solomon Iſles | Aſia | ——— | Pacif. Ocean | 5 50 S.<br>11 15 S. | 171 05 W.<br>178 35 W. | |
| Salonechi | Eu. | Turkey | Archipelago | 40 41 N. | 23 13 E. | |
| I. Salvages | Africa | Canaries | N. Atl. Ocean | 30 00 N. | 15 49 W. | |
| I. Salvages Upper | Am. | North Main | Hudſ. Straits | 61 48 N. | 66 20 W. | 9h. 00m. |
| Lower | | | | 62 32½ N. | 70 48½ W | 11 10 |
| I. Samos | Aſia | Natolia | Archipelago | 37 46 N. | 27 13 E. | |
| C. Sambrough | Am. | Nova Scotia | Weſtern Oc. | 44 33 N. | 63 20 W. | |
| Sandwich | Eu. | England | Downs | 51 20 N. | 1 20 E. | 11 30 |
| Sandwich Iſland | Aſia | N. Hebrides | Pacif. Ocean | 17 41 S. | 168 38 E. | |
| Sandwich Harbour | Aſia | Malicola | Pacif. Ocean | 16 25 S. | 167 58 E. | |
| Sandwich's Bay | Am. | St. Georgia | Atl. Ocean | 54 42 S. | 36 4 W. | |
| I. Sanguin | Aſia | Philip. Iſles | Pacif. Ocean | 3 50 N. | 122 30 E. | |
| I. Sanien | Eu. | Norway | North Ocean | 69 30 N. | 14 30 E. | |
| Santa Cruz | Africa | Barbary | Atl. Ocean | 30 30 N. | 9 35 W. | |
| I. Sardinia N. limit | | | | 41 15 N. | 9 31 E. | |
| S. pt. C. Tavolaro | Eu. | Italy | Medit. Sea | 38 54 N. | 9 15 E. | |
| Cagliari | | | | 39 25 N. | 9 38 E. | |
| Oriſtagni | | | | 39 53 N. | 9 01 E. | |
| Sarena | Am. | Chili | Pacif. Ocean | 29 40 S. | 71 15 W. | |
| Saunders's Iſle | Am. | Sandwich L. | Atl. Ocean | 58 00 S. | 26 53 W. | |
| C. Saunders | Am. | St. Georgia | Atl. Ocean | 54 6 S. | 36 53 W. | |
| Scanderoon | Aſia | Syria | Levant | 36 35 N. | 36 25 E. | |
| Scarborough head | Eu. | England | Germ. Ocean | 54 18 N. | 00 00 | 3 45 |
| I. Scarpanto | Aſia | Natolia | Archipelago | 35 45 N. | 27 40 E. | |
| I. Scatarie, N. E. pt. | Am. | Acadia | Weſt. Ocean | 46 01 N. | 61 57 W. | |
| Scaw | Eu. | Denmark | Sound | 57 34 N. | 10 54 E. | |
| I Schelling | Eu. | D. Neth. | Germ. Ocean | 53 27 N. | 5 30 E. | |
| I. Scio C. St. Nicholas | | | | 38 38 N. | 26 12 E. | |
| Scio | Aſia | Natolia | Archipelago | 38 24 N. | 26 29 E. | |
| C. Blanco | | | | 38 08 N. | 26 20 E. | |
| Scilly Iſles | Eu. | England | St. Geo. Ch. | 50 00 N. | 6 45 W. | 3 45 |
| Scolt Head | Eu. | England | Germ. Ocean | 53 00 N. | 0 44 E. | 6 20 |
| Scots Settlement | Am. | Terra Firma | Carribbe. Sea | 8 45 N. | 76 35 W. | |
| I. Sea | Eu. | Turkey | Archipelago | 37 38 N. | 24 53 E. | |
| Seames | Eu. | France | B. Biſcay | 48 00 N. | 4 51 W. | |
| I. Sebaldes | Am. | Patagonia | S. Atl. Ocean | 50 53 S. | 59 35 W. | |
| C. Sebaſtian | Am. | California | Pacif. Ocean | 43 00 N. | 126 00 W. | |
| C. St. Sebaſtian | Africa | Madagaſcar | Indian Ocean | 12 30 S. | 46 30 E. | |
| St. Sebaſtian | Eu. | Spain | B. Biſcay | 43 16 N. | 2 05 W. | |
| Port Segura | Am. | Braſil | Atl. Ocean | 16 57 S. | 39 45 W. | |
| R. Senegal | Africa | Negroland | Atl. Ocean | 15 53 N. | 16 26 W. | 10 30 |
| I. Serranilha | Am. | Weſt Indies | Atl. Ocean | 16 20 N. | 79 40 W. | |
| I. Serigo | Eu. | Turkey | Archipelago | 36 09 N. | 23 24 E. | |
| I. Sertes | Africa | Canaries | Atl. Ocean | 32 35 N. | 16 20 W. | |
| R. Seſtos | Africa | Guinea | Atl. Ocean | 5 48 N. | 8 13 W. | |
| Seven Capes | Africa | Barbary | Medit. Sea | 37 30 N. | 6 15 W. | |
| Seven Stones, or Iſles | Eu. | England | St. Geo. Ch. | 50 10 N. | 6 40 W. | 4 30 |
| R. Severn, Ent. | Eu. | England | St. Geo. Ch. | 51 41 N. | 3 05 W. | 6 0 |
| R. Severn | Am. | New Wales | Hudſon's Bay | 56 12 N. | 88 57 W. | |
| R. Seyn, Ent. | Eu. | France | Eng. Channel | 49 36 N. | 0 30 E. | 9 00 |
| Seynhead | Eu. | France | Eng. Channel | 49 44 N. | 0 34 E. | |
| Sheerneſs | Eu. | England | R. Thames | 51 25 N. | 0 50 E. | 0 00 |
| Shepherd's Iſles | Aſia | N. Hebrides | Pacif. Ocean | 17 00 S. | 168 47 E. | |
| Siam | Aſia | India | Bay Siam | 14 18 N. | 100 55 E. | |
| R. Siam Ent. | Aſia | India | Bay Siam | 13 15 N. | 100 47 E. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ' | ° ' | |
| Siara | Am. | Braſil | Atl. Ocean | 3 18 S. | 39 50 W. | |
| Iſl. Sicily: E. end, Meſſina | Eu. | Italy | Medit. Sea | 38 10 N. | 15 58 E. | |
| Catanca | | | | 37 22 N. | 15 21 E. | |
| Syracuſe | | | | 37 04 N. | 15 31 E. | |
| S. end, C. Paſſara | | | | 36 35 N. | 15 22 E. | |
| Alicata | | | | 37 11 N | 14 07 E. | |
| W. end, C. Bocco | | | | 37 51 N. | 21 43 E. | |
| Palermo | | | | 38 10 N. | 13 43 E. | |
| Sierra Leona | Africa | Guinea | Atl. Ocean | 8 30 N. | 12 07 W. | 8h. 15m. |
| Sillabar Road | Aſia | I. Sumatra | Indian Ocean | 4 00 S. | 102 50 E. | |
| Str. Sincapore | Aſia | Malacca | Indian Ocean | 1 00 N. | 104 30 E. | |
| R. Sinda, or Indus, mouth | Aſia | India | Indian Ocean | 24 30 N.<br>25 45 N. | 63 10 E.<br>62 40 E. | |
| Po. Shabak | Africa | Abyſſinia | Red Sea | 18 58 N. | 38 24 E. | |
| Shark, or Seahorſe point | Am. | New Wales | Hudſon's Bay | 64 05 N. | 82 12 W. | |
| Shields | Eu. | England | Germ. Ocean | 55 02 N. | 1 20 W. | |
| Shelvock's Iſle | Am. | California | Pacif. Ocean | 23 15 N. | 117 35 W. | |
| Shillocks | Eu. | Ireland | Weſt. Ocean | 51 30 N. | 11 05 W. | 5 0 |
| I. Shetland, limits | Eu. | Scotland | Weſt. Ocean | 60 47 N.<br>59 54 N. | 0 10 W.<br>1 31 W. | <br>3 00 |
| Shoreham | Eu. | England | Eng. Channel | 50 55 N. | 00 17 E. | 10 30 |
| I. Sky, N. point | Eu. | Scotland | Weſt. Ocean | 57 50 N. | 6 30 W. | |
| S. point | | | | 57 15 N. | 6 16 W. | 5 30 |
| Sleepers Iſles | Am. | New Britain | Hudſon's Bay | 60 00 N.<br>58 35 N. | 81 30 W. | |
| Great Sleeper | | | | 60 10 N. | 82 00 W. | |
| The Sleepers lie in a chain from the Great Sleeper down to Lat. 58° 50' N & Long. 82° 20' W. | | | | | | |
| Sline Head | Eu. | Ireland | Weſt. Ocean | 53 20 N. | 2 15 W. | |
| R. Slude | Am. | New Britain | Hudſon's Bay | 53 24 N. | 78 50 W. | |
| Sluyce | Eu. | D. Neth. | Germ. Ocean | 51 19 N. | 3 50 E. | |
| C. Smith | Am. | Labradore | Hudſon's Bay | 60 48 N. | 80 55 W. | |
| Smyrna | Aſia | Natolia | Archipelago | 38 28 N. | 27 25 E. | |
| I. Socatora | Africa | Anian | Indian Ocean | 12 15 N. | 52 55 E. | |
| C. Solomon | Eu. | I. Candia | Medit. Sea | 34 57 N. | 27 06 E. | |
| R. Somme | Eu. | France | Eng. Channel | 50 18 N. | 1 40 E. | 11 00 |
| Sound Royal | Eu. | Iceland | North Ocean | 66 22 N. | 15 15 W. | |
| Southampton | Eu. | England | Eng. Channel | 50 55 N. | 1 00 W. | 0 00 |
| C. Southampton | Am. | New Wales | Hudſon's Bay | 61 54 N. | 86 14 W. | |
| South Cape | Aſia | Diemen's la. | Pacif. Ocean | 42 40 S. | 130 05 E. | |
| C. Spartivento | Eu. | Italy | Medit. Sea | 37 50 N. | 16 41 E. | |
| C. Spartel | Africa | Barbary | Atl. Ocean | 35 46 N. | 5 53 W. | |
| I. Spirito Sancto | Am. | Braſil | Atl. Ocean | 20 24 S. | 39 55 W. | |
| Spurn | Eu. | England | Germ. Ocean | 53 35 N. | 0 30 E. | 5 15 |
| I. Stampalia | Aſia | Natolia | Archipelago | 36 25 N. | 26 55 E. | |
| I. Stancho | Aſia | Natolia | Archipelago | 36 50 N. | 27 30 E. | |
| Start point | Eu. | England | Eng. Channel | 50 09 N. | 3 46 W. | 6 45 |
| I. States: C. St. John | Am. | Patagonia | Atl. Ocean | 54 45 S. | 60 35 W. | |
| C. St. Bartholomew | | | | 55 08 S. | 60 45 W. | |
| Stavenger | Eu. | Norway | Weſt. Ocean | 58 47 N. | 6 45 E. | |
| C. Stephens | Aſia | N. Zealand | Pacif. Ocean | 40 36 S. | 174 05 E. | |
| Stetin | Eu. | Germany | Baltic Sea | 53 36 N. | 15 25 E. | |
| C. Stillo | Eu. | Italy | Medit. Sea | 38 23 N. | 17 07 E. | |
| Port Steven | Am. | Chili | Pacif. Ocean | 46 50 S. | 82 36 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Stockholm | Eu. | Sweden | Baltic Sea | 59 22 N. | 18 12 E. | |
| Stockton | Eu. | England | Germ. Ocean | 54 33 N. | 1 15 W. | 5h. 15m. |
| Straelſund | Eu. | Germany | Baltic Sea | 54 23 N. | 14 10 E. | |
| Strangford Bay | Eu. | Ireland | Iriſh Sea | 54 23 N. | 5 40 W. | 10 30 |
| I. Stromboli | Eu. | Italy | Medit. Sea | 38 42 N. | 15 48 E. | |
| Succeſs Bay | Am. | T. del Fuego | Atl. Ocean | 54 50 S. | 65 20 W. | |
| Suez Town | Africa | Egypt | Red Sea | 29 50 N. | 33 27 E. | |
| Sukadana | Aſia | I. Borneo | Indian Ocean | 1 00 S. | 110 40 E. | |
| I. Sumatra NW. end<br>SE. end | Aſia | India | Indian Ocean | 5 15 N.<br>5 07 S. | 95 55 E.<br>106 20 E. | |
| Sunderland | Eu. | England | Germ. Ocean | 54 55 N. | 1 00 W. | 3 30 |
| Str. Sunda | Aſia | Siam | Indian Ocean | 6 10 S. | 105 35 E. | |
| Surinam | Am. | Terra Firma | Atl. Ocean | 6 30 N. | 55 30 W. | |
| Surat | Aſia | India | Indian Ocean | 21 10 N. | 72 25 E. | |
| I. Surroy | Eu. | Lapland | North Ocean | 71 00 N. | 22 00 E. | |
| Swaken | Africa | Abyſſinia | Red Sea | 19 30 N. | 37 38 E. | |
| Swally Road | Aſia | India | Arabian Sea | 21 55 N. | 72 00 E. | |
| Swanſey | Eu. | Wales | St. Geo. Cha. | 51 40 N. | 4 25 W. | |
| Sweetnoſe | Eu. | Lapland | North Ocean | 68 08 N. | 34 42 E. | |
| Swin, a ſand | Eu. | England | Ent. Thames | 51 37 N. | 1 12 E. | 12 00 |
| Syracuſe | Eu. | I. Sicily | Medit. Sea | 37 04 N. | 15 31 E. | |
| Syriam | Aſia | Pegu | B. Bengal | 16 00 N. | 96 40 E. | |
| T | | | | | | |
| Tadouſac Fort | Am. | Canada | R. St. Lawr. | 48 00 N. | 67 35 W. | |
| I. Tamarica | Am. | Braſil | Atl. Ocean | 7 56 S. | 35 05 W. | |
| Tamarin Town | Africa | I. Socatora | Indian Ocean | 12 30 N. | 53 14 E. | 9 00 |
| B. Tanaſſarin | Aſia | Malacca | B. Bengal | 12 00 N. | 98 48 E. | |
| I. Tandoxima | Aſia | Japan | Pacif. Ocean | 30 30 N. | 130 40 E. | |
| Tangier | Africa | Barbary | Atl. Ocean | 35 55 N. | 5 45 W. | |
| Tanna | Aſia | N. Hebrides | Pacif. Ocean | 19 32 S. | 169 45 E. | 3 00 |
| Taoukaa | Aſia | Society Iſles | Pacif. Ocean | 14 31 S. | 145 4 W. | |
| Tarento | Eu. | Italy | Medit. Sea | 40 43 N. | 17 31 E. | |
| C. Tat'nam | Am. | New Wales | Hudſon's Bay | 57 35 N. | 91 30 W. | |
| R. Tees, mouth | Eu. | England | Germ. Ocean | 54 36 N. | 0 52 W. | 3 00 |
| Tegoantepec | Am. | Mexico | Pacif. Ocean | 14 45 N. | 96 23 W. | |
| Tellichery | Aſia | India | Malabar Coaſt | 11 42 N. | 75 30 E. | |
| C. Telling | Eu. | Ireland | Weſt. Ocean | 54 40 N. | 10 07 W. | |
| I. Tenedos | Aſia | Natolia | Archipelago | 39 57 N. | 26 14 E. | |
| I. Teneriff (Peak) | Africa | Canaries | Atl. Ocean | 28 13 N. | 16 24 W. | 3 00 |
| C. Tenes | Africa | Barbary | Medit. Sea | 36 26 N. | 1 53 E. | |
| I. Tercera | Eu. | Azores | Atl. Ocean | 38 45 N. | 27 01 W. | |
| Terra Nieva | Am. | N— Main | Hudſ. Straits | 62 4 N. | 67 2 W. | 9 50 |
| Tervere | Eu. | D. Neth. | Germ Ocean | 51 38 N. | 3 35 E. | 0 45 |
| Tetuan | Africa | Barbary | Medit. Sea | 35 27 N. | 4 50 W. | |
| I. Texel | Eu. | D. Neth. | Germ. Ocean | 53 10 N. | 4 59 E. | 7 30 |
| C. St. Thadæus | Aſia | Siberia | North Ocean | 62 10 N. | 175 05 E. | |
| R. Thames, mouth | Eu. | England | Germ. Ocean | 51 28 N. | 1 10 E. | 1 30 |
| C. St. Thomas | Africa | Caffers | Atl. Ocean | 24 54 S. | 15 25 E. | |
| I. St. Thomas | Africa | Guinea | Atl. Ocean | 00 00 | 1 00 E. | |
| St. Thomas | Aſia | India | B. Bengal | 13 00 N. | 80 00 E. | |
| C. Three Points | Am. | Terra Firma | Atl. Ocean | 10 51 N. | 62 41 W. | |
| C. Three Points | Africa | Guinea | Atl. Ocean | 4 48 N. | 1 21 W. | |
| South Thule | Am. | Sandwich Ia. | Atl. Ocean | 59 34 S. | 27 45 W. | |
| I. Tidore | Aſia | Molucca Is. | Indian Ocean | 0 35 N. | 126 40 E. | |
| I. Timor NE. pt.<br>SW. pt. | Aſia | Molucca Is. | Indian Ocean | 8 20 S.<br>10 23 S. | 127 40 E.<br>123 55 E. | |
| Tinmouth | Eu. | England | Germ. Ocean | 55 03 N. | 1 17 W. | 3 00 |
| I. Tino | Eu. | Turkey | Archipelago | 37 33 N. | 25 43 E. | |
| I. Tobago | Am. | Carribbee | Atl. Ocean | 11 15 N. | 60 27 W. | |
| Tobolſki | Aſia | Siberia | Inland | 58 12 N. | 68 20 E. | |
| B. Todos Sanctos | Am. | Braſil | Atl. Ocean | 13 05 S. | 38 45 W. | |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| Tonquin | Aſia | India | Pacif. Ocean | 20 50 N. | 105 55 E. | |
| Tonſberg | Eu. | Norway | Sound | 58 50 N. | 10 05 E. | |
| Topſham | Eu. | England | Eng. Channel | 50 37 N. | 3 27 W. | 6h.00m. |
| Torbay | Eu. | England | Eng. Channel | 50 34 N. | 3 36 W. | 5 15 |
| Tornea | Eu. | Sweden | G. Bothnia | 65 51 N. | 24 16 E. | |
| R. Tortoſa | Eu. | Spain | Medit. Sea | 40 47 N. | 1 03 E. | |
| I. Tortola | Am. | Antl. Iſle | Atl. Ocean | 18 24 N. | 65 00 W. | |
| I. Tory | Eu. | Ireland | Weſt. Ocean | 55 09 N. | 8 30 W. | 5 30 |
| Toulon | Eu. | France | Medit. Sea | 43 07 N. | 6 02 E. | |
| C. Trefalga | Eu. | Spain | Atl. Ocean | 36 08 N. | 5 58 W. | |
| I. Tremiti | Eu. | Italy | Medit. Sea | 42 09 N. | 15 40 E. | |
| C. de Tres forcas | Africa | Barbary | Medit. Sea | 35 30 N. | 2 11 W. | |
| I. Trailles | Aſia | India | Indian Ocean | 19 30 S. | 101 25 E. | |
| I. Trinity | Am. | Braſil | Atl. Ocean | 20 25 S. | 23 35 W. | |
| I. Trinidada, E. pt. | Am. | Terra Firma | Atl. Ocean | 10 38 N. | 60 27 W. | |
| Trinity Bay, Ent. | Am. | Newfoundl. | Atl. Ocean | 48 30 N. | 52 35 W. | |
| Trieſt | Eu. | Carniola | Adriat. Sea | 45 51 N. | 14 03 E. | |
| Trinquemali | Aſia | I. Ceylon | Indian Ocean | 8 50 N. | 83 24 E. | |
| Tripoli | Aſia | Syria | Levant | 34 53 N. | 36 07 E. | |
| Tripoly | Africa | Barbary | Medit. Sea | 32 54 N. | 13 10 E. | |
| G. Triſte | Am. | Terra Firma | Atl. Ocean | 10 19 N. | 67 41 W. | |
| I. Triſtian d'Acunha | Africa | Caffers | S. Atl. Ocean | 37 12 S. | 13 23 W. | |
| Iſ. Tromſound | Eu. | Lapland | North Ocean | 70 20 N. | 19 00 E. | |
| Truxilla | Am. | Peru | Pacif. Ocean | 8 00 S. | 78 35 W. | |
| Tunder | Eu. | Denmark | Weſt. Ocean | 55 00 N. | 9 35 E. | |
| Tunis | Africa | Barbary | Medit. Sea | 36 47 N. | 10 16 E. | |
| Turin | Eu. | Italy | R. Po | 45 05 N. | 7 45 E. | |
| I. Turks | Am. | Bahama | Atl. Ocean | 21 18 N. | 71 05 W. | |
| Turtle Iſland | Aſia | ——— | Pacif. Ocean | 19 49 S. | 177 52 W. | |
| V | | | | | | |
| Valencia | Eu. | Spain | Medit. Sea | 39 30 N. | 0 40 W. | |
| St. Valery | Eu. | France | Eng. Channel | 50 11 N. | 1 42 E. | 10 30 |
| Valona | Eu. | Turkey | Medit. Sea | 40 55 N. | 21 15 E. | |
| Valpariſo | Am. | China | Pacif. Ocean | 33 03 S. | 72 14 W. | |
| Van Diemen's land | Aſia | N. Holland | Indian Ocean | 43 38 S. | 146 27 E. | |
| Vannes | Eu. | France | B. Biſcay | 47 39 N. | 2 41 W. | 3 45 |
| C. Vela | Am. | Terra Firma | Atl. Ocean | 12 15 N. | 71 20 W. | |
| P. Venus | Aſia | Otaheite | Pacif. Ocean | 17 29 S. | 149 31 W. | 10 38 |
| Venice | Eu. | Italy | Medit. Sea | 45 27 N. | 12 9 E. | |
| Vera Cruz | Am. | New Spain | G. Mexico | 19 12 N. | 97 25 W. | |
| C. Verd | Africa | Negroland | Atl. Ocean | 14 45 N. | 17 28 W. | |
| Uhma | Eu. | Sweden | G. Bothnia | 63 45 N. | 21 10 E. | |
| Vicegapatam | Aſia | India | B. Bengal | 17 30 N. | 84 02 E. | |
| C. Victory | Am. | Patagonia | Pacif. Ocean | 52 15 S. | 74 28 W. | |
| Vienna | Eu. | Germany | R. Danube | 48 11 N. | 16 28 E. | |
| Vigo | Eu. | Spain | Atl. Ocean | 42 14 N. | 8 23 W. | |
| B. St. Vincent | Am. | Paraguay | Atl. Ocean | 23 55 S. | 45 11 W. | |
| C. St. Vincent | Eu. | Portugal | Atl. Ocean | 37 01 N. | 8 58 W. | |
| I. St. Vincent | Africa | C. Verd | Atl. Ocean | 17 47 N. | 24 44 W. | |
| I. St. Vincent | Am. | Carribbee | Atl. Ocean | 13 05 N. | 61 05 W. | |
| R. St. Vincent | Africa | Guinea | Eth. Ocean | 4 50 N. | 7 41 W. | |
| C. Virgins | Am. | Patagonia | Atl. Ocean | 52 23 S. | 67 50 W. | |
| I. Virgins | Am. | Antil. Iſle | Atl. Ocean | 18 18 N. | 64 14 W. | |
| Virgin Rocks | Am. | Newfoundl. | Atl. Ocean | 46 30 N. | 51 30 W. | |
| Umba | Eu. | Ruſſia | Inland | 66 40 N. | 34 15 E. | |
| C. Volo | Eu. | Turkey | Archipelago | 39 07 N. | 23 23 E. | |
| R. Voltas | Africa | Guinea | Atl. Ocean | 5 52 N. | 1 10 E. | |
| C. Voltas | Africa | Caffers | Atl. Ocean | 28 04 S. | 16 18 E. | |
| Upſal | Eu. | Sweden | R. Sala | 59 52 N. | 17 47 E. | |
| Uraniberg | Eu. | Denmark | Baltic Sea | 55 54 N. | 12 57 E. | |
| I. Uſhant | Eu. | France | Eng. Channel | 48 30 N. | 5 00 W. | 4 30 |

| Names of Places. | Cont. | Countries. | Coaſt. | Latitude. | Longitude. | H. Water. |
|---|---|---|---|---|---|---|
| | | | | ° ′ | ° ′ | |
| I. Uſtica | Eu. | Italy | Medit. Sea | 38 43 N. | 13 33 E. | |
| I. Vulcano | Eu. | Italy | Medit. Sea | 38 29 N. | 15 33 E. | |
| W | | | | | | |
| Prin. Wales's Iſles | Aſia | New Guinea | Endeavour St. | 10 26 S. | 141 00 E. | |
| R. Wager | Am. | New Wales | Hudſon's Bay | 65 28 N. | 87 25 W. | 6h. 00m. |
| Wallis's Iſle | Aſia | ——— | Pacif. Ocean | 13 18 S. | 176 20 W. | |
| C. Walſingham | Am. | New Britain | Hudſon's Str. | 62 39 N. | 77 48 W. | 12 00 |
| Wardhus | Eu. | Lapland | North Ocean | 70 23 N. | 51 12 E. | |
| Warſaw | Eu. | Poland | R. Viſtula | 52 14 N. | 21 5 E. | |
| Waterford | Eu. | Ireland | St. Geo. Ch. | 52 07 N. | 7 42 W. | 6 30 |
| Watling Iſle | Am. | Bahama | Atl. Ocean | 23 42 N. | 74 22 W. | |
| Wells | Eu. | England | Germ. Ocean | 53 07 N. | 1 00 E. | 6 00 |
| Weſtern Iſles | Aſia | Diemen's la. | Pacif. Ocean | 43 36 S. | 147 00 E. | |
| Weſtern Iſles S. point | Eu. | Scotland | Weſt. Ocean | 56 46 N. | 7 40 W. | |
| Weſtern Iſles N. point | | | | 58 35 N. | 6 37 W. | |
| Iſ. Weſtmania | Eu. | Iceland | Weſt. Ocean | 63 55 N. | 17 30 W. | |
| Iſ. Weſtrol | Eu. | Lapland | North Ocean | 69 15 N. | 24 00 E. | |
| Wexford | Eu. | Ireland | St. Geo. Ch. | 52 13 N. | 6 56 W. | |
| Weymouth | Eu. | England | Eng. Channel | 52 40 N. | 2 34 W. | 7 20 |
| Whale's Back | Eu. | Iceland | Weſt. Ocean | 63 44 N | 17 05 W. | |
| Whale's Head | Eu. | Greenland | North Ocean | 77 18 N. | 21 30 E. | |
| Whale Rock | Eu. | Azores | Atl. Ocean | 38 50 N. | 24 41 W. | |
| Whitby | Eu. | England | Germ. Ocean | 54 30 N. | 0 50 W. | 3 00 |
| Whitehaven | Eu. | England | Iriſh Sea | 54 25 N. | 3 15 W. | |
| Whitſuntide I. | Aſia | N. Hebrides | Pacif. Ocean | 16 44 S. | 168 25 E. | |
| Wicklow | Eu. | Ireland | St. Geo. Cha. | 52 50 N. | 6 30 W. | |
| Willis's Iſles | Am. | S. Georgia | Atl. Ocean | 54 00 S. | 38 25 W. | |
| Windaw | Eu. | Courland | Baltic Sea | 57 08 N. | 22 20 E. | |
| I. Wight N. end | Eu. | England | Eng. Channel | 50 47 N. | 1 11 W. | 0 00 |
| I. Wight S. end | | | | 50 34 N. | 1 10 W. | |
| I. Wight E. end | | | | 50 41 N. | 1 00 W. | |
| I. Wight W. end | | | | 50 41 N. | 1 23 W. | |
| Iſ. Prince William | Aſia | ——— | Pacif. Ocean | 16 45 S. | 177 55 E. | |
| Wiliam Henry I. | Aſia | Society Iſles | Pacif. Ocean | 19 00 S. | 141 6 E. | |
| Winchelſea | Eu. | England | Eng. Channel | 50 58 N. | 0 50 E. | 0 45 |
| Wintertoneſs | Eu. | England | Germ. Ocean | 53 02 N. | 1 22 E. | 9 00 |
| Wiſbuy in I. Gotland | Eu. | Sweden | Baltic Sea | 57 40 N. | 19 50 E. | |
| C. Wrath | Eu. | Scotland | Weſt. Ocean | 58 40 N. | 4 50 E. | |
| Wybourg | Eu. | Finland | G. Finland | 60 55 N. | 30 20 E. | |
| Y | | | | | | |
| Yamboa | Aſia | Arabia | Red Sea | 24 25 N. | 38 54 E. | |
| Yarmouth | Eu. | England | Germ. Ocean | 52 55 N. | 1 40 E. | 9 45 |
| Yas de Amber | Africa | Zanguebar | Indian Ocean | 0 00 | 47 15 E. | |
| Yellow River | Aſia | China | Pacif. Ocean | 34 06 N. | 120 10 E. | |
| Ylo | Am. | Peru | Pacif. Ocean | 17 36 S. | 71 08 W. | |
| Cape York | Aſia | N. Holland | Endeavour St. | 10 41 S. | 141 39 E. | |
| York Fort | Am. | New Wales | Hudſon's Bay | 57 02 N. | 92 47 W. | 9 10 |
| York, New | Am. | N. England | Atl. Ocean | 40 43 N. | 74 04 W. | 3 0 |
| Youghall | Eu. | Ireland | St. Geo. Ch. | 51 46 N. | 8 06 W. | 4 30 |
| Z | | | | | | |
| Zacatula | Am. | Mexico | Pacif. Ocean | 17 10 N. | 105 00 W. | |
| Zachee | Am. | Antilles | Atl. Ocean | 18 24 N. | 67 52 W. | |
| I. Zant | Eu. | Italy | Adriatic Sea | 37 50 N. | 21 30 E. | |
| I. Zanzebar | Africa | Zanguebar | Indian Ocean | 6 55 S. | 40 10 E. | |
| Zara | Eu. | Dalmatia | Medit. Sea | 44 15 N. | 16 65 E. | |
| N. Zealand N.W. point | Aſia. | ——— | Pacif. Ocean | 34 28 S. | 172 44 E. | |
| N. Zealand S. point | | | | 47 20 S. | 167 50 E. | |
| Zenan city | Aſia | Arabia | Inland | 16 20 N. | 47 44 E. | |
| Zuric Sea | Eu. | D. Neth. | Germ. Ocean | | | 3 00 |

Beſides the times of high-water in the preceding table, the following times ſerve for coaſts of conſiderable extent, and will ſerve nearly for the places on thoſe coaſts.

Finmark, or NNW. coaſt of Lapland, 1 h. 30 m. Jutland Iſles 0 h. 0 m.
Frieſland coaſt 7 h. 30 m. Zealand coaſt 1 h. 30 m.
Flanders coaſt 0 h. 0 m. Picardy and Normandy coaſts 10 h. 30 m.
Biſcay, Gallician, and Portugal coaſts 3 h. 00 m.
Iriſh W. coaſt 3 h. 00 m. Iriſh S. coaſt 5 h. 15 m.
Africa W. coaſt 3 h. 0 m. America W. coaſt 3 h. 0 m.
America E. coaſt 4 h. 30 m.

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